On the Capacity of Wireless 1-Hop Intersession Network Coding — A Broadcast Packet Erasure Channel Approach

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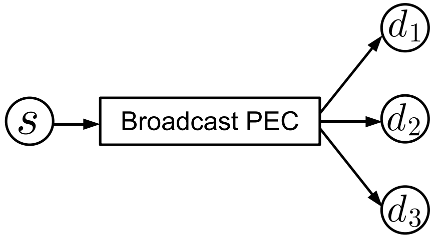
Presented in ISIT, 06/17/2010

Sponsored by NSF CCF-0845968 and CNS-0905331.

Two Ingredients

- Packet Erasure Channels:



  - Input: $X \in \mathsf{GF}(2^b)$ for large b.
  - A packet X either arrives perfectly (with the help of CRC), or is considered as erasure and discarded. (No hybrid ARQ).

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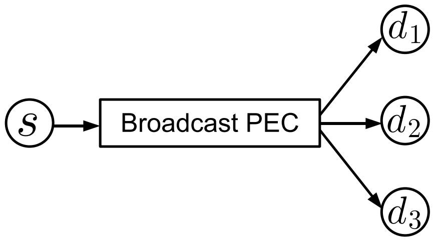
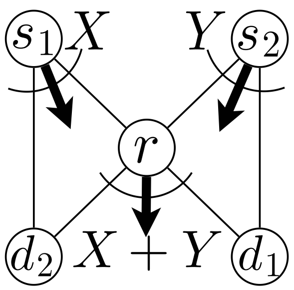


- The COPE protocol — 2-hop relay networks [Katti *et al.* 06]

4 transmissions w/o coding vs. 3 transmissions w. coding
  - r sends $[X+Y]$; d_1 decodes X by subtraction.
  - Empirically, 40–200% throughput improvement.



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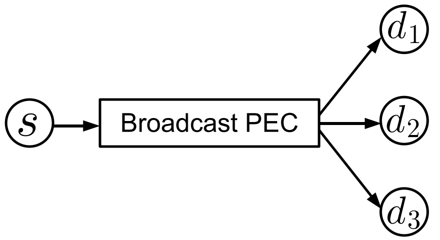

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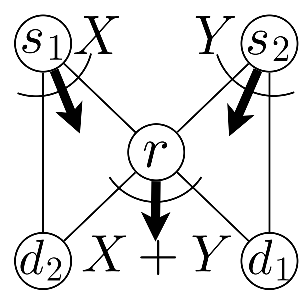

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  - Empirically, 40–200% throughput improvement.

- Our goal: Finding the Shannon capacity of COPE-like protocols — Non-trivial for random broadcast PECs and $M > 2$ sessions.

Our Setting

- Memoryless broadcast PECs: Ex: A 1-to-2 PEC is governed by the success probabilities $p_{s\to 12}, p_{s\to 12^c}, p_{s\to 1^c2}, p_{s\to 1^c2^c}$.

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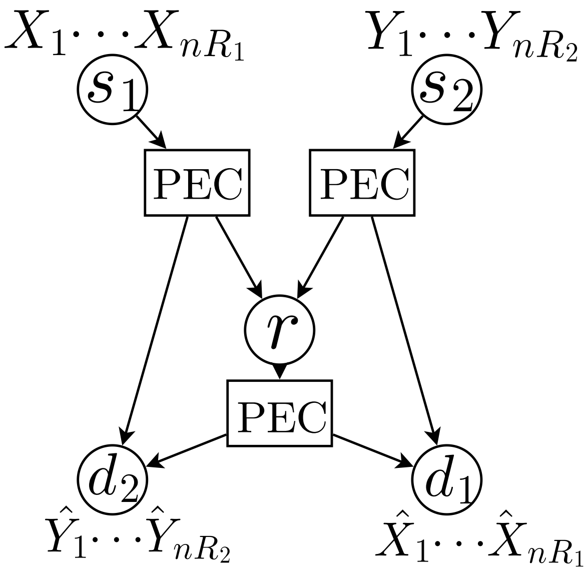

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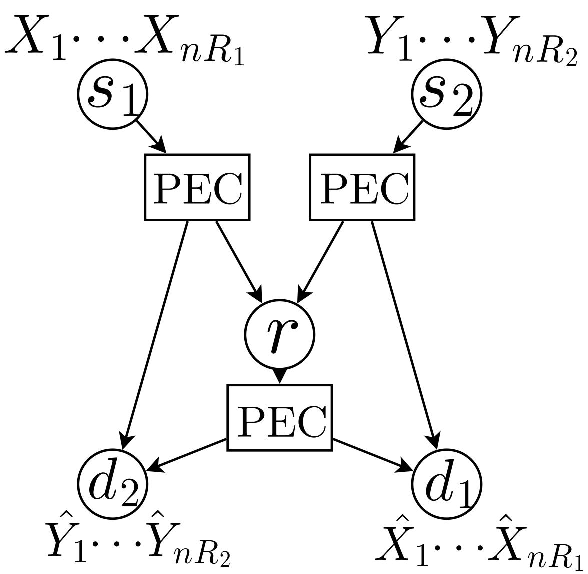

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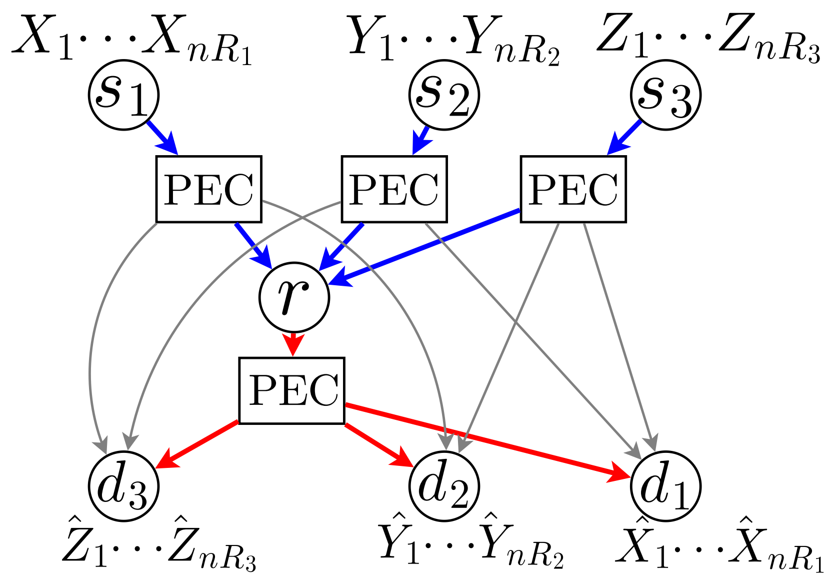

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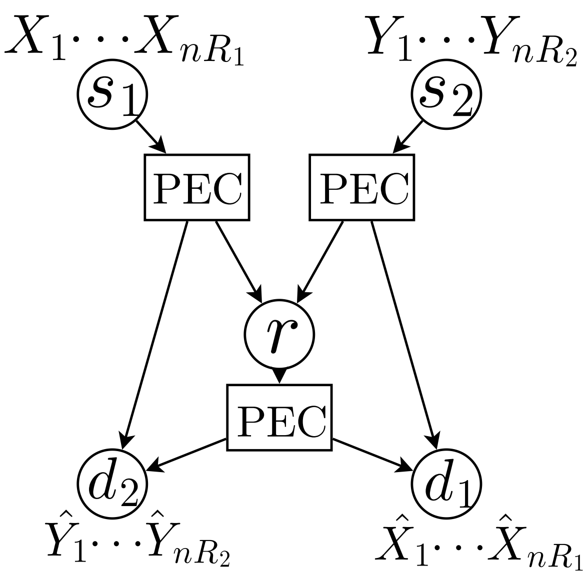

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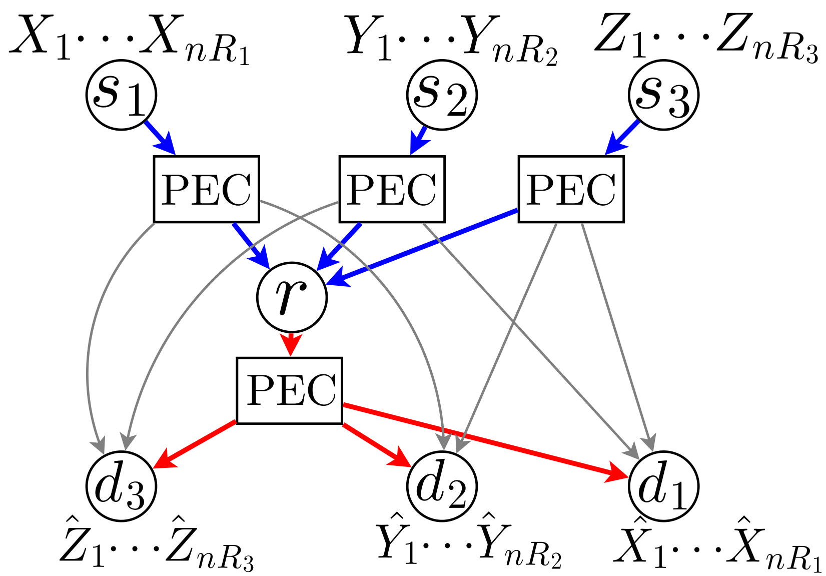

- Sequentially, s_1 to s_M, and r each can send n packets.
- Our goal: Find the largest $(R_1, R_2, \cdots, R_M)$ vector one can achieve, given that the PEC parameters are known to all nodes.

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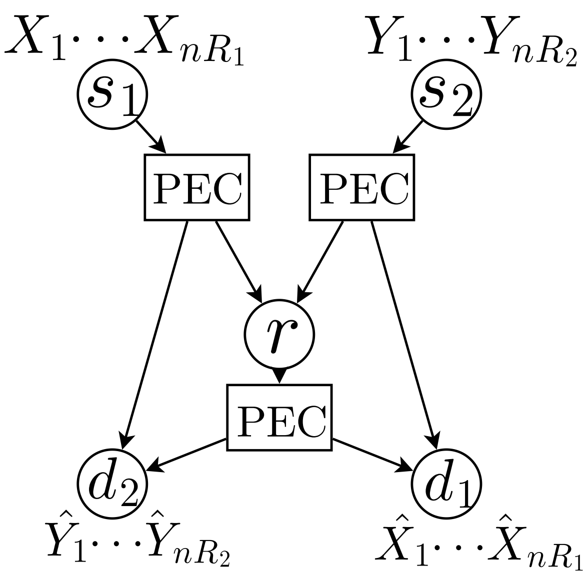

PEC parameters for $M = 2$:

Joint Prob.:

$$p_{s_1\to 2r}, p_{s_1\to 2r^c}, p_{s_1\to 2^c r}, p_{s_1\to 2^c r^c};$$
$$p_{s_2\to 1r}, p_{s_2\to 1r^c}, p_{s_2\to 1^c r}, p_{s_2\to 1^c r^c};$$
$$p_{r\to 12}, p_{r\to 12^c}, p_{r\to 1^c2}, p_{r\to 1^c2^c}.$$

Marginal Prob.:

$$p_{r;1} \stackrel{\Delta}{=} p_{r\to 12} + p_{r\to 12^c}$$

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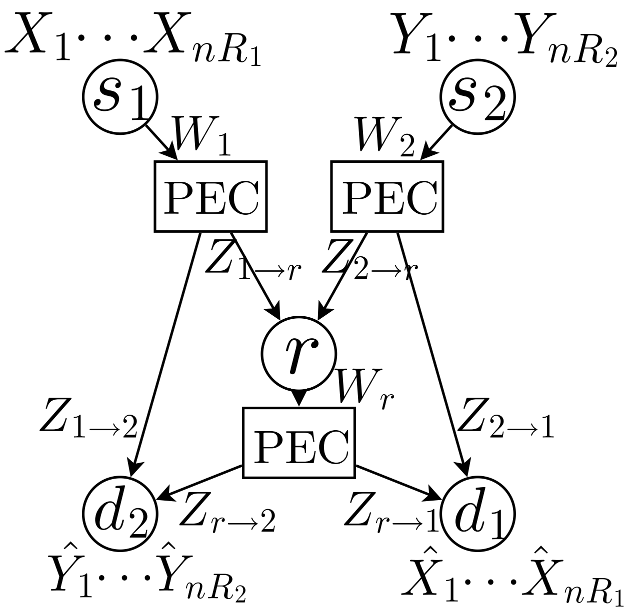
Formal Definition of Feasibility

A network code is defined by five functions:

$$\mathbf{W}_1 = f_{s_1}(\mathbf{X}),\ \mathbf{W}_2 = f_{s_2}(\mathbf{Y}),$$ sequential tx: s_1 and s_2 first

$$\mathbf{W}_r = f_r(\mathbf{Z}_{1\to r}, \mathbf{Z}_{2\to r}, \mathbf{1}_{\{Z_{1\to 2}(\cdot)=*\}}, \mathbf{1}_{\{Z_{2\to 1}(\cdot)=*\}}),$$ overheating status feedback before r's tx

$$\hat{\mathbf{X}} = f_{d_1}(\mathbf{Z}_{r\to 1}, \mathbf{Z}_{2\to 1}),\ \hat{\mathbf{Y}} = f_{d_2}(\mathbf{Z}_{r\to 2}, \mathbf{Z}_{1\to 2}).$$ joint decoding



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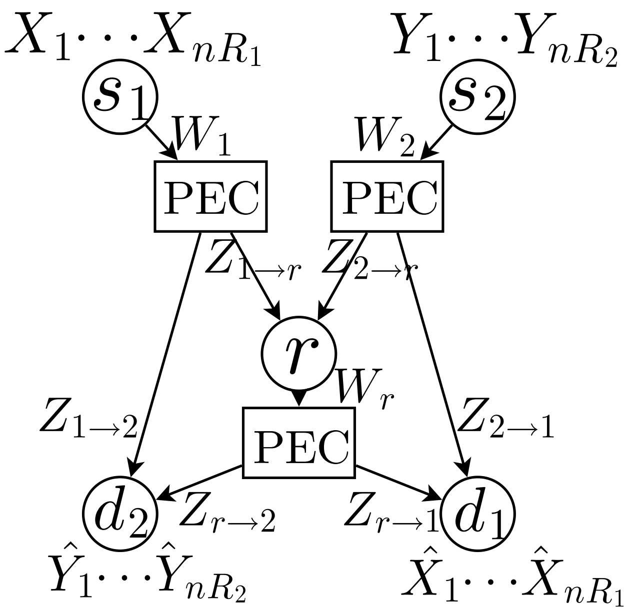
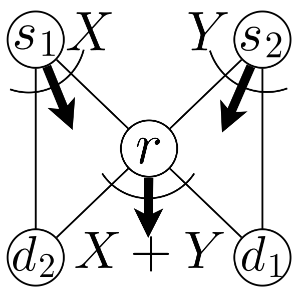
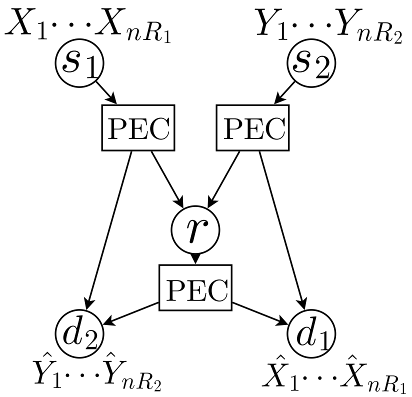


Definition 1 (R_1, R_2) *is achievable if* $\forall \epsilon > 0$, *there exist a sufficiently large* n, *a sufficiently large finite field* $\mathsf{GF}(2^b)$, *and a corresponding network code, such that for independently and uniformly distributed* $\mathbf{X}$ *and* $\mathbf{Y}$*:*

$$\mathsf{P}\left(\hat{\mathbf{X}} \neq \mathbf{X}\right) + \mathsf{P}\left(\hat{\mathbf{Y}} \neq \mathbf{Y}\right) < \epsilon.$$

Feature by Feature Comparison





COPE		Our Setting
Listening/Overhearing		
XOR coding		
Opportunistic		
Learning neighbors' states		
Periodic feedback		

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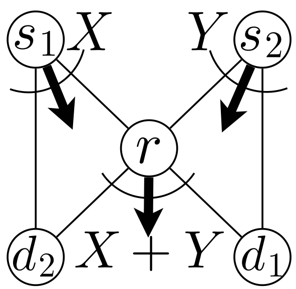

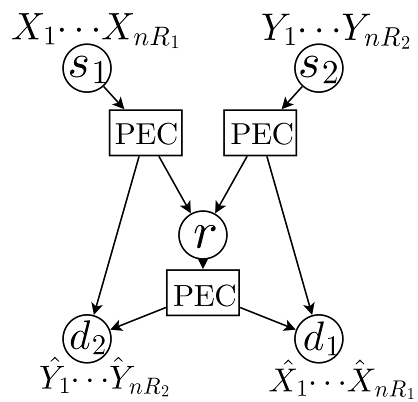

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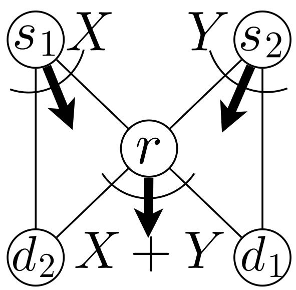

