Fast Resource Allocation for Network-Coded Traffic
A Coded-Feedback Approach

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Content

- Existing results on resource allocation for wireline networks.
- The throughput and delay benefit of network-coded multicast traffic.
- A new primal approach that takes full advantage of network coded traffic.
- ♡ A fast min-cut algorithm based on coded feedback.
- Simulation & comparison to existing works.
  - Comparable performance to the back-pressure algorithms.
  - Many desirable features of a primal approach and for network-coded traffic.
- Conclusion.

The Setting

- Wireline networks:
  $G = (V, E)$, w. edge capacity c_e (packets/sec).
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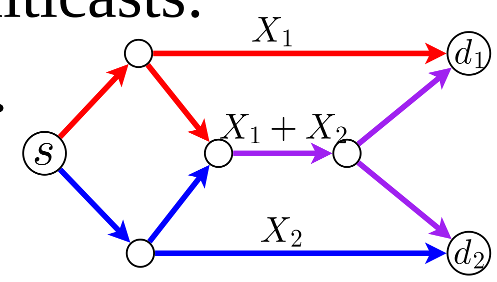
- $R(\mathbf{r}^{(i)})$: The $(s_i, \{d_{i,j}\})$ multicast rate supported by the rate allocation $\mathbf{r}^{(i)}$.
- Utility maximization by convex optimization:

$$\max_{r_e^{(i)} \geq 0} \quad \sum_i U_i(R(\mathbf{r}^{(i)}))$$

$$\text{subject to} \quad \sum_i r_e^{(i)} \leq c_e, \ \forall e \in E$$

Benefits of (Intrasession) Network Coding

- Strict throughput improvement for multiple multicasts: For the same $\mathbf{r}^{(i)}$, $R_{\text{coding}}(\mathbf{r}^{(i)}) > R_{\text{routing}}(\mathbf{r}^{(i)})$.



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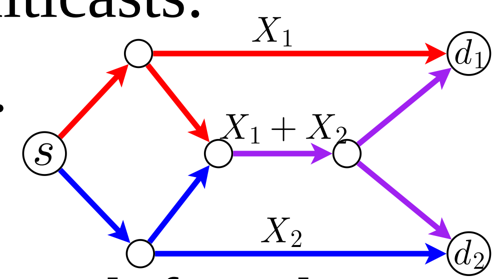
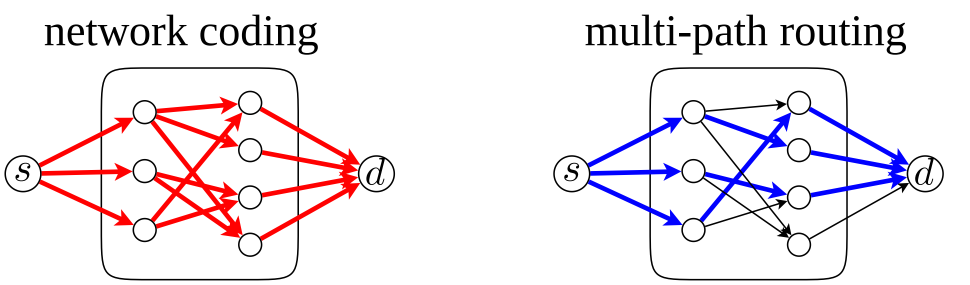


- Even for unicasts:
  Autonomous random mixing $\Longrightarrow$ No need to search for "the edge-disjoint paths" (the max flow)
  - Low-complexity,
  - Instant ON.



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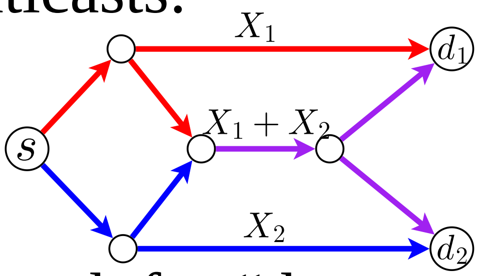

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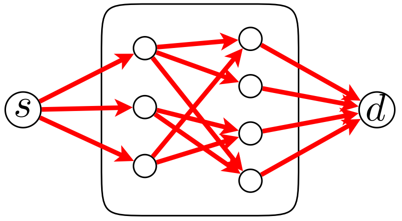
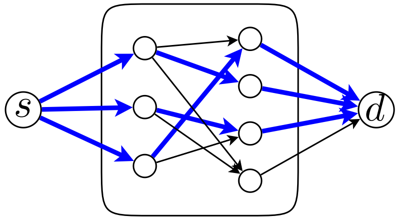


network coding



multi-path routing

- Autonomous random mixing may easily waste bandwidth.

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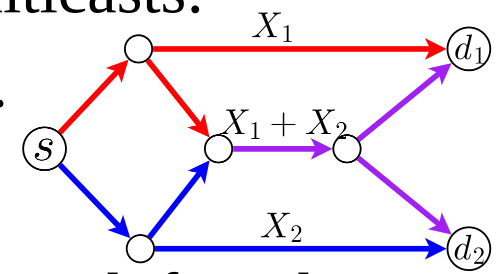

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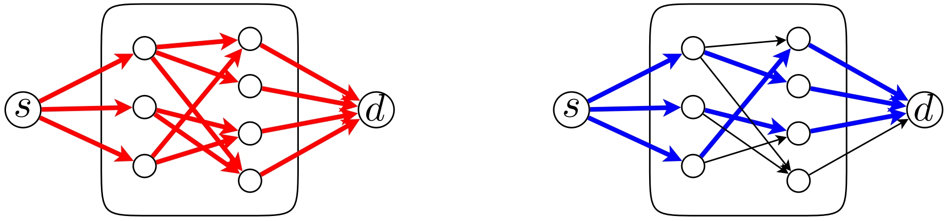

- Autonomous random mixing may easily waste bandwidth.
- Rate allocation is critical.

Existing Rate Control Results

- The dual approaches (solving the dual problem): [Lun *et al.* 06], [Wu *et al.* 06], [Chen *et al.* 07], [Khreishah *et al.* 08], and many more.
  - Disconnection between the dual (price) and the primal (rate) variables.
  - Convergence of the dual $\not\Rightarrow$ convergence of the primal
  - The utility may not be monotonically increasing.
  - Rate assignment > capacity. Queue build-up. Delay?!
- The primal approaches (directly solving the $\mathbf{r}^{(i)}$): [Xi *et al.* 05], [Wu *et al.* 06].
  - The utility is monotonically increasing.
  - Rate assignment $\leq$ capacity. No queue build-up.

A Subgradient Approach

- The min-cut/max-flow theorem [Ahlswede *et al.* 00]: $R(\mathbf{r}^{(i)})$ is the min-cut/max-flow value $\mathrm{mcMF}(\mathbf{r}^{(i)})$.

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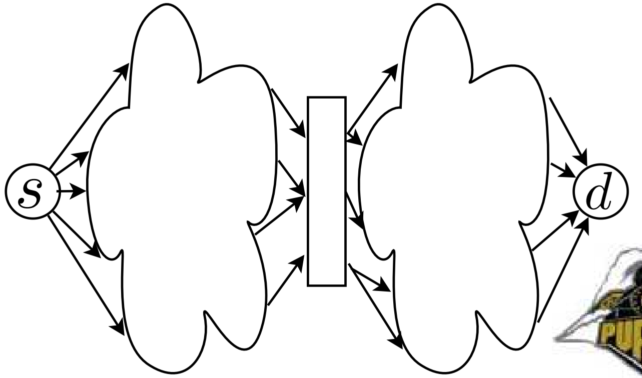
  $$\text{subject to} \quad \sum_i r_e^{(i)} \leq c_e, \ \forall e \in E$$
- A subgradient approach [Wu *et al.* 06]:
  - Denote the session min-cut by $C_{\min}^{(i)}$ (based on $\mathbf{r}^{(i)}$).
  - The subgradient of the $F(\mathbf{r})$ is

$$\frac{\partial F(\mathbf{r})}{\partial r_e^{(i)}} = U_i'(\mathrm{mcMF}(\mathbf{r}^{(i)}))1_{\{e \in C_{\min}^{(i)}\}}$$



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  -
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- Step 3: Subgradient update for the primal variables.

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- Use coded feedback. Two-way messages: source $\leftrightarrow$ dest.
  - The information is interpreted at the intermediate nodes.
  - The min-cut can be obtained very efficiently with low cost.