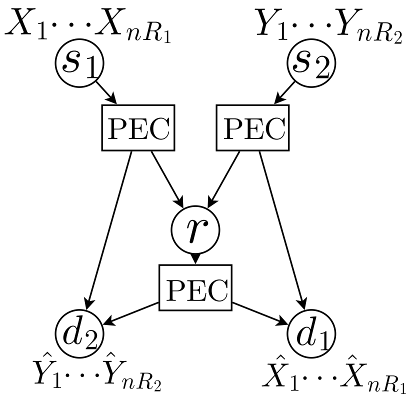

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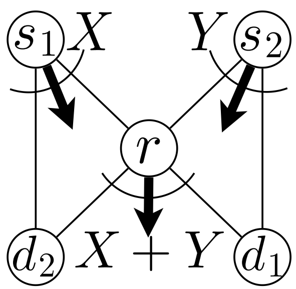

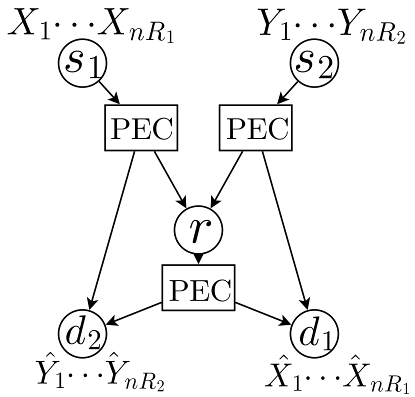

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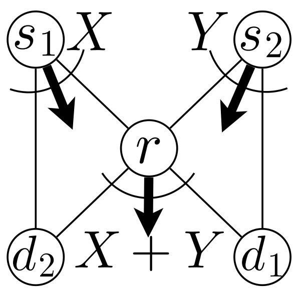

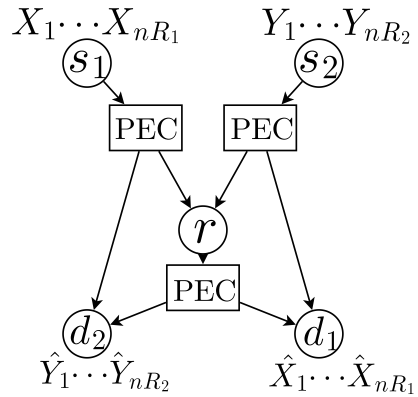

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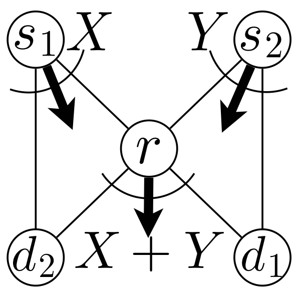

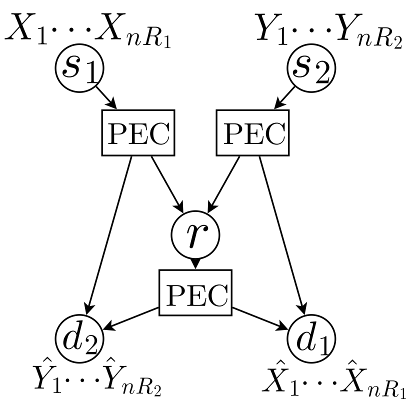

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Periodic feedback	>	One-time feedback before tx of the relay Remark 1: Feedback increases the capacity [Ozarow et al. 84], [Xue et al. 08], [Georgiadis et al. 09]. Remark 2: Improvement is small when the number of receivers is small [W, Allerton 10].

Comparison to Existing Literatures

# of sessions	COPE-like Protocols (Broadcast PECs w. **MSI**)	Gaussian broadcast channels w. **MSI**
M=2	Full capacity region	Full capacity region [Wu 07]
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General M	Outer and inner bounds that are numerically close	?

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  - Random overhearing, rate-adaptation for $r \rightarrow \{d_i\}$: [Chaporkar *et al*. 07].
  - Random overhearing, deterministic $r \rightarrow \{d_i\}$: [Rayanchu *et al*. 08].
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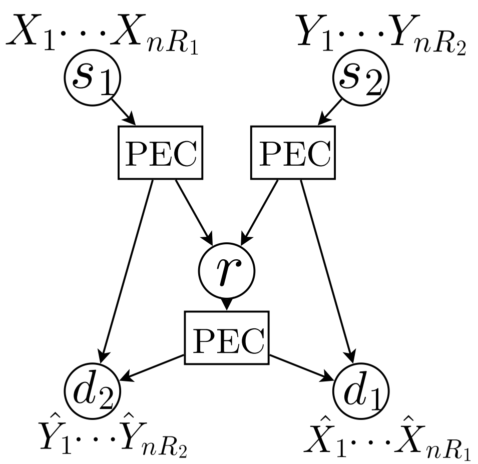
- Index coding [Bar-Yossef *et al.* 06], [Alon *et al.* 08], [Chaudhry *et al.* 08].

Round-Based Policies

- Each round: s_1 to s_M first and then r. Totally $(M+1) \cdot n$ pkts.

M=2



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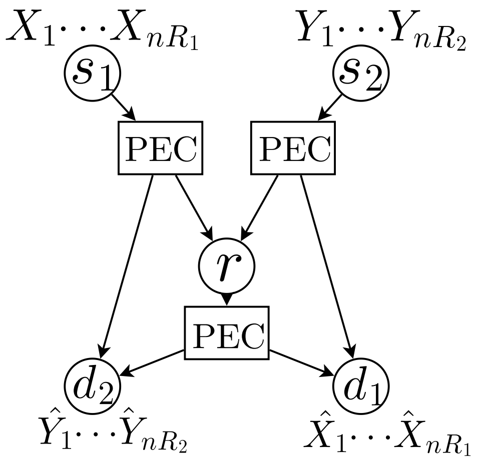

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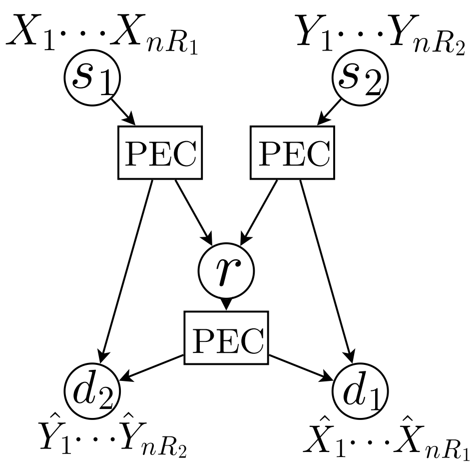

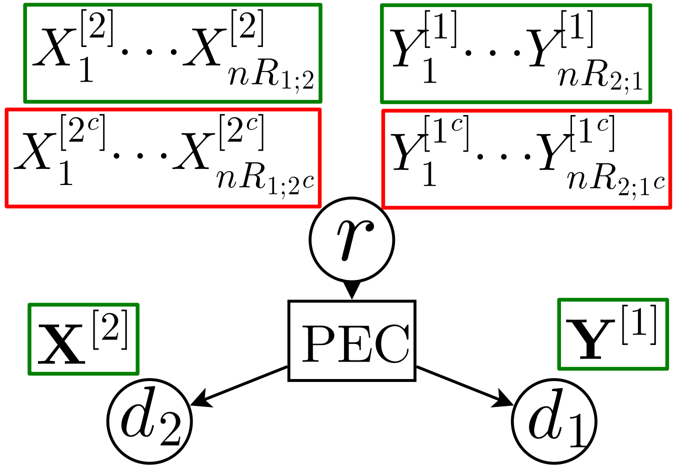

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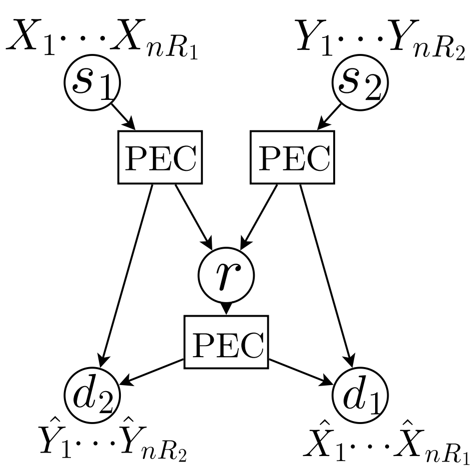
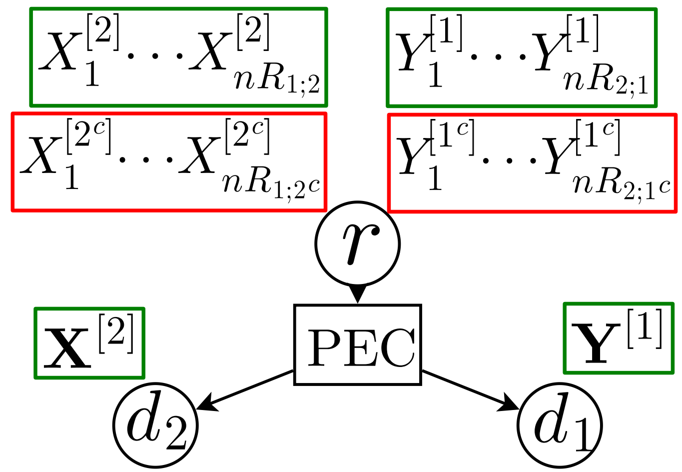
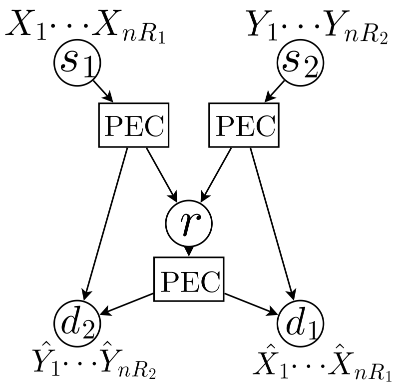


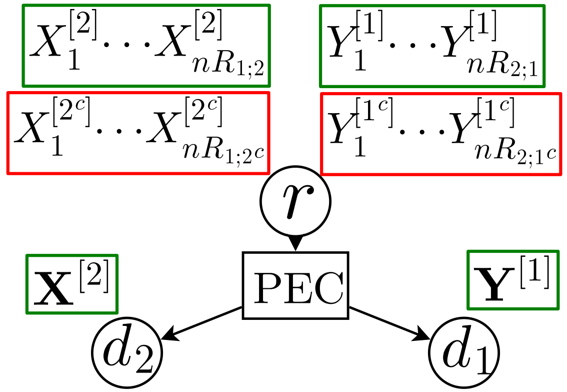


- No feedback is allowed during the transmission of the last n packets by relay r.

The Capacity Regions for $M = 2$

M=2





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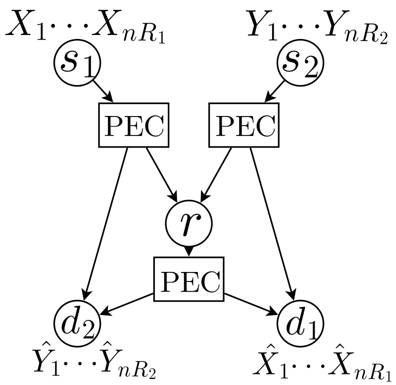

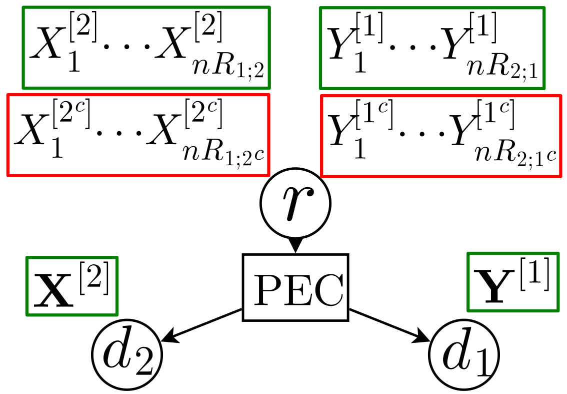

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- The capacity region for $M = 2$, [W, Asilomar 09]:

$$R_1 \leq \min\left(p_{s_1;r}, p_{r;1} - (R_2 - p_{s_2;1})^+\right)$$

$$R_2 \leq \min\left(p_{s_2;r}, p_{r;2} - \frac{p_{r;2}}{p_{r;1}}(R_1 - p_{s_1;2})^+\right)$$

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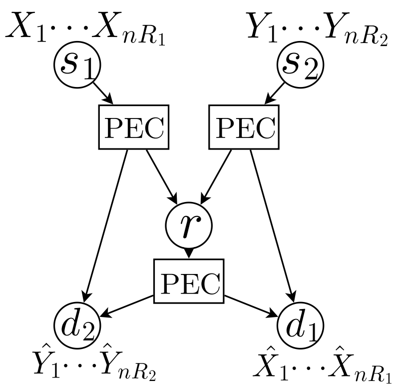

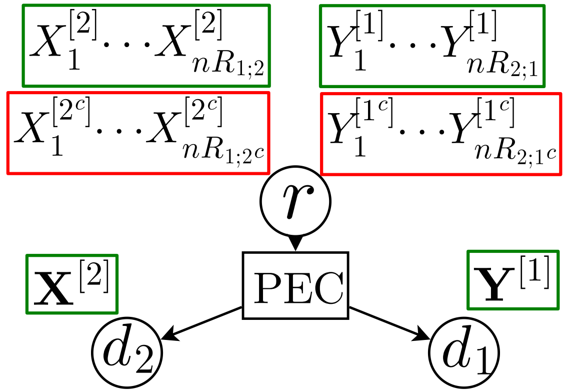

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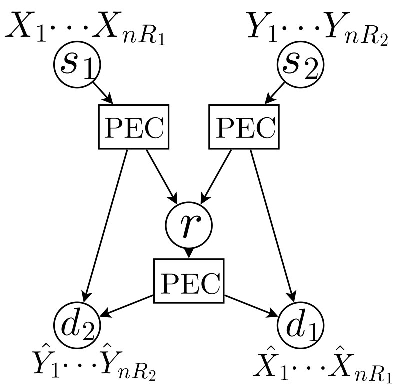

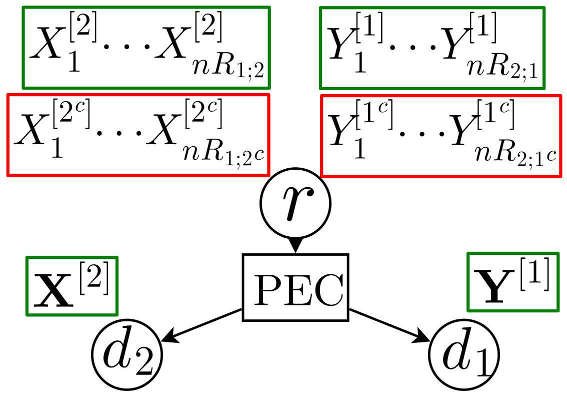

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  - Ensure transmission from s_i to r.
  - Paying the price of carrying the information for the other session.