A Coding-Based Perspective

- $G' = (V', E')$ [Chou *et al.* 03] [Li *et al.* 03].
- Unit edge-capacity (1 packet per use). Allow parallel edges.
- Each session has a *generation* size $n = 32$–100.
- Minimize the generation flushing time D to maximize transmission rate.
- A link e of rate $r_e^{(i)} \Rightarrow \lfloor r_e^{(i)} D \rfloor$ parallel edges.
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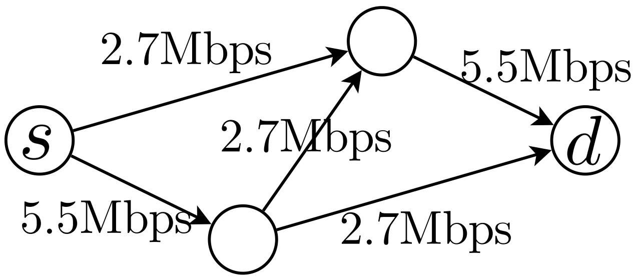

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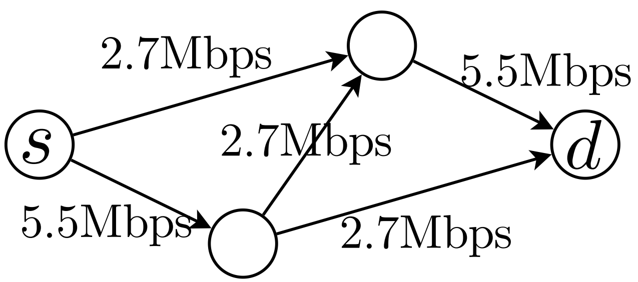
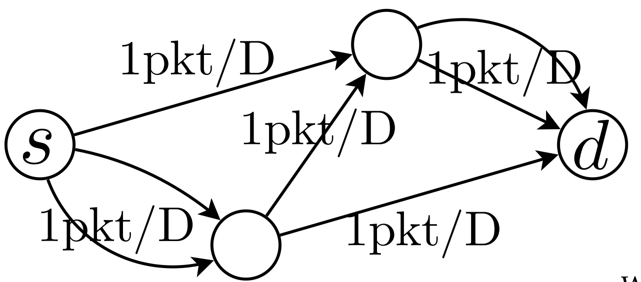
1 packet = 1kB, D ≈2.97ms





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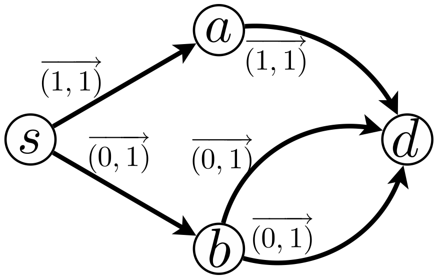

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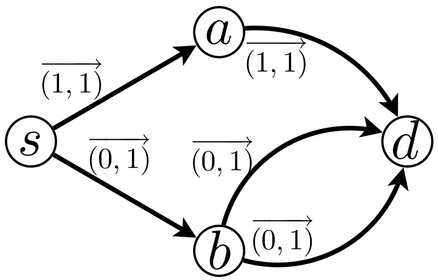

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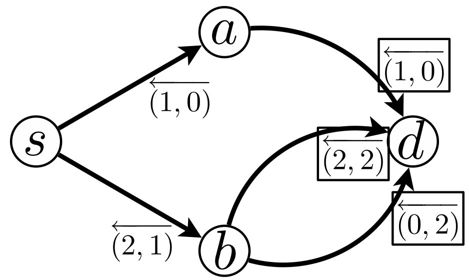

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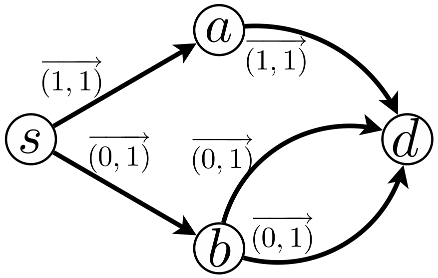