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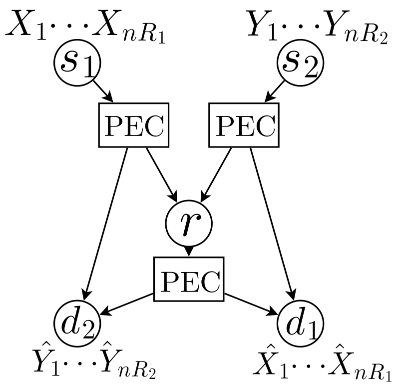

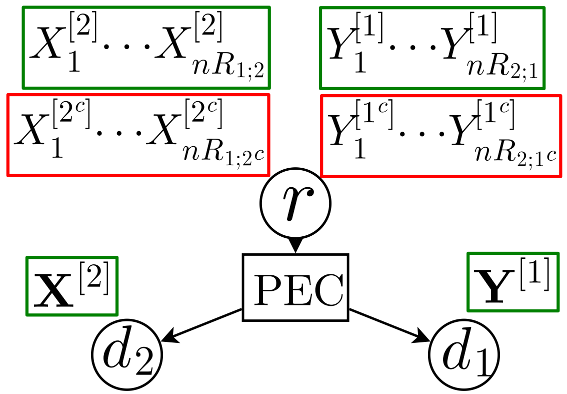

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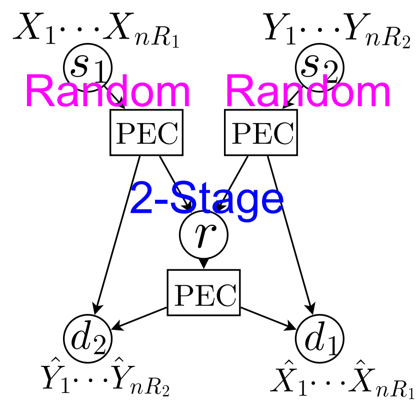
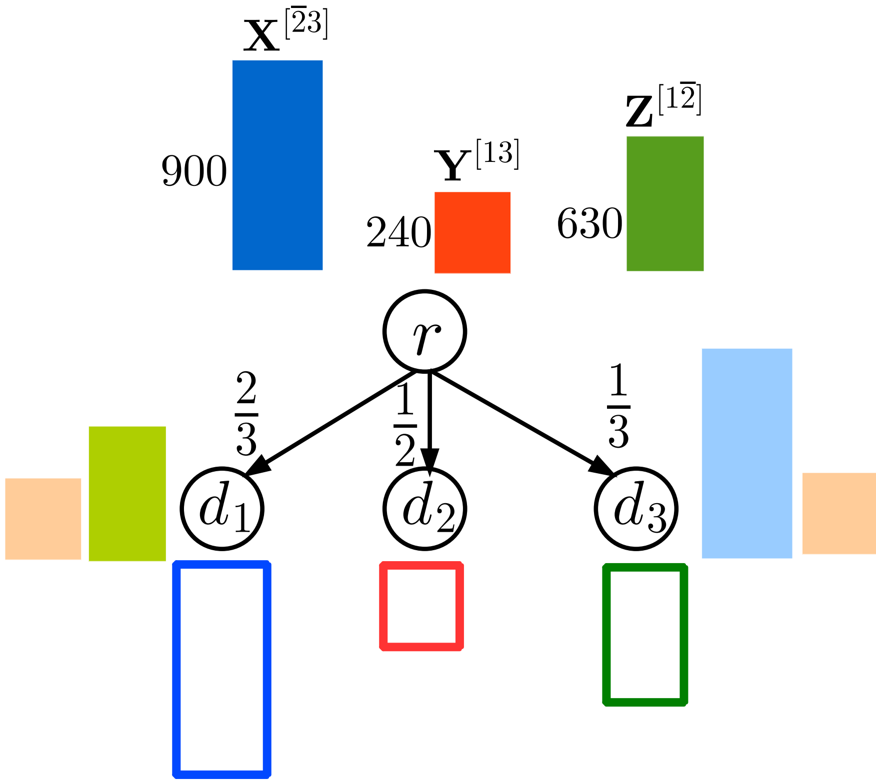
$$R_2 \leq \min\left(p_{s_2;r}, p_{r;2} - \frac{p_{r;2}}{p_{r;1}}(R_1 - p_{s_1;2})^+\right)$$



  - Ensure transmission from s_i to r.
  - Paying the price of carrying the information for the other session.

An Illustrative Example of $M = 3$



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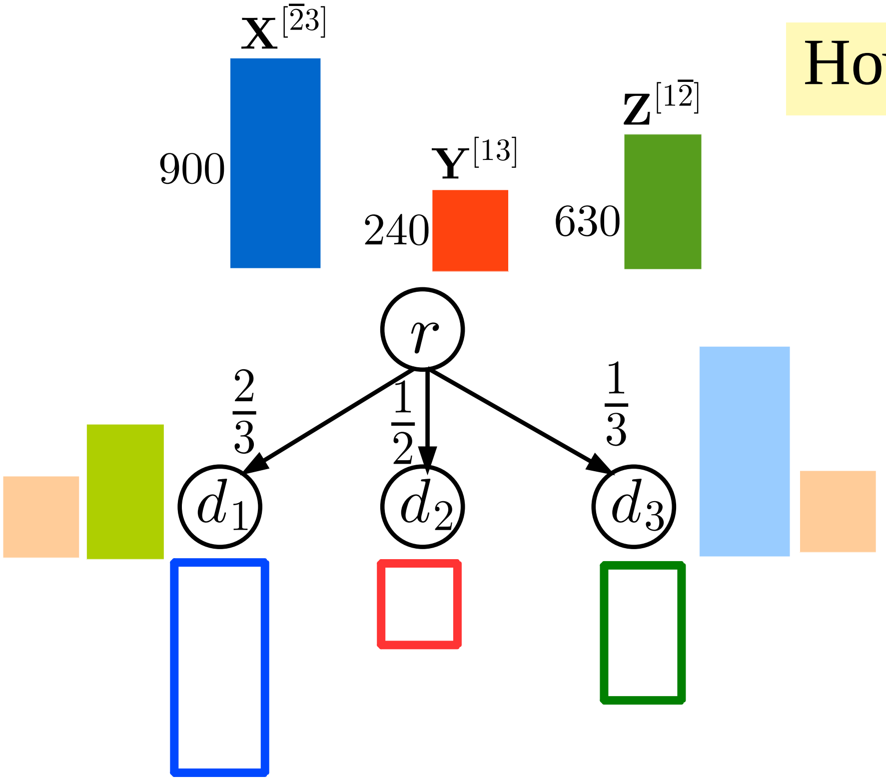

How many time slots to finish transmission?

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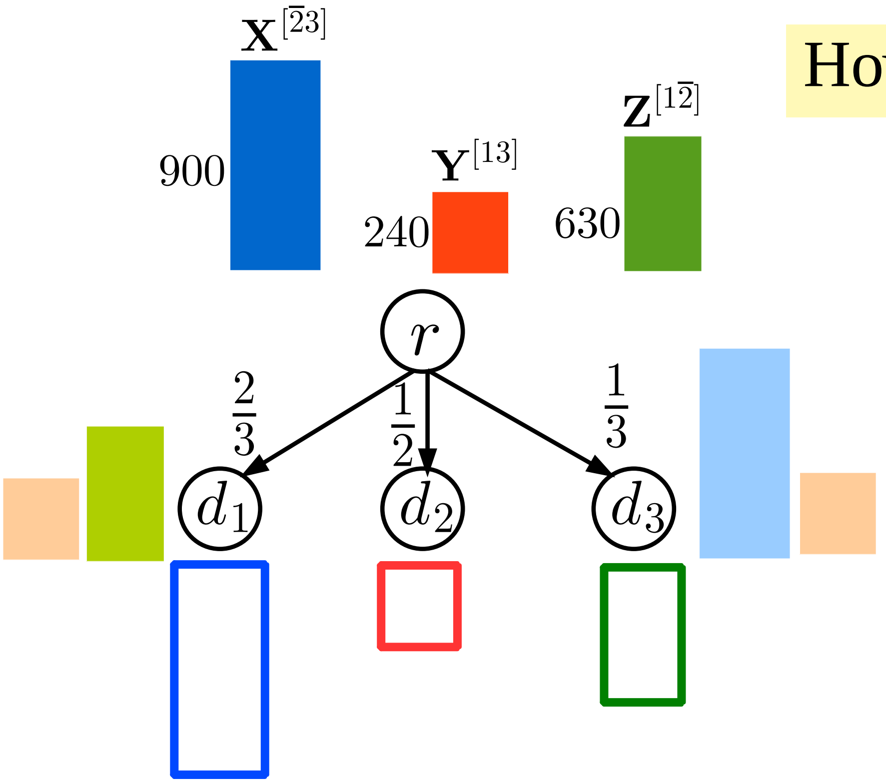

How many time slots to finish transmission?

Solution 1 — Time sharing:

$$\frac{900}{2/3} + \frac{240}{1/2} + \frac{630}{1/3} = 3720$$

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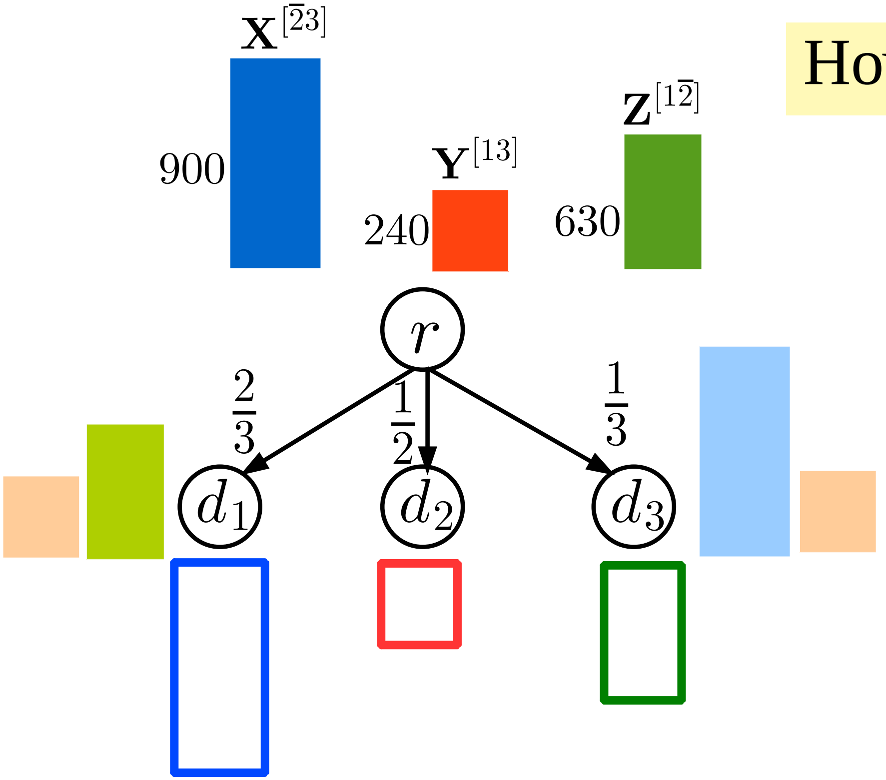

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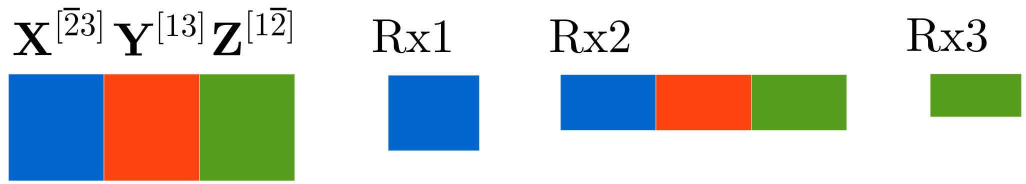
Solution 2 — Random mixing:



$$\max\left(\frac{900}{2/3}, \frac{900+240+630}{1/2}, \frac{630}{1/3}\right) = 3540$$

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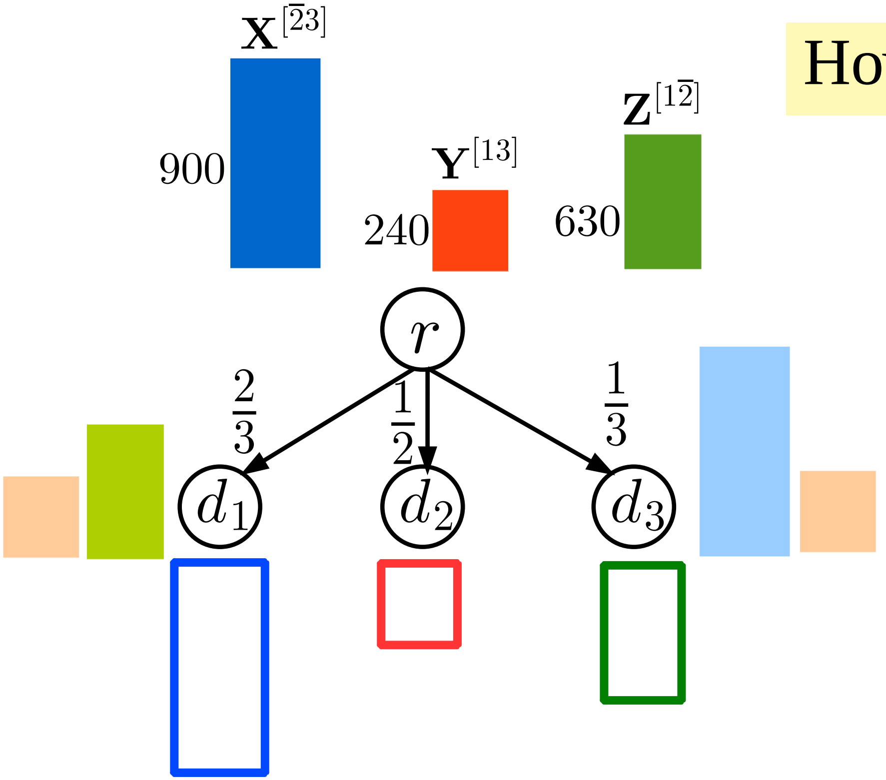

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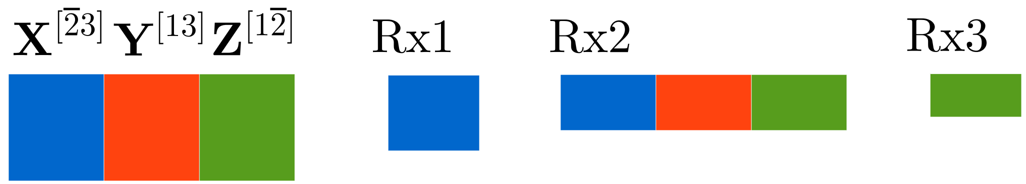

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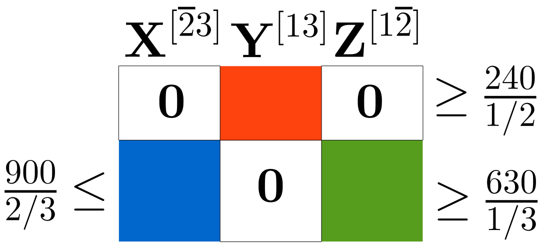

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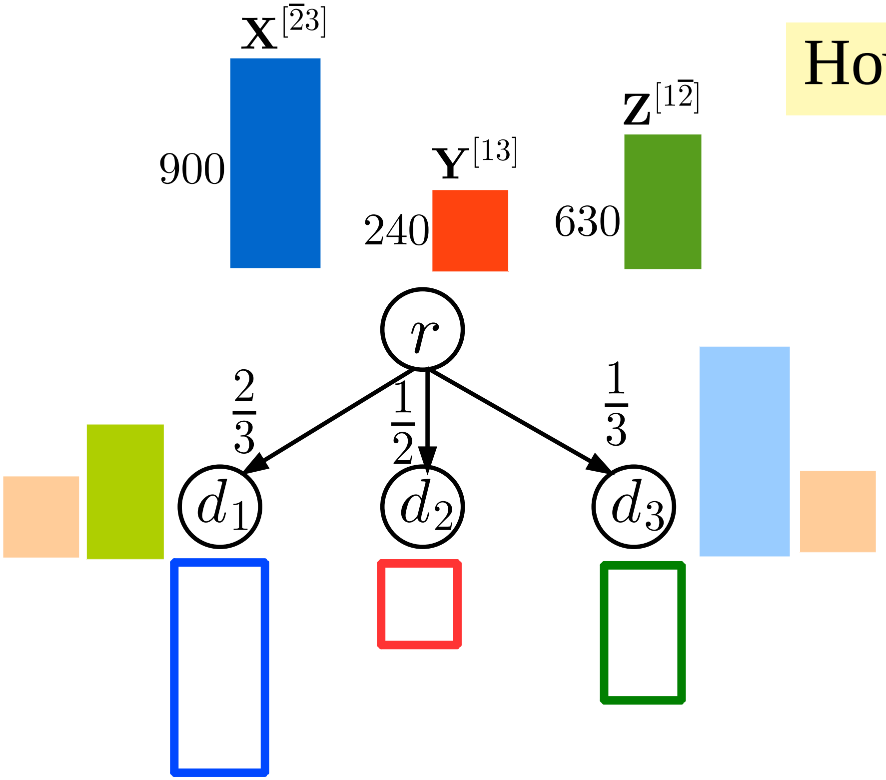

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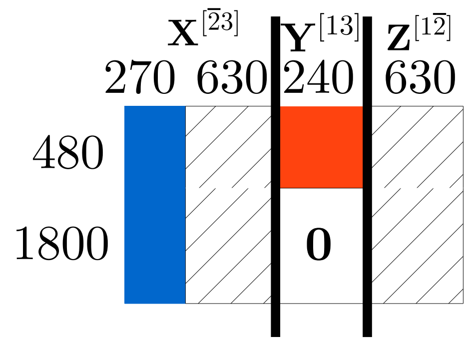

Totally 2280 time slots.

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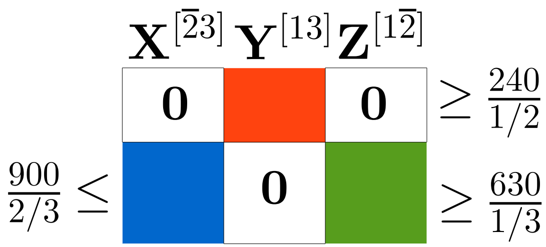

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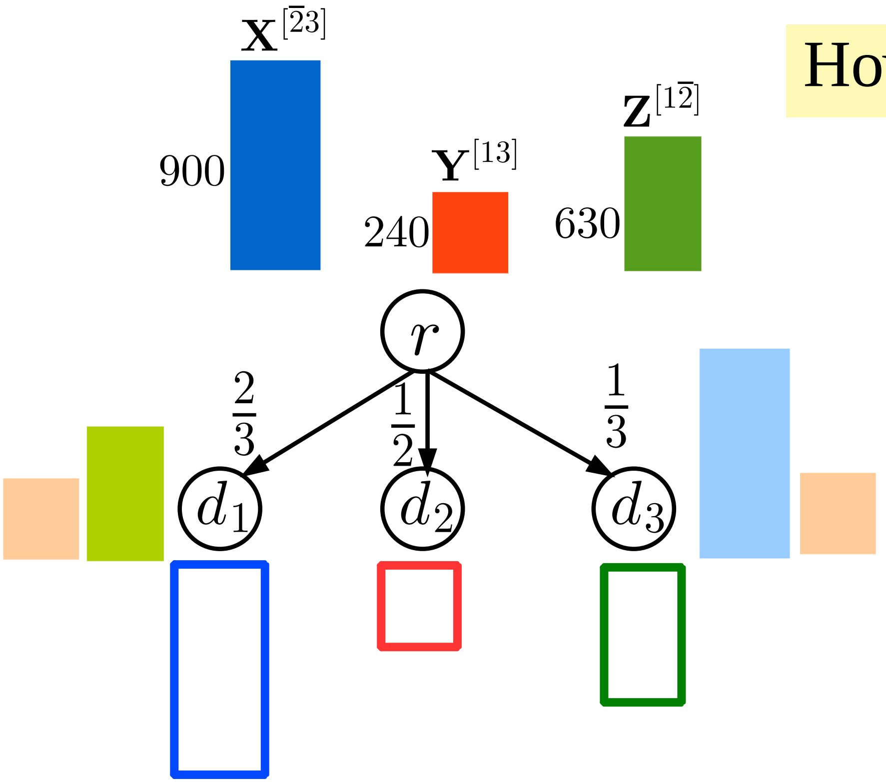

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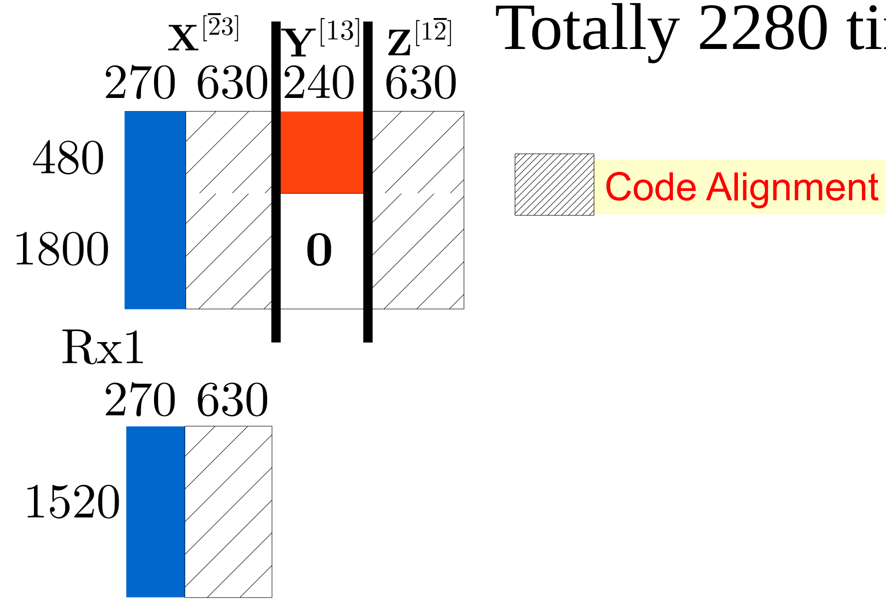

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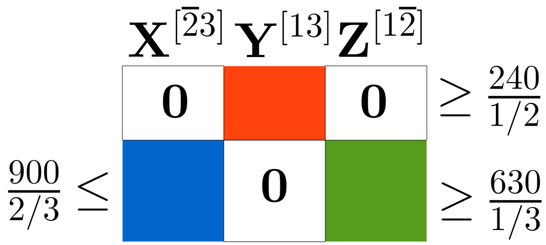

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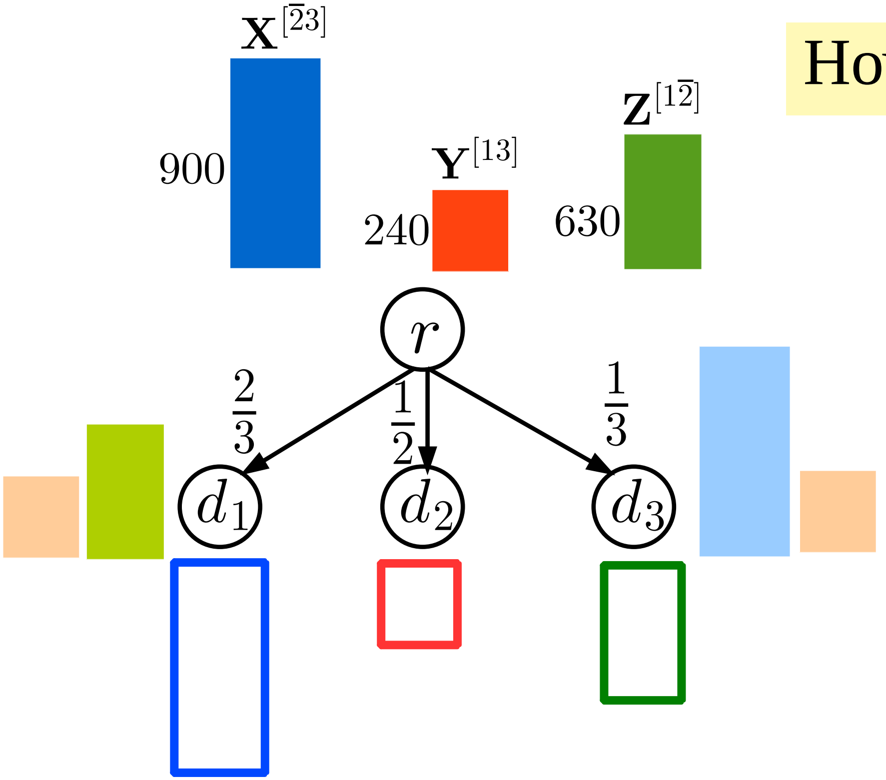

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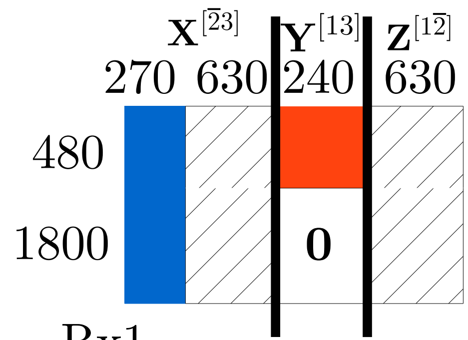

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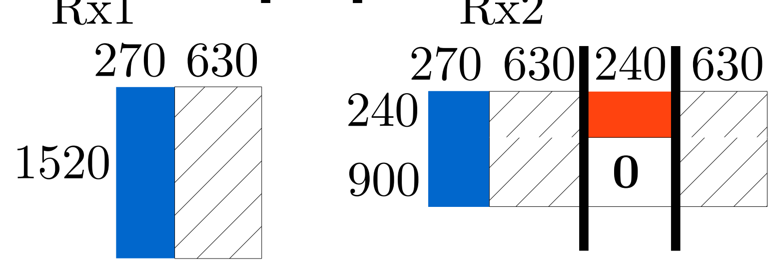
Code Alignment

Instead of decoding X and Z, we decode only $X + Z$.



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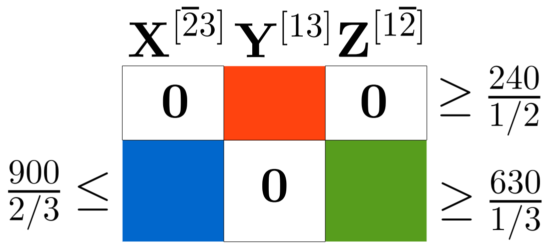

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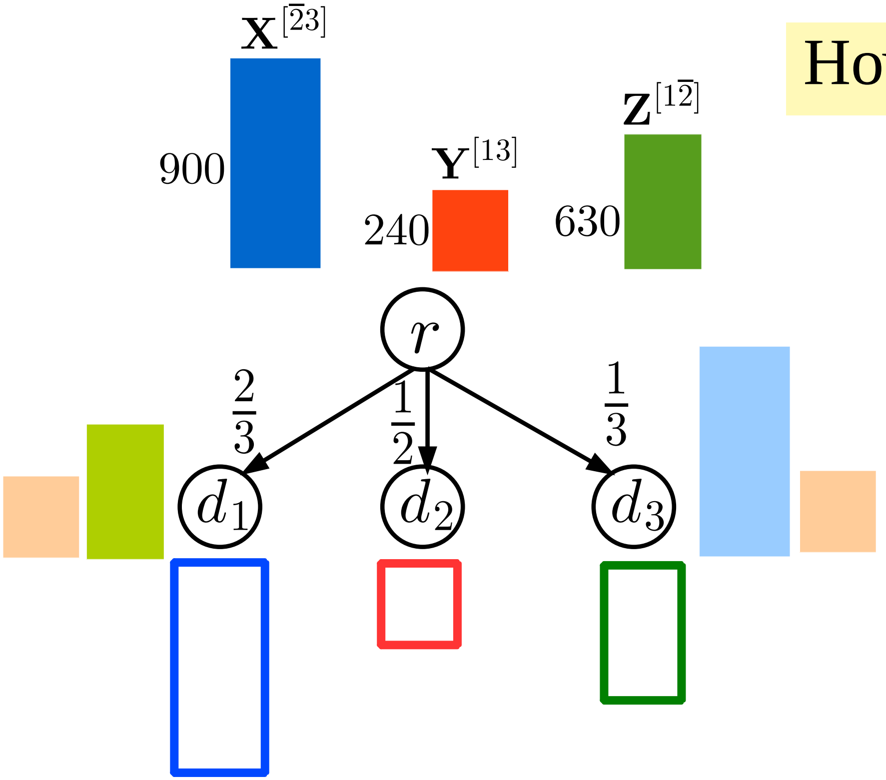

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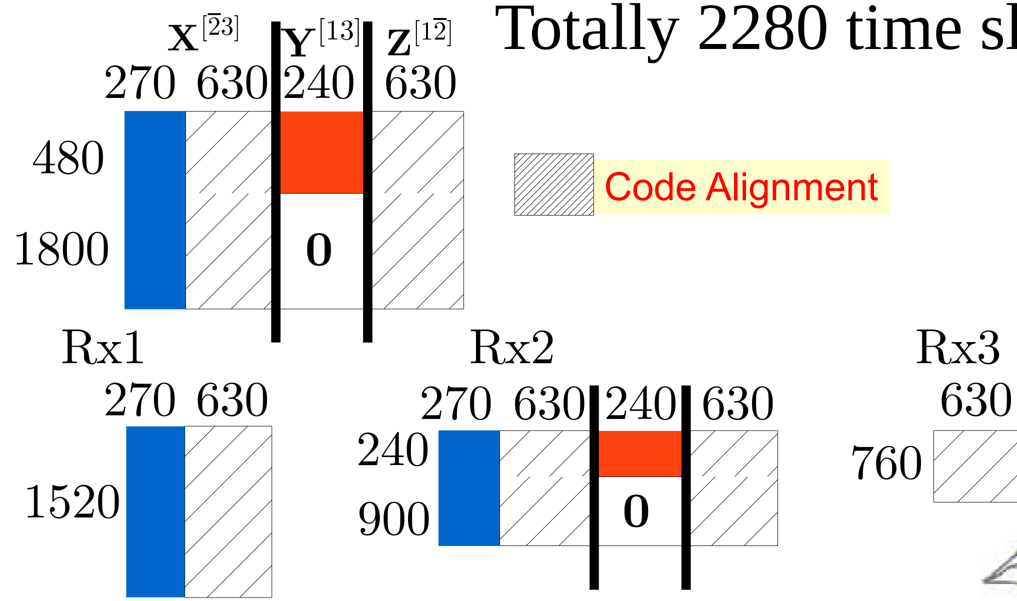

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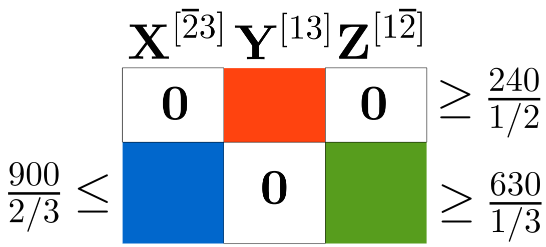

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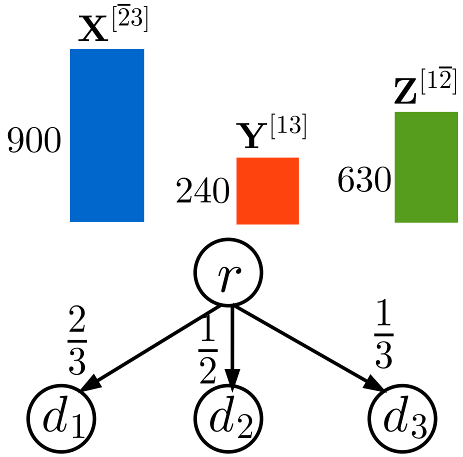

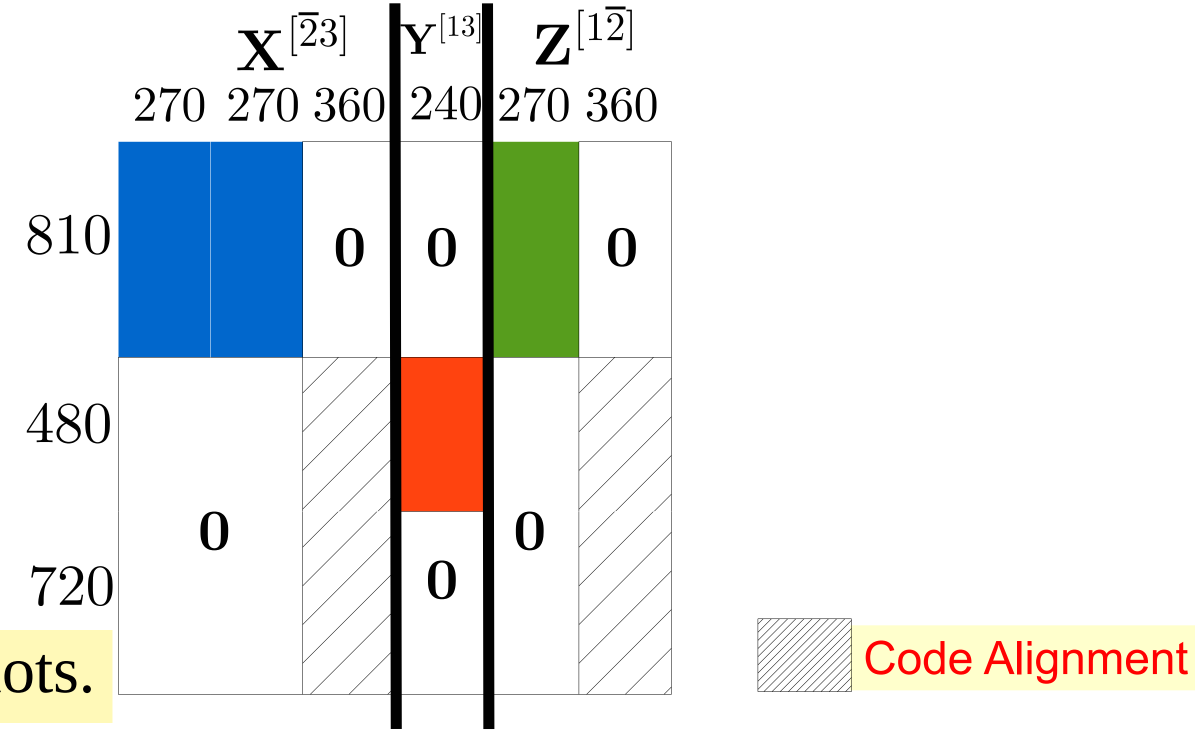

Totally 2010 time slots.