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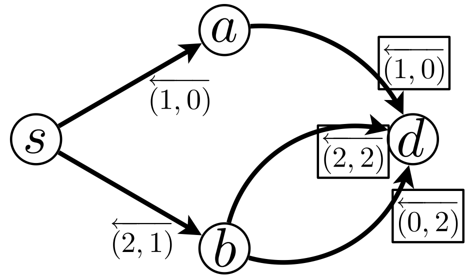

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$$\mathbf{m}_{\text{downstream}} = \Gamma_v \mathbf{m}_{\text{upstream}}, \quad \mathbf{q}_{\text{upstream}} = \Gamma_v^{\mathrm{T}} \mathbf{q}_{\text{downstream}}.$$

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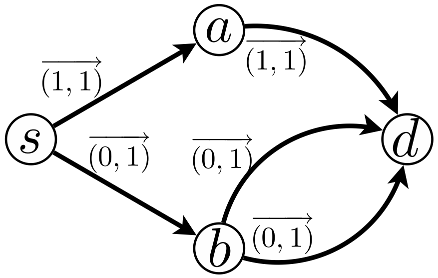

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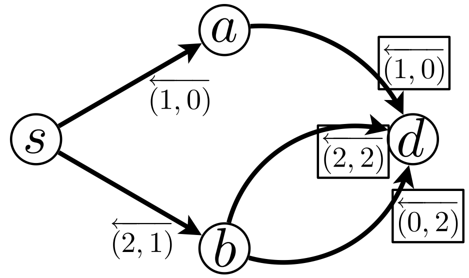

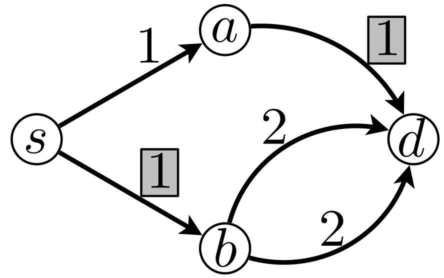

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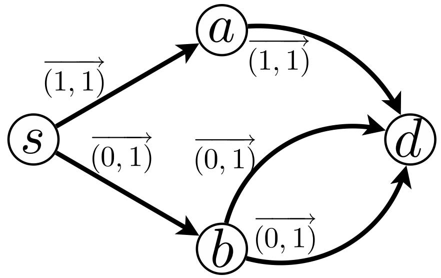

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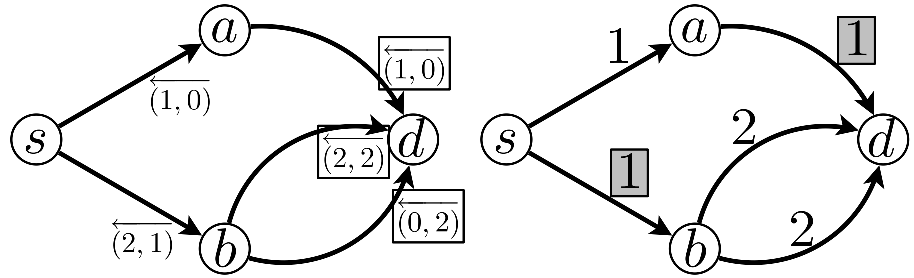

- Step 0: Run (random) network coding.
- Step 1: Send **generalized coded ACK q** that acknowledge the linear independence of the incoming packets $\mathbf{m}$ and satisfy.

$$\left(\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}2\\2\end{bmatrix}\begin{bmatrix}0\\2\end{bmatrix}\right)\begin{pmatrix}[1\quad 1]\\ [0\quad 1]\end{pmatrix} \quad \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Fully distributed, each edge makes it own decision.

- Step 2: Propagate $\mathbf{q}$ back to s with the same local kernel Γ_v.

$$\mathbf{m}_{\text{downstream}} = \Gamma_v \mathbf{m}_{\text{upstream}}, \qquad \mathbf{q}_{\text{upstream}} = \Gamma_v^T \mathbf{q}_{\text{downstream}}.$$

- Step 3: Check whether $\mathbf{m}\cdot\mathbf{q} = 1$ and return the first such cut.