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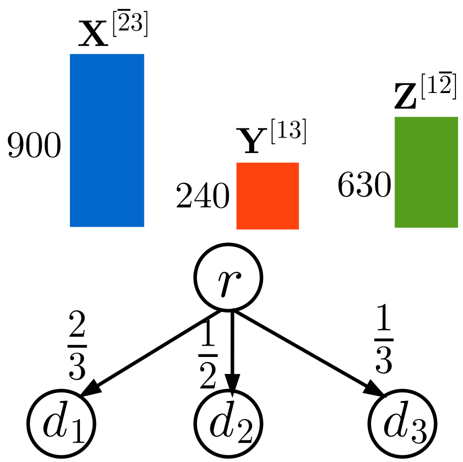

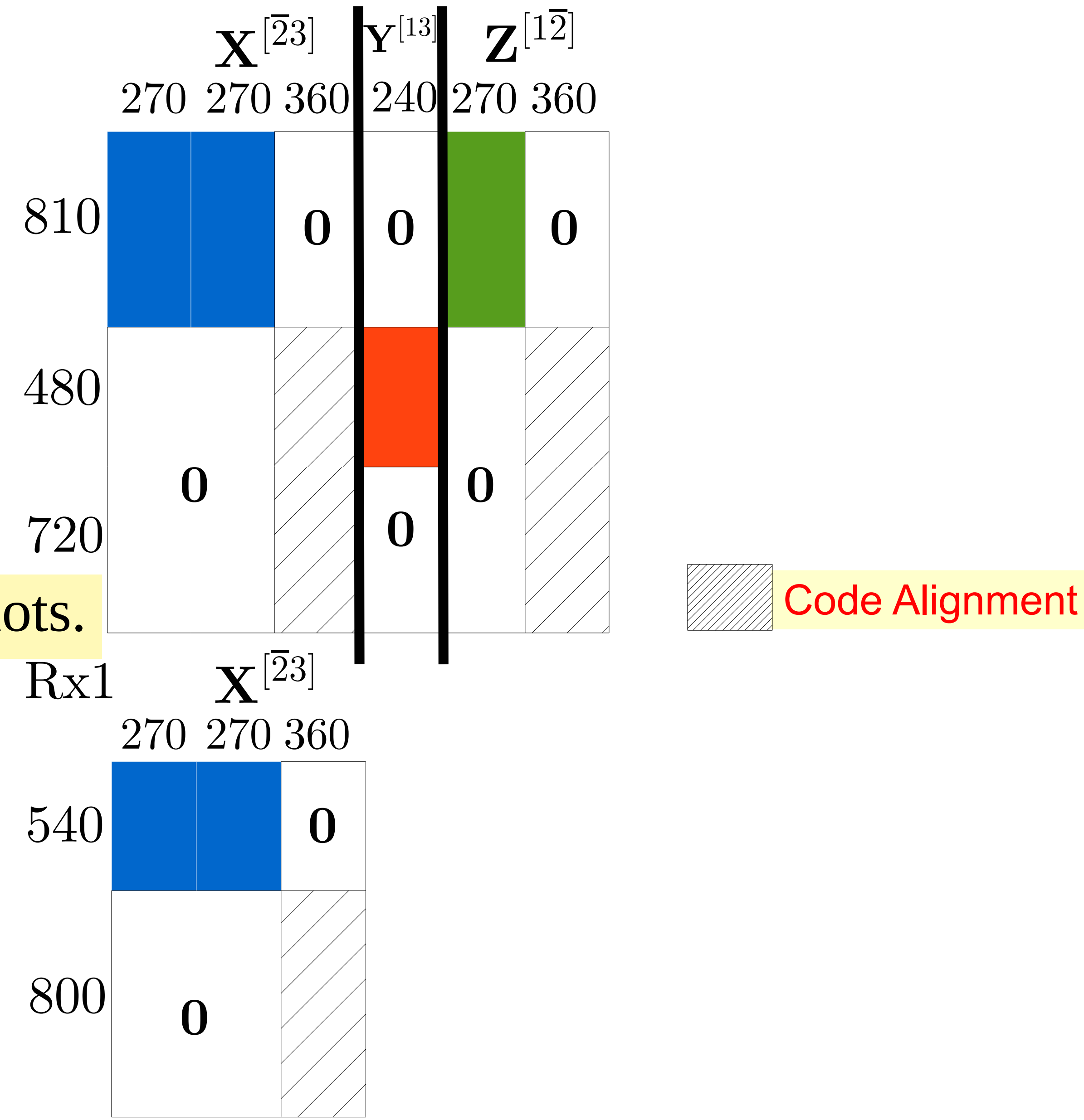

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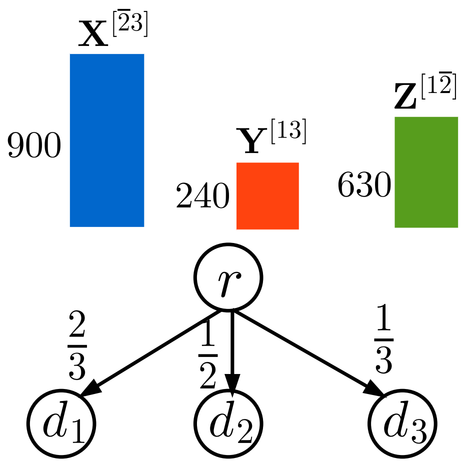

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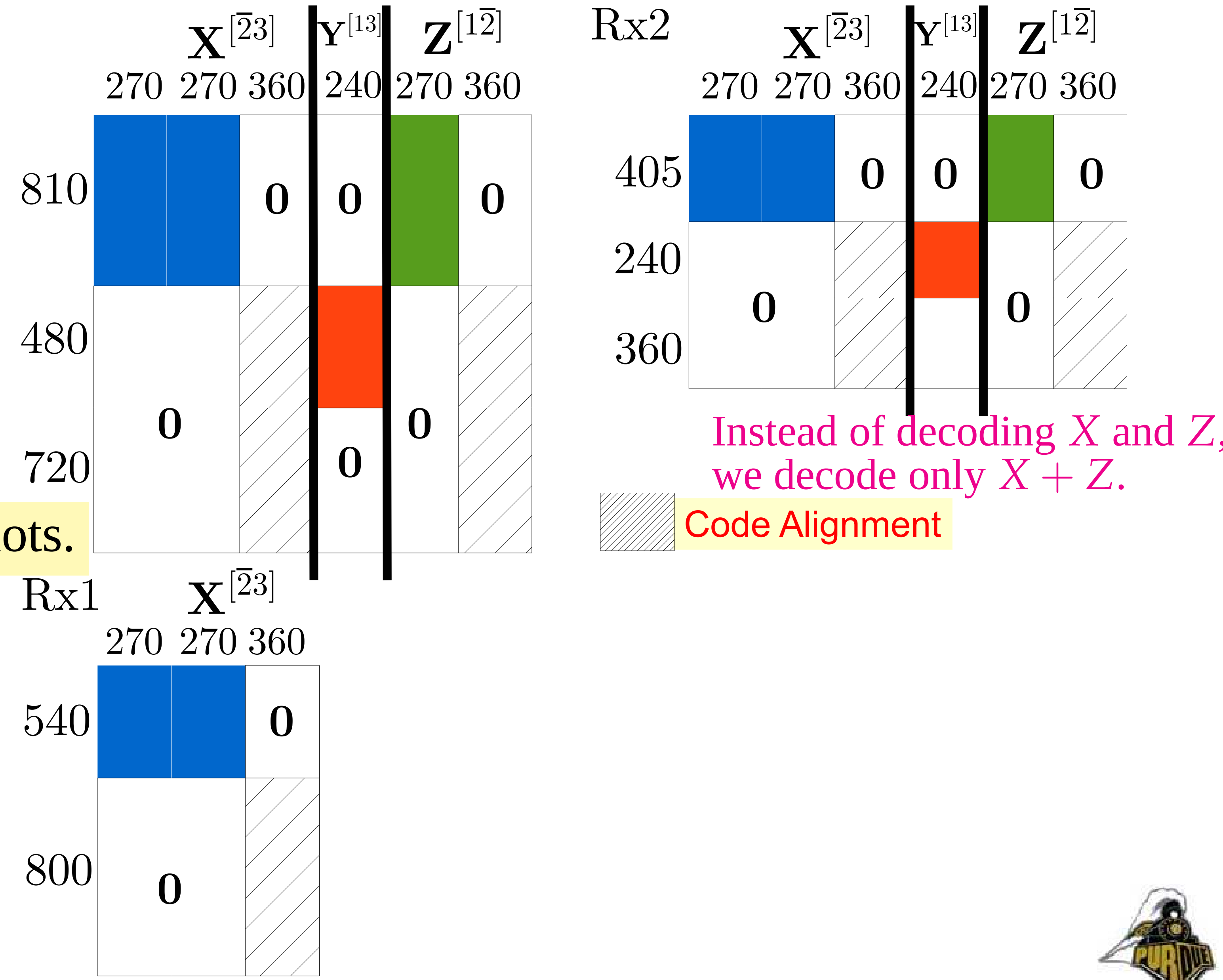

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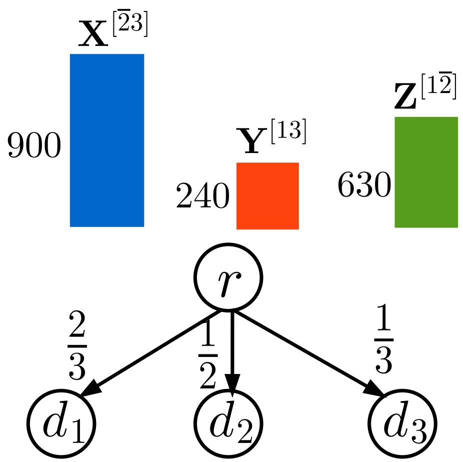

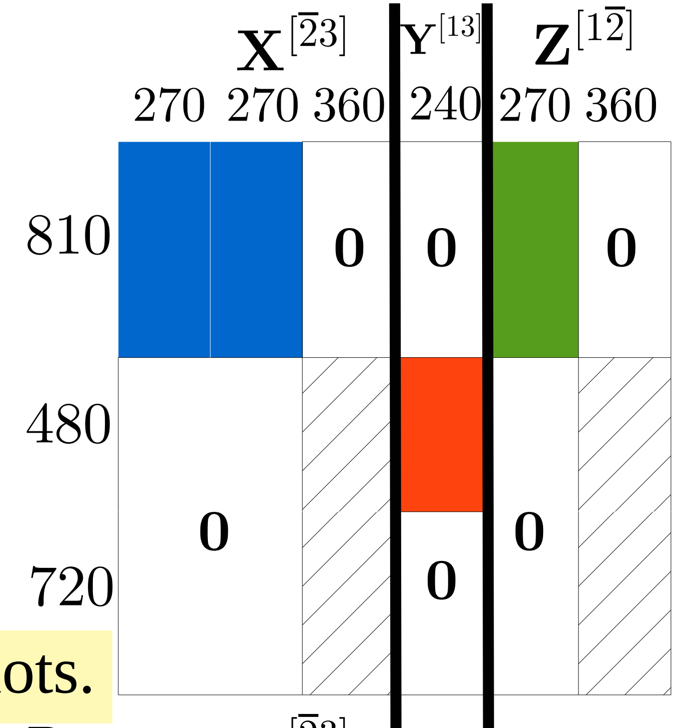

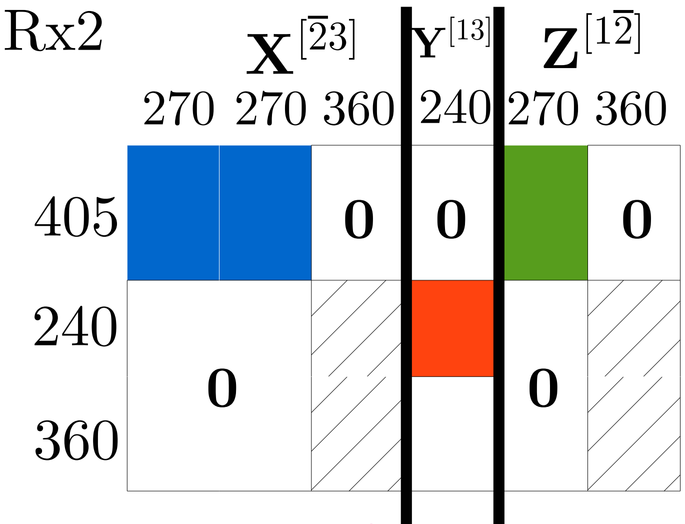

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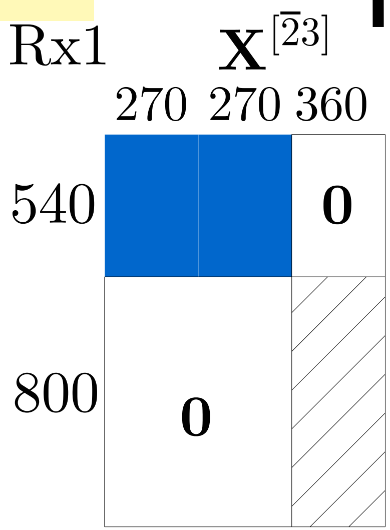
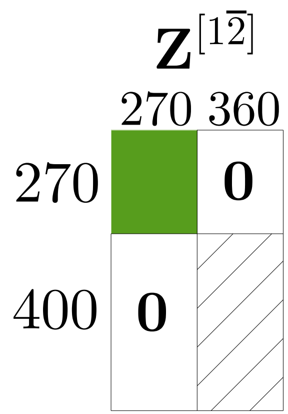
Rx1

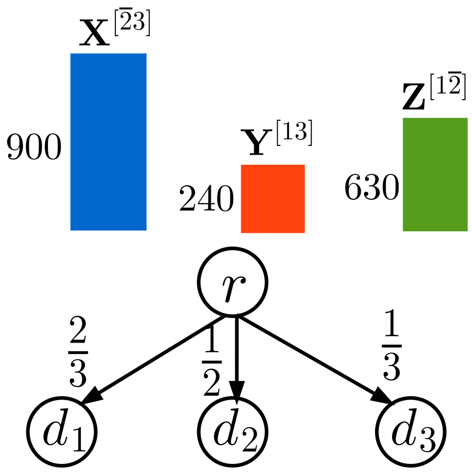


Rx3



Is The Hybrid Scheme Optimal?

Can we finish tx in <2010 slots?



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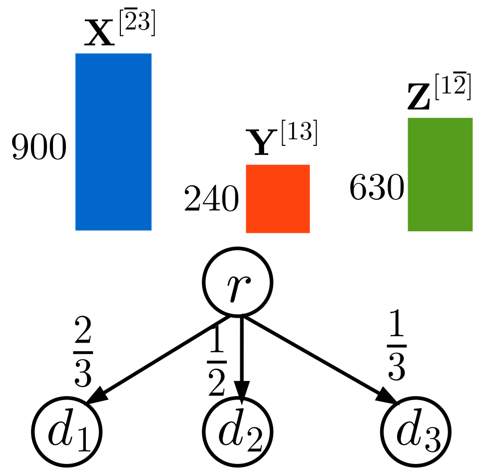
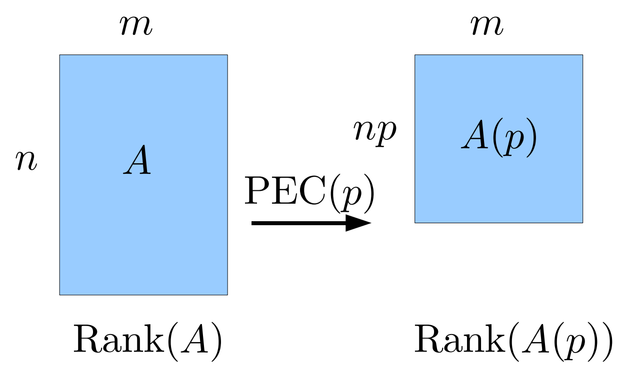


The cut-set bounds do not work.

We need new outer bounding arguments.

Information Concavity Property

- Rank-concavity:

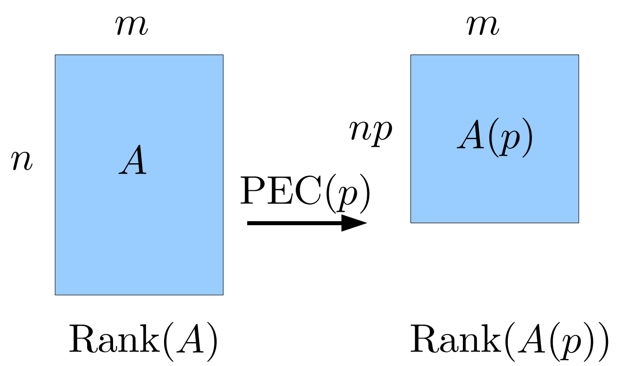


Information Concavity Property

- Rank-concavity:

At least p fraction of $\text{Rank}(A)$ basis vectors of A will be passed to $A(p)$.

$\Rightarrow \text{Rank}(A(p)) \geq p \cdot \text{Rank}(A)$.



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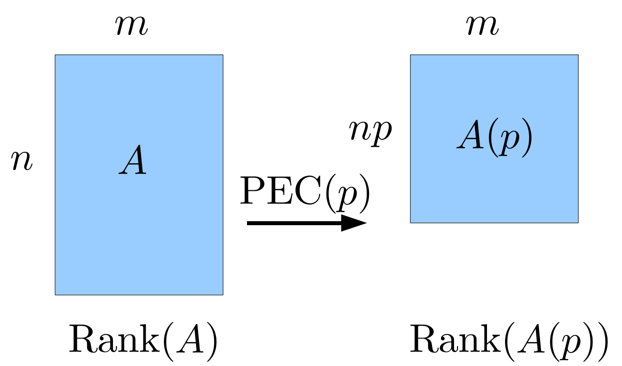

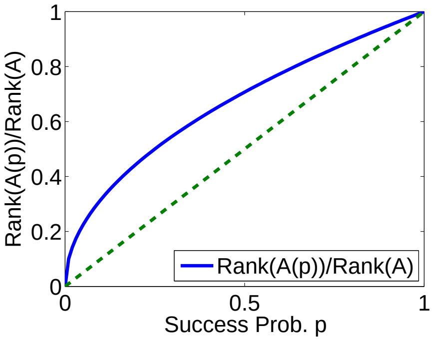

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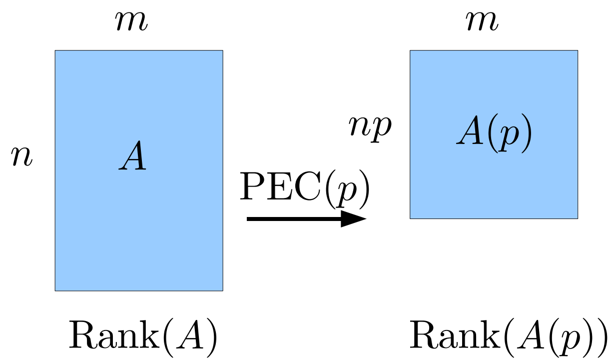
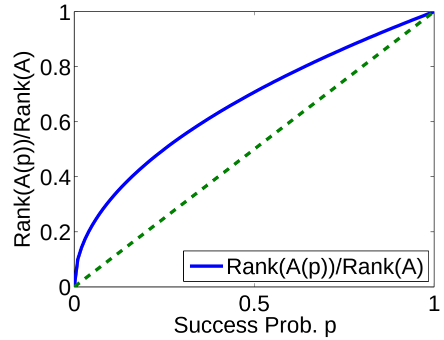
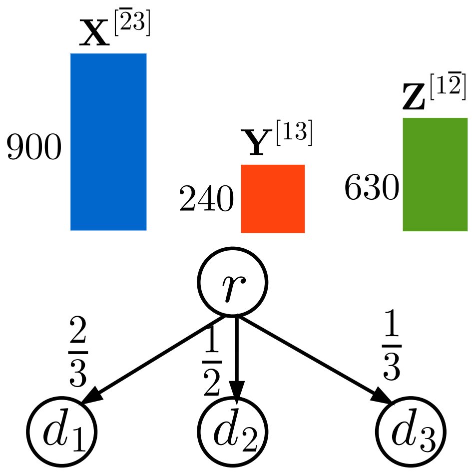




- When focusing on the mutual info. instead, the info. concavity argument can be generalized for non-linear codes.

Is The Hybrid Scheme Optimal?



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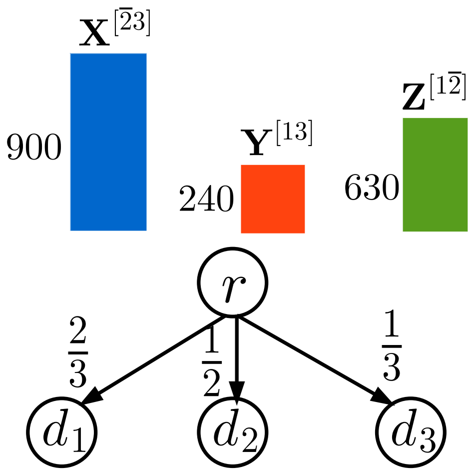
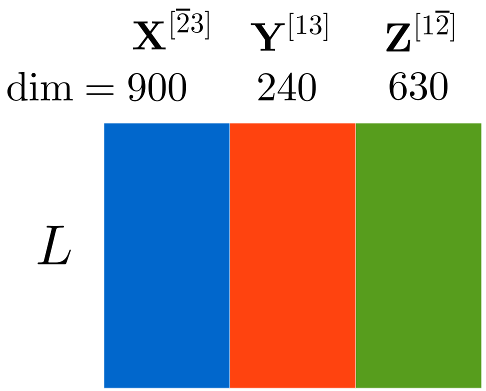


Any network code.



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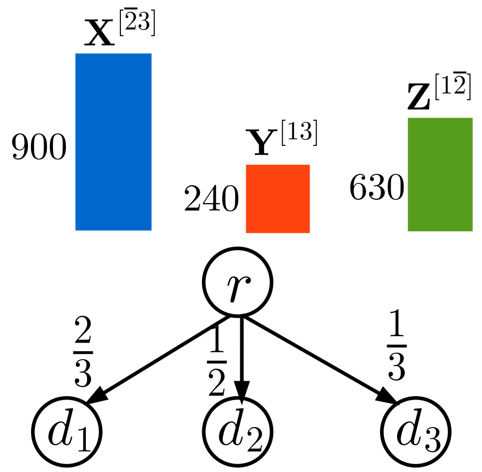

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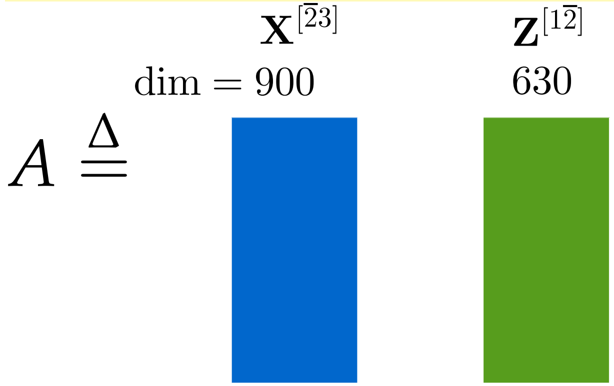

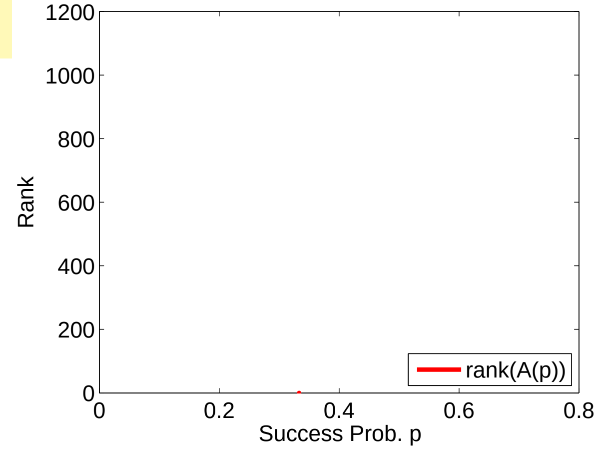

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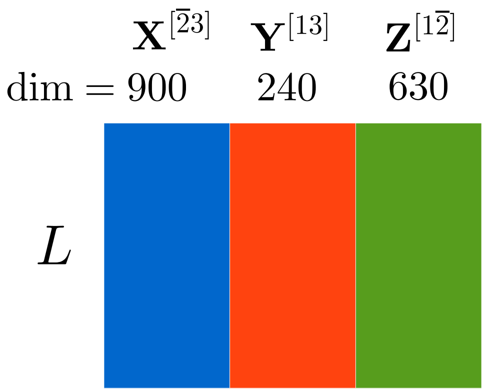

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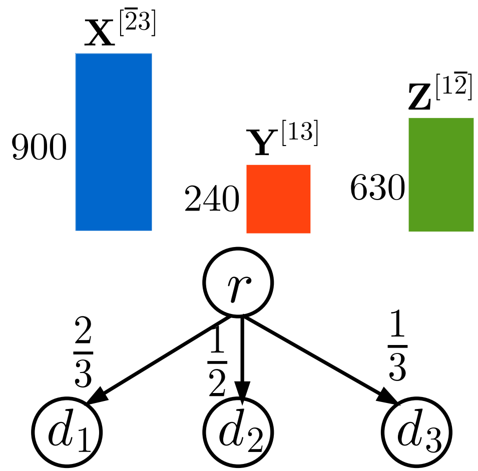

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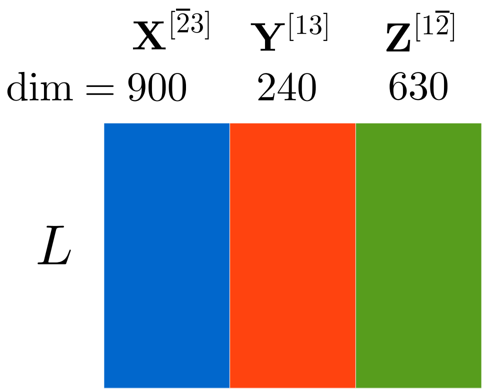

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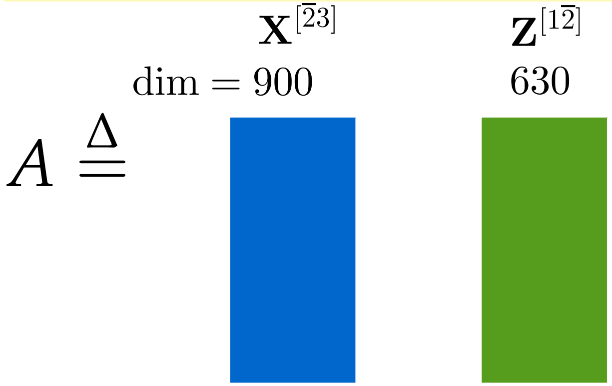

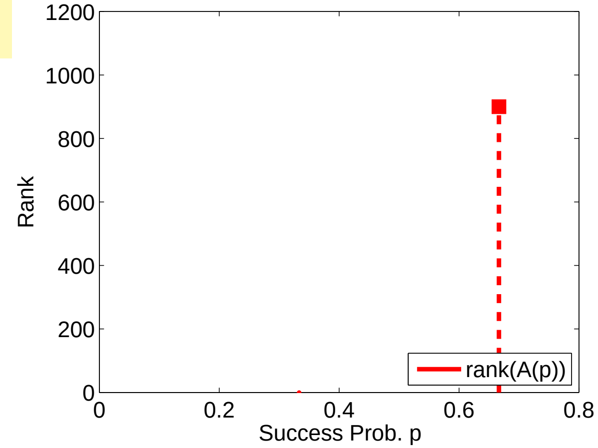

Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900$.

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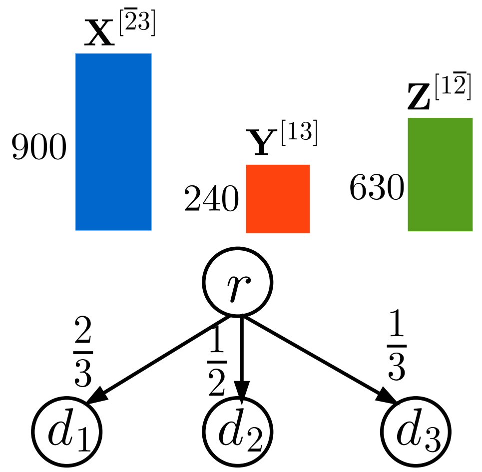

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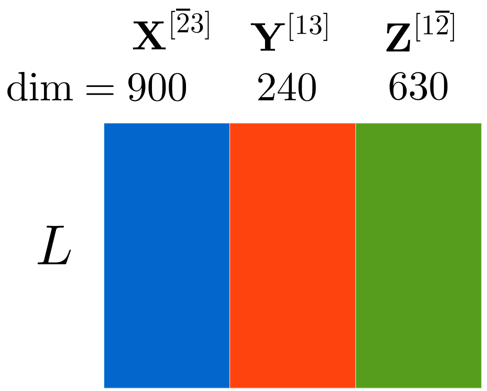

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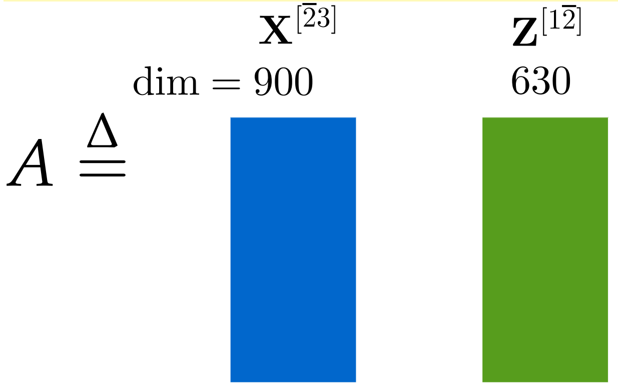

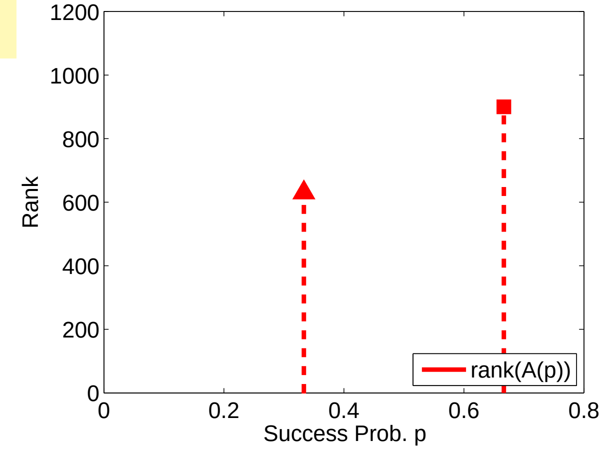

Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900.$

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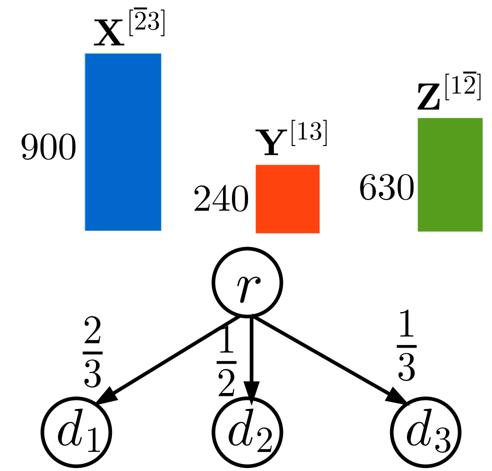

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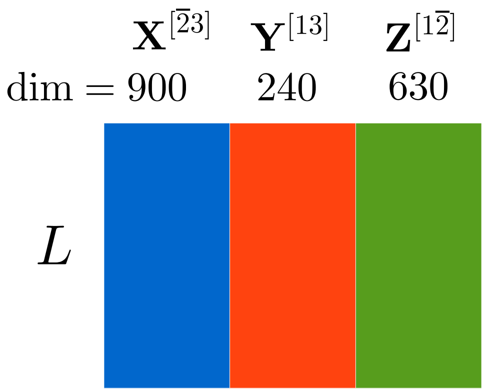

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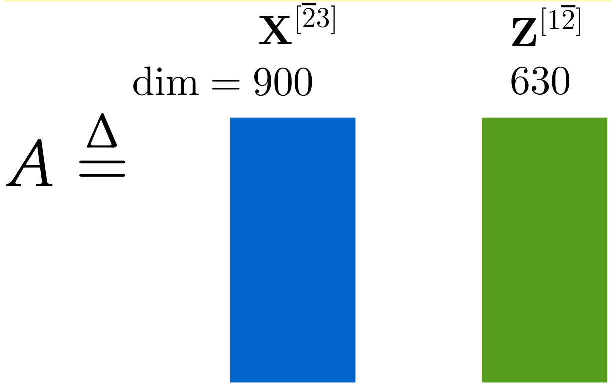

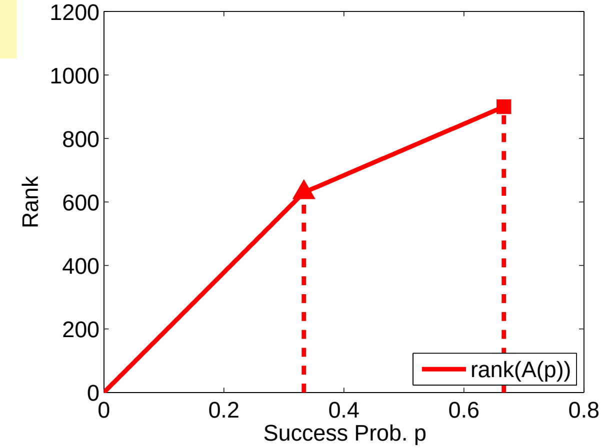

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Concavity of information transmission.

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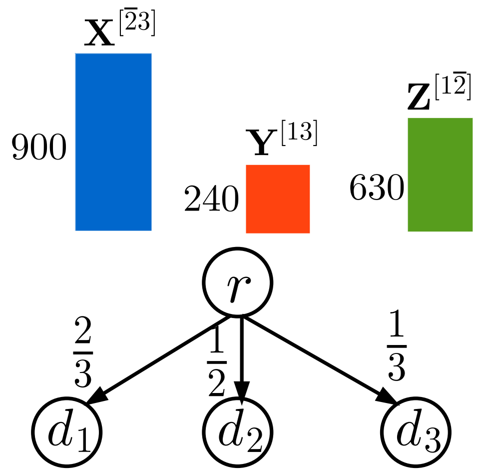
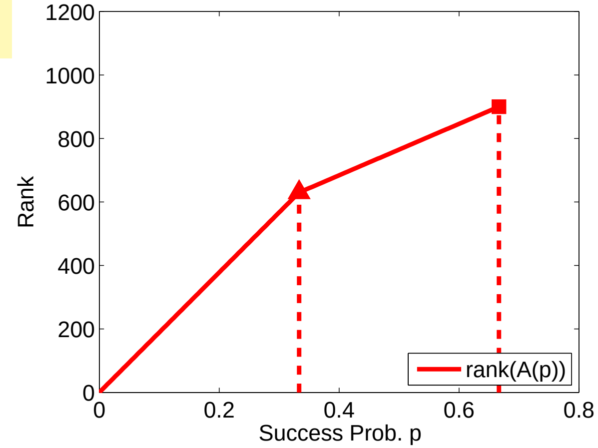


Can we finish tx in <2010

$\mathbf{X}^{[\bar{2}3]}$ $\mathbf{Z}^{[1\bar{2}]}$

dim = 900 630

$A \triangleq$



Any network code.