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  - The proposed scheme: each control message is $\mathcal{O}(n^2)$ bytes. The overall time: $\mathcal{O}\left((n^2 t_{\mathrm{tran}} + t_{\mathrm{prop}})|V|\right)$.
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  - 10Mbps, $n = 100$, 1000km. $n^2 t_{\text{tran}} = 8\text{ms}$. $t_{\text{prop}} \approx 3.3\text{ms}$. (Ignore the queuing and processing delays)

The Randomized Subgradient Method

- The Prob(a min-cut is found) is large, but not one.
- **Proposition 3** *For any* $\epsilon > 0$, *there exist* $2^b, D > 0$ *such that the combination of the subgradient method and the randomized min-cut algorithm will satisfy:*

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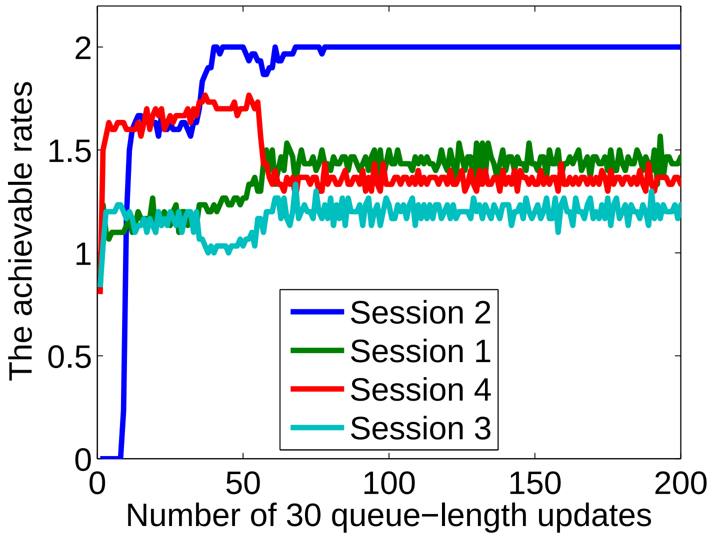
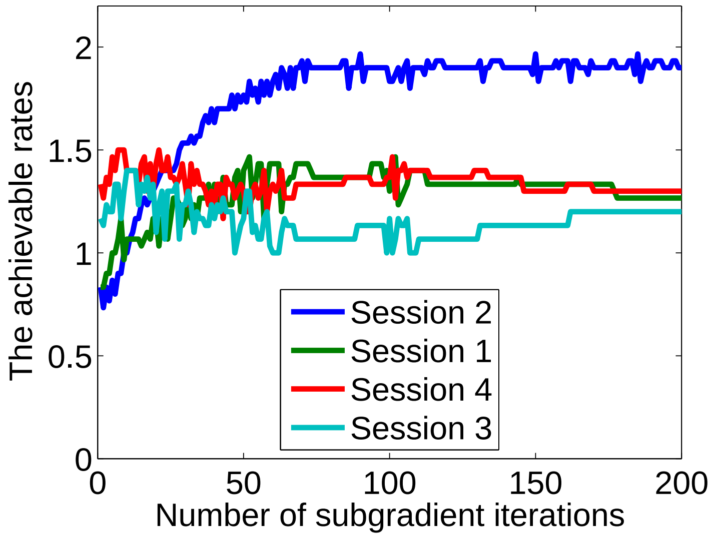
- The control messages are part of the traffic: Low overhead and fast reaction to network changes.