$\mathbf{X}^{[\bar{2}3]}$ $\mathbf{Y}^{[13]}$ $\mathbf{Z}^{[1\bar{2}]}$

dim = 900 240 630

L

Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900$.

Decodability at $d_3 \Rightarrow \text{Rank}(A(1/3)) \geq 630$.

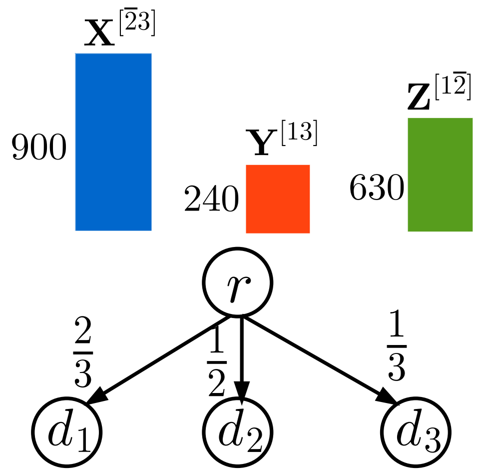
Concavity of information transmission.

$\mathbf{X}^{[\bar{2}3]}$ $\mathbf{Y}^{[13]}$ $\mathbf{Z}^{[1\bar{2}]}$

dim = 900 240 630

$B \triangleq$

Is The Hybrid Scheme Optimal?



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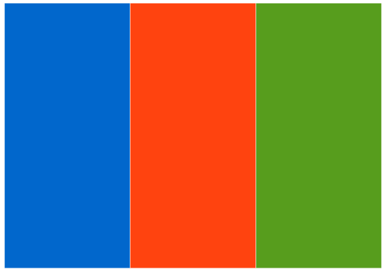

Can we finish tx in <2010

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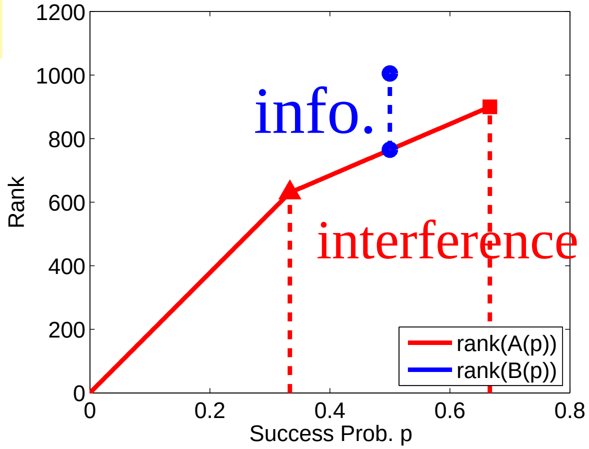
$A \triangleq$



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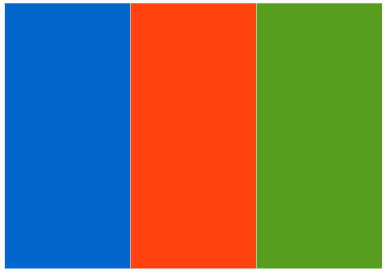
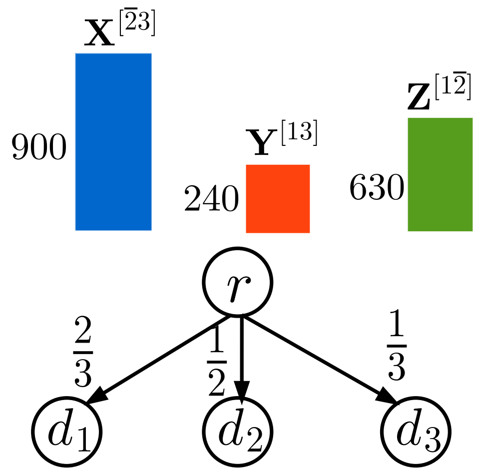


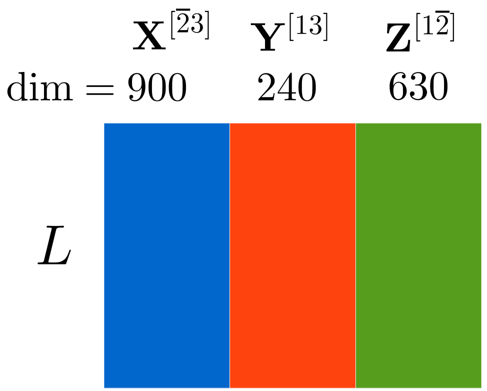
info. interference.

Decodability at $d_2 \Rightarrow \text{Rank}(B(1/2)) \geq 240 + \text{Rank}(A(1/2)) = 1005$.

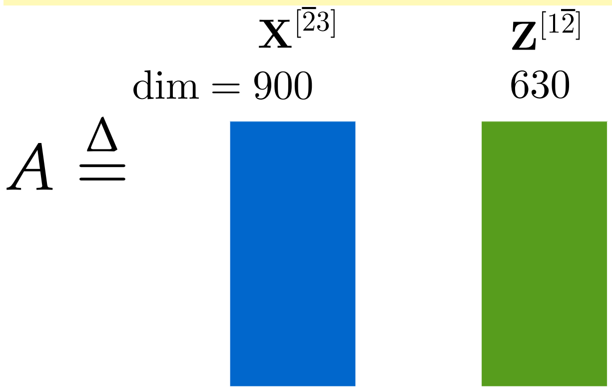
Is The Hybrid Scheme Optimal?

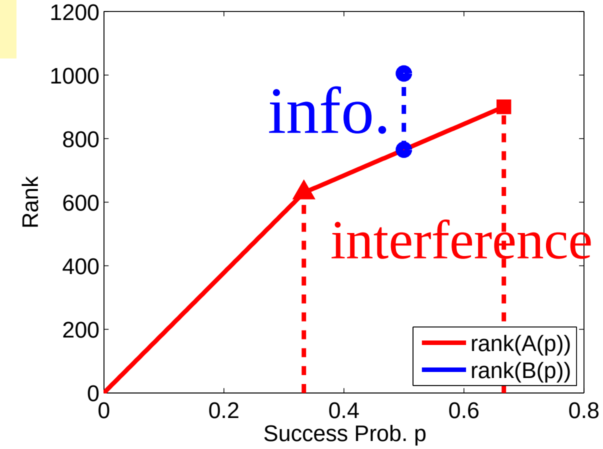


Any network code.



Can we finish tx in <2010





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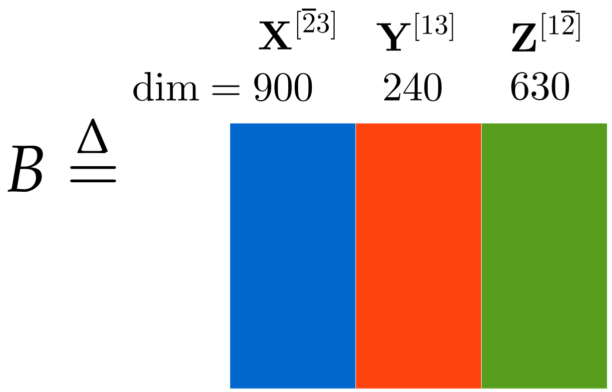
Concavity of information transmission.



Total time slots at d_2:

$L \cdot \frac{1}{2} \geq \text{Rank}(B(1/2)) \geq 1005$

$\Rightarrow L \geq 2010.$

info. interference.

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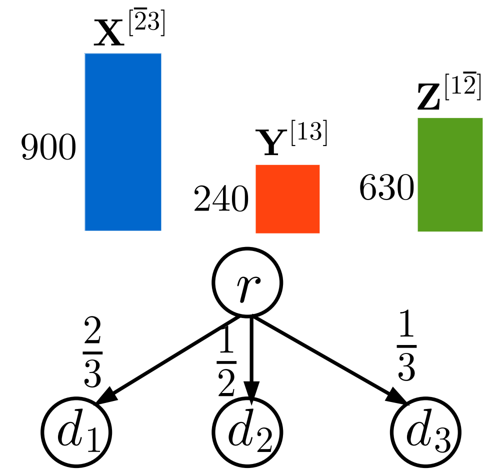
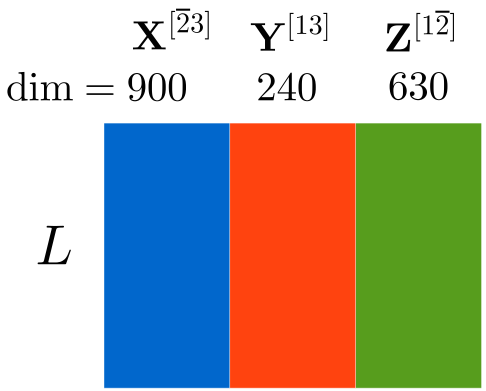
Is The Hybrid Scheme Optimal?



Any network code.



Can we finish tx in <2010

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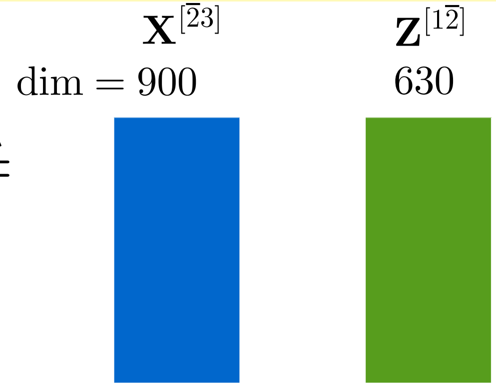
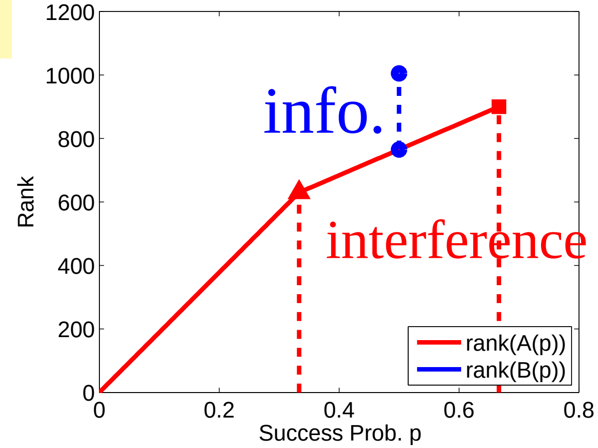




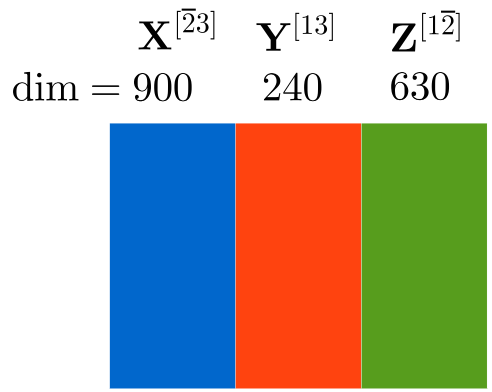
Decodability at $d_1 \Rightarrow$ Ran

The same arguments hold for non-linear codes as well.

Decodability at $d_3 \Rightarrow \text{Rank}(A(1/3)) \geq 630.$

Concavity of information transmission.

$B \triangleq$



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info. interference.

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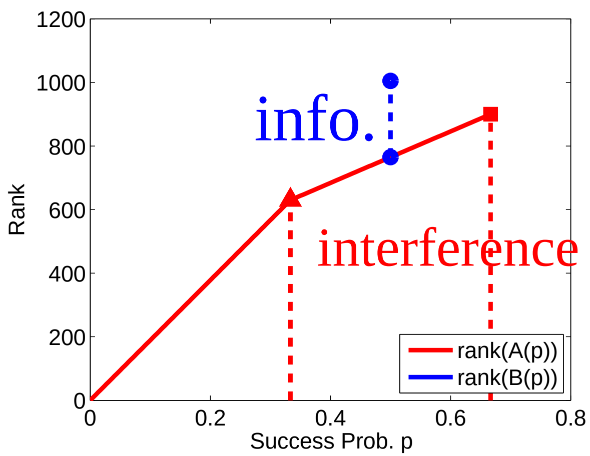
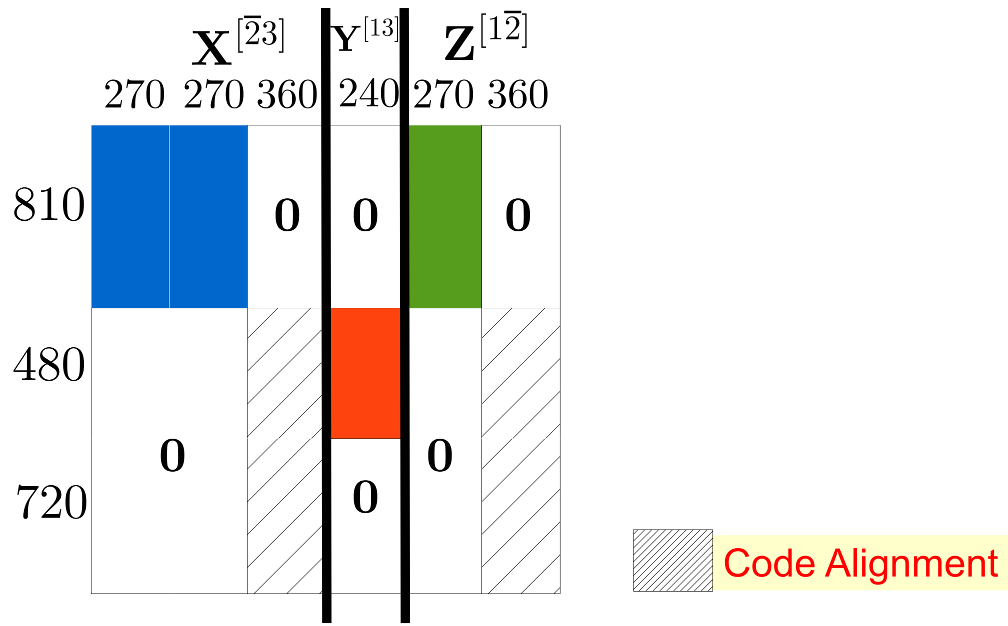
M-Session Cap. Region

- Outer bound: Interference quantification + information concavity
- Inner bound: Hybrid schemes with stage-based approaches + code alignment.





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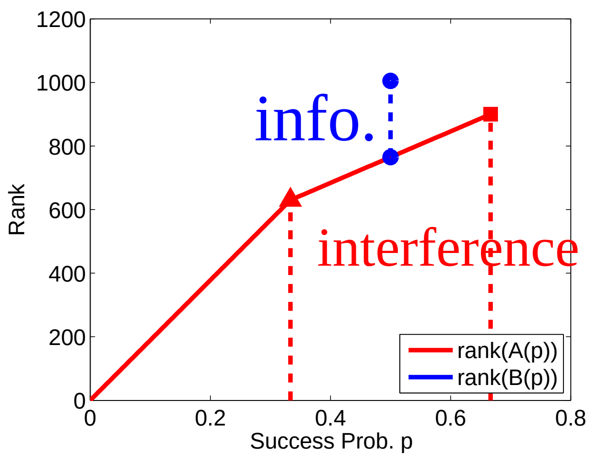
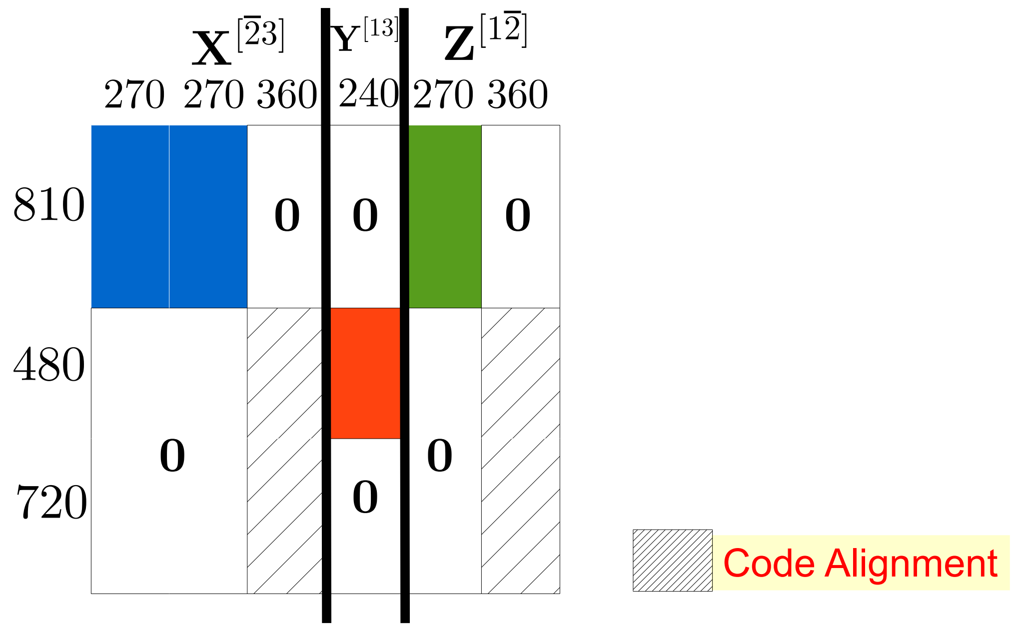
- $M = 3$: It is proven that the outer and inner bounds always meet $\Rightarrow$ capacity.