Numerical Experiments

- A 4×4 Network. Back-pressure versus Coding solutions

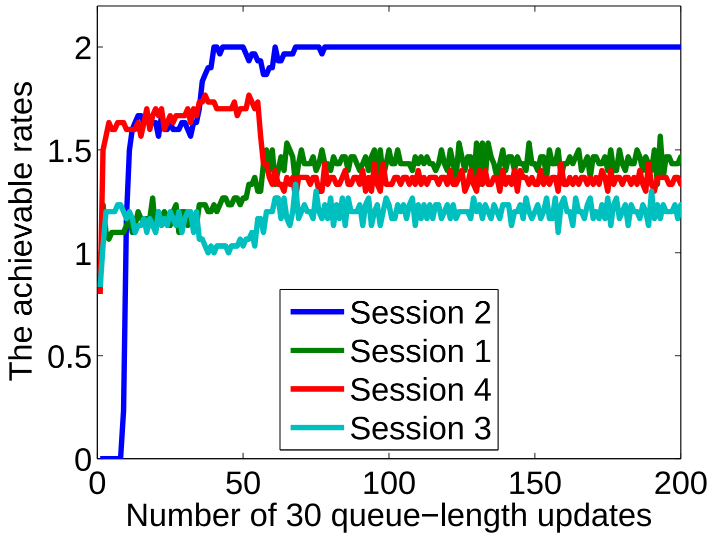


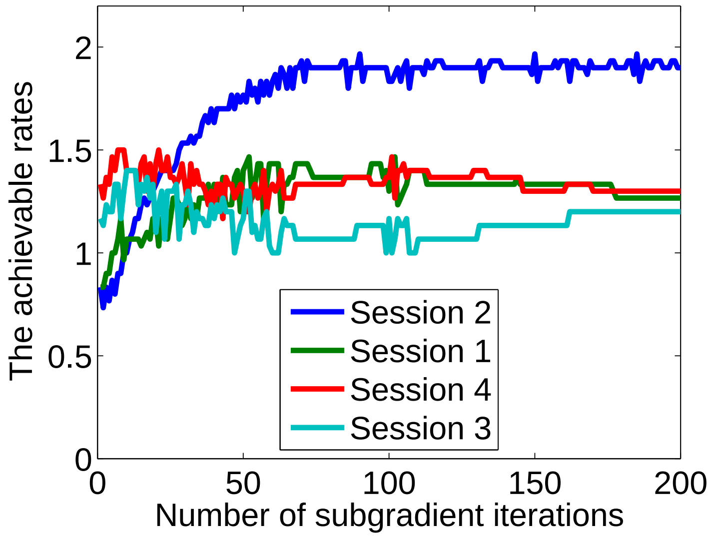


Back Pressure	Coding
Instant feedback after every packet	Batch feedback after every generation

Numerical Experiments

- A 4 $\times$ 4 Network. Back-pressure versus Coding solutions





Back Pressure	Coding
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Queue/delay dynamics	No queues. Buffers only packets of the same generation. Fixed delay.

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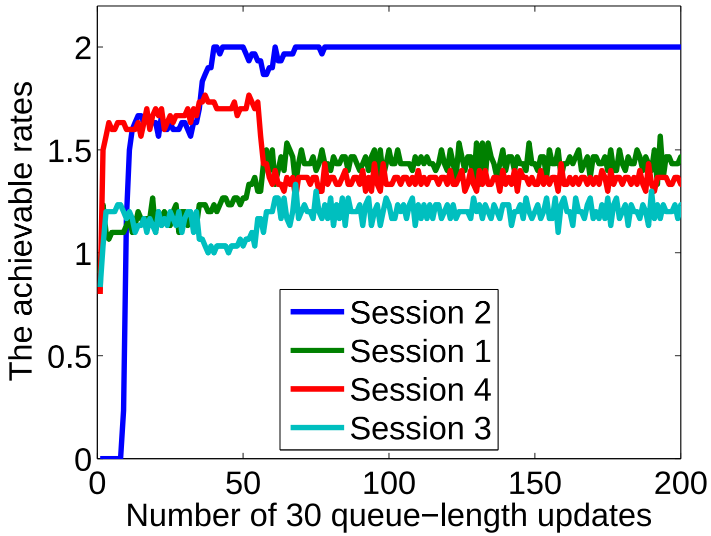

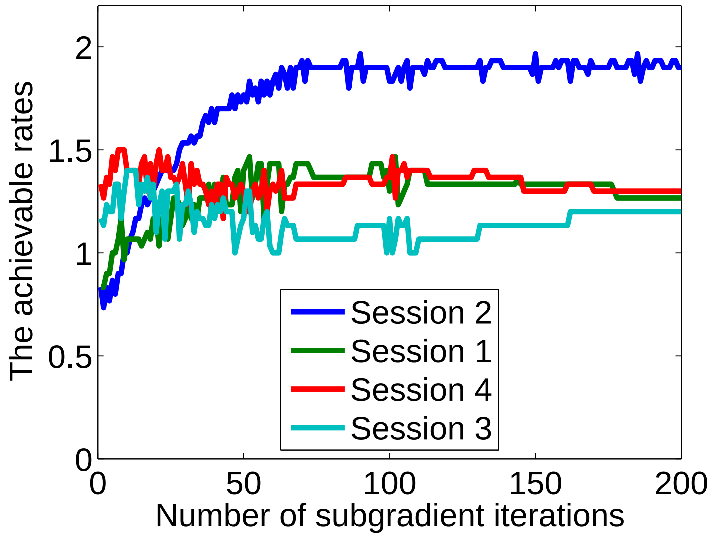

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Queue build-up	Quick Start

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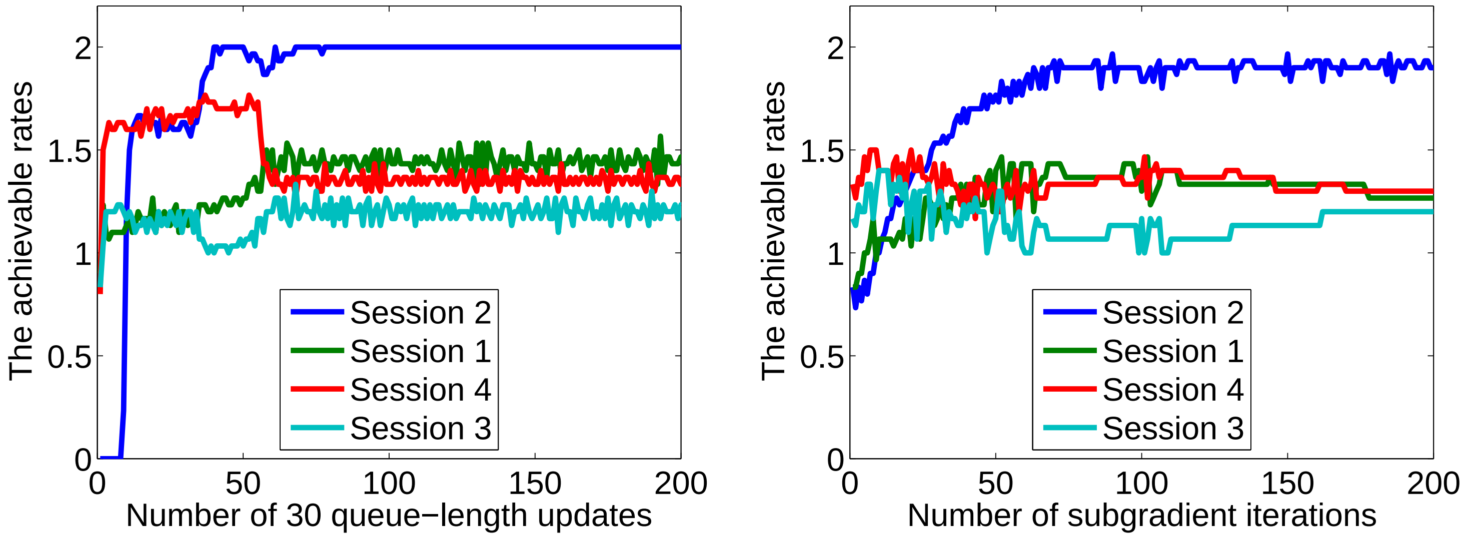


Back Pressure (dual)	Coding (primal)
Instant feedback after every packet	Batch feedback after every generation
Queue/delay dynamics	No queues. Buffers only packets of the same generation. Fixed delay.
Queue build-up	Quick Start

Quick Start of Network Coding

- The network coding primal solution.
  - New sessions instantly grab the fair share the edge capacity.

$$\forall e, \quad r_e^{(\text{new})} \leftarrow \frac{1}{\text{no. sessions that use } e} c_e$$

$$r_e^{(\text{existing})} \leftarrow (1 - \frac{1}{\text{no. sessions that use } e}) r_e^{(\text{existing})}$$

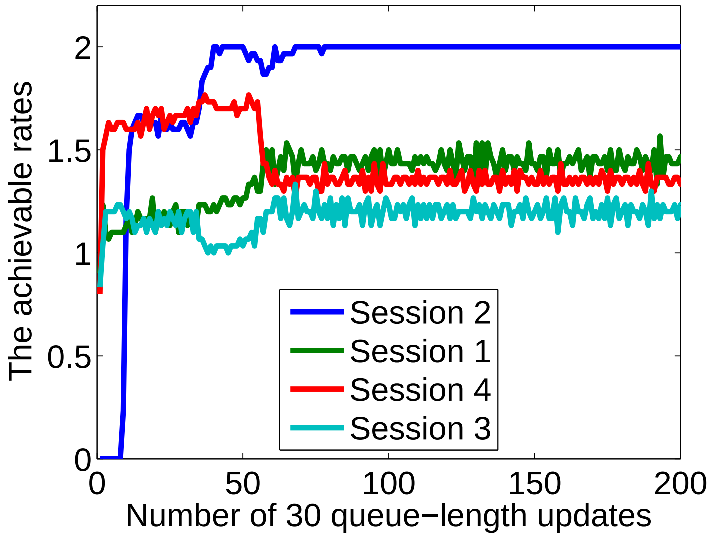
  - There is no need to wait for queue build-up.
  - The rates are optimized thereafter from the new starting point.

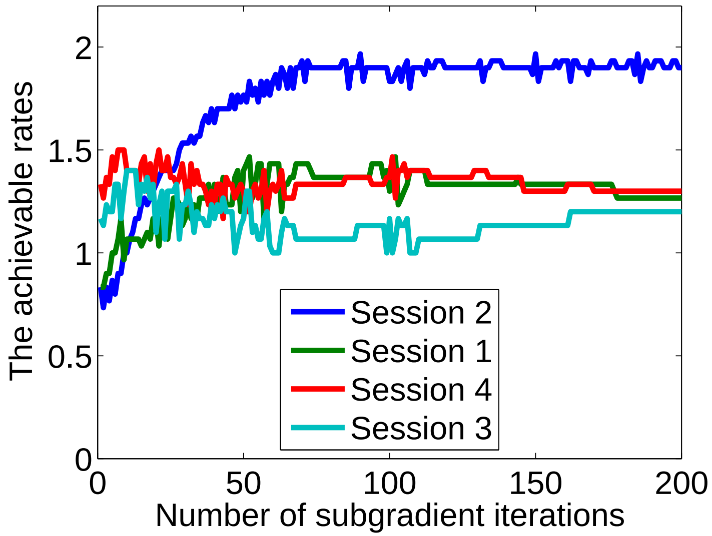
Conclusion

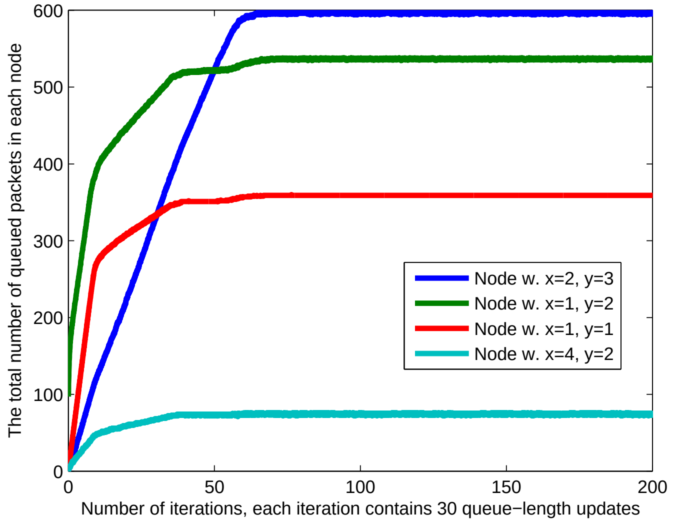
- We present a new primal approach that fully takes advantage of the network coded traffic.
  - The benefits of primal approaches. Not relying on queue lengths.
  - Low computation complexity, low overhead.
- The convergence time outperforms the existing results for many practical scenarios. $\mathcal{O}\left((n^2 t_{\text{tran}} + t_{\text{prop}})|V|\right)$ vs. $\mathcal{O}\left((t_{\text{tran}} + t_{\text{prop}})|V|^2\right)$
- Network coding enables the quick start feature with fairness.
- Future direction: Applications to wireless networks, for which (coding-based) batch feedback is more favorable than (back-pressure) packet-by-packet feedback.

Delay Comparison

- A 4×4 Network. Back-pressure versus Coding solutions







A fixed generation size $n = 60$. Each nodes only needs to buffer at most two consecutive generations. The delay is fixed to D, the time between flushing generations.