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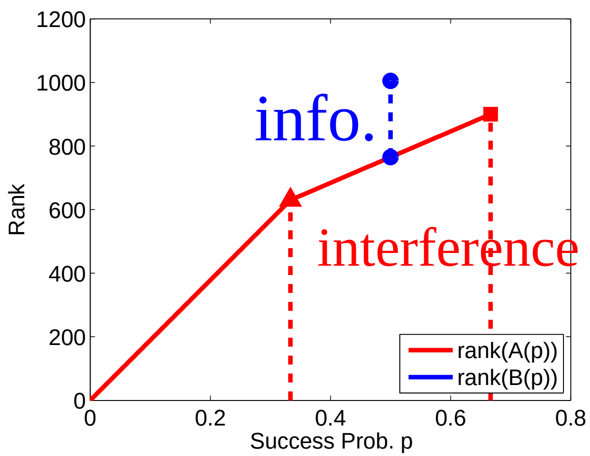

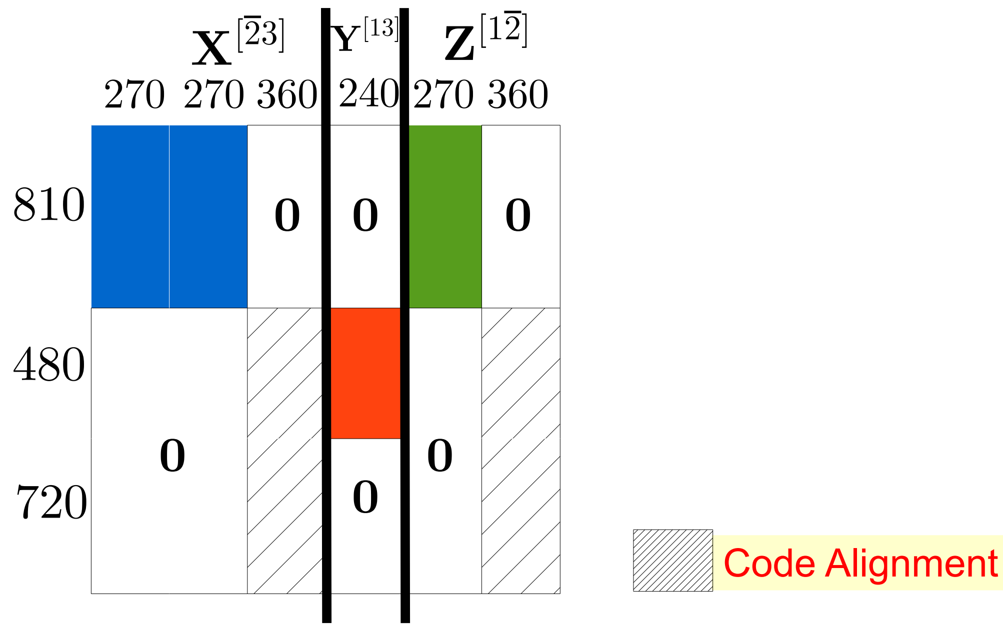

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$$R_1 \leq \min(p_{s_1;r}, p_{r;d_1} - \max\left((R_2 - p_{s_2;d_1})^+ + (R_3 - p_{s_3;d_1\cup d_2})^+, (R_2 - p_{s_2;d_1\cup d_3})^+ + (R_3 - p_{s_3;d_1})^+\right)),$$

$$R_2 \leq \min(p_{s_2;r}, p_{r;d_2} - \max\Bigg(\frac{p_{r;d_2}}{p_{r;d_1}}\left((R_1 - p_{s_1;d_2})^+ + (R_3 - p_{s_3;d_1\cup d_2})^+\right),$$

$$\left(\frac{p_{r;d_2}}{p_{r;d_1}} - \frac{p_{r;d_2} - p_{r;d_3}}{p_{r;d_1} - p_{r;d_3}}\right)(R_1 - p_{s_1;d_2\cup d_3})^+ + \frac{p_{r;d_2} - p_{r;d_3}}{p_{r;d_1} - p_{r;d_3}}(R_1 - p_{s_1;d_2})^+$$

$$+\left(1 - \frac{p_{r;d_1} - p_{r;d_2}}{p_{r;d_1} - p_{r;d_3}}\right)(R_3 - p_{s_3;d_1\cup d_2})^+ + \frac{p_{r;d_1} - p_{r;d_2}}{p_{r;d_1} - p_{r;d_3}}(R_3 - p_{s_3;d_2})^+,$$

$$\frac{p_{r;d_2}}{p_{r;d_1}}(R_1 - p_{s_1;d_2\cup d_3})^+ + (R_3 - p_{s_3;d_2})^+\Bigg)),$$

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$$\frac{p_{r;d_3}}{p_{r;d_1}}(R_1 - p_{s_1;d_2\cup d_3})^+ + \frac{p_{r;d_3}}{p_{r;d_2}}(R_2 - p_{s_2;d_3})^+\Bigg)).$$

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The capacity region is governed by linear inequalities.

$$+\left(1 - \frac{p_{r;d_1} - p_{r;d_2}}{p_{r;d_1} - p_{r;d_3}}\right)(R_3 - p_{s_3;d_1 \cup d_2})^+ + \frac{p_{r;d_1} - p_{r;d_2}}{p_{r;d_1} - p_{r;d_3}}(R_3 - p_{s_3;d_2})^+,$$

$$\left.\frac{p_{r;d_2}}{p_{r;d_1}}(R_1 - p_{s_1;d_2 \cup d_3})^+ + (R_3 - p_{s_3;d_2})^+\right)),$$

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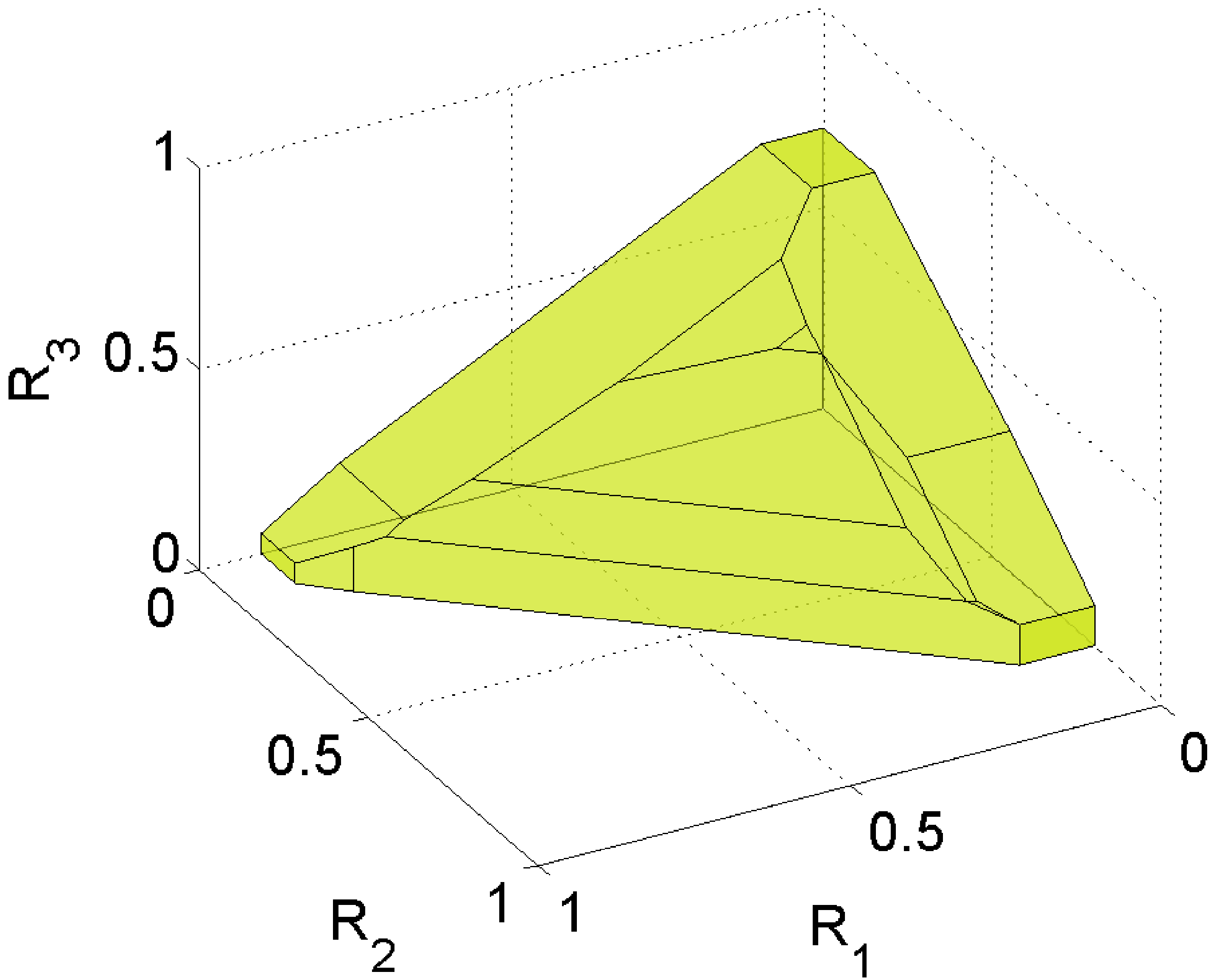
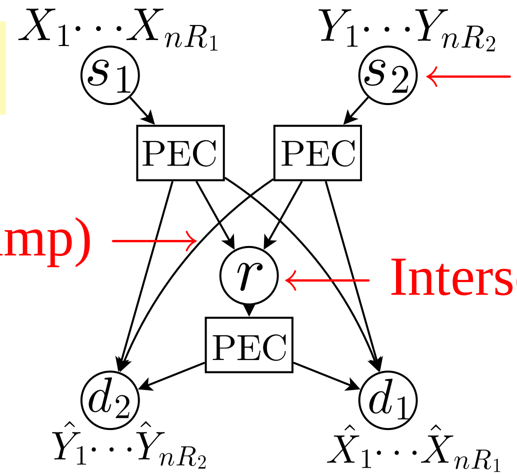
$$\left.\frac{p_{r;d_3}}{p_{r;d_1}}(R_1 - p_{s_1;d_2 \cup d_3})^+ + \frac{p_{r;d_3}}{p_{r;d_2}}(R_2 - p_{s_2;d_3})^+\right)).$$

A 3-User Cap. Illustration



The Throughput Improvements

Competing technologies:



Cross-layer (time allocation).

Opp. Routing (direct-jump)

Intersession NC (Mixing at r).

The Throughput Improvements

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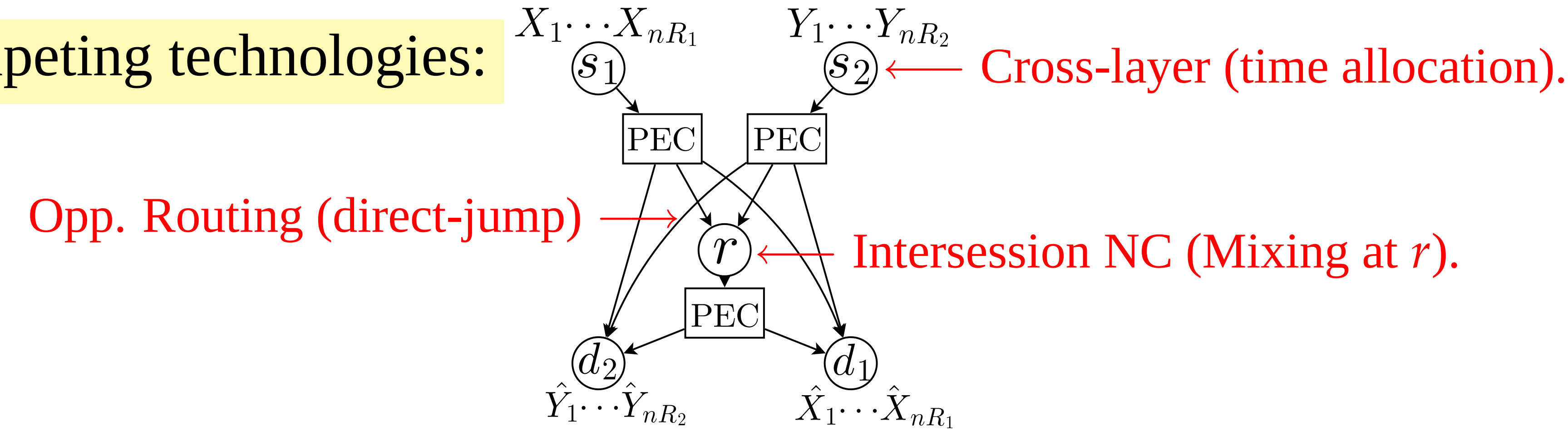
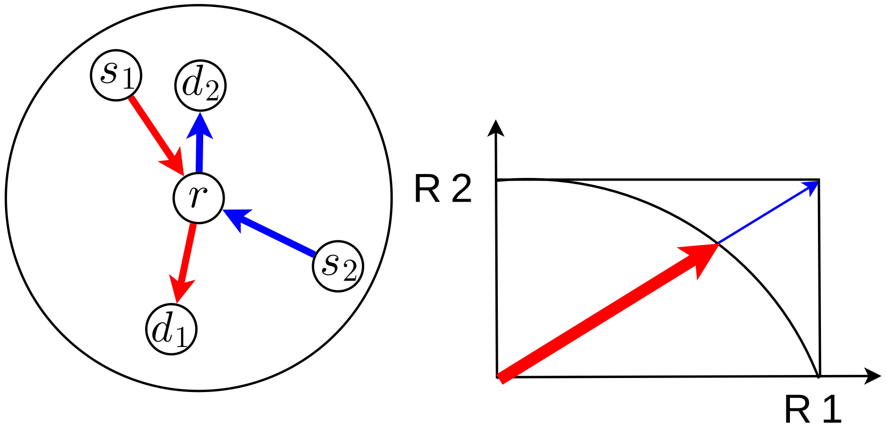
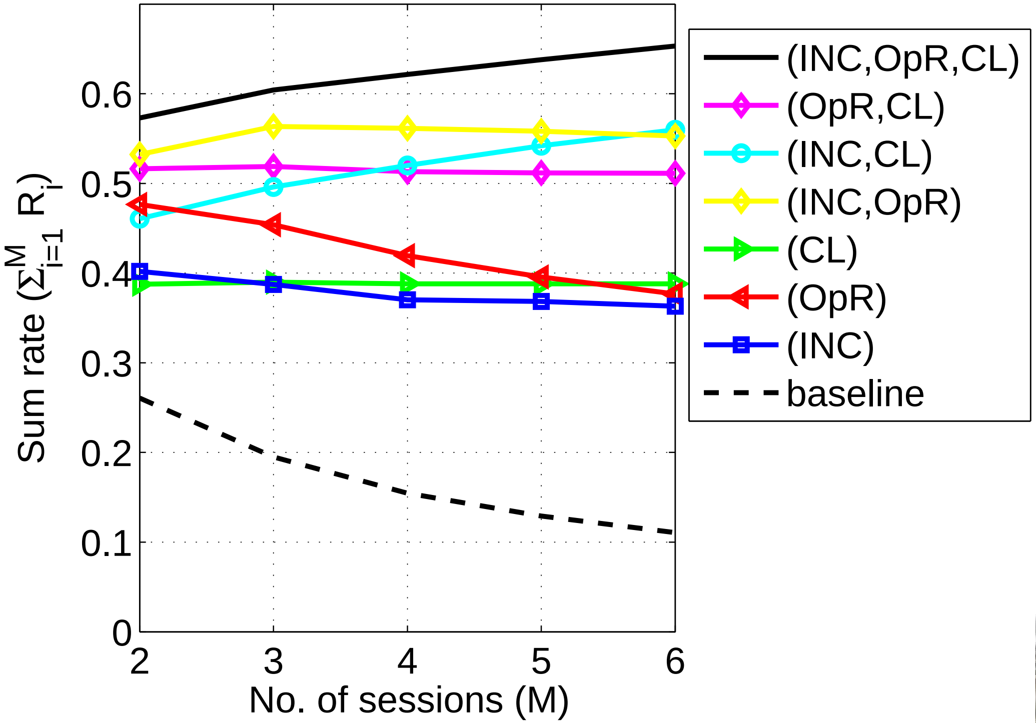


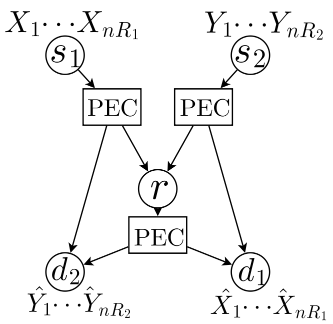
2-hop random networks, Rayleigh fading, proportional fairness.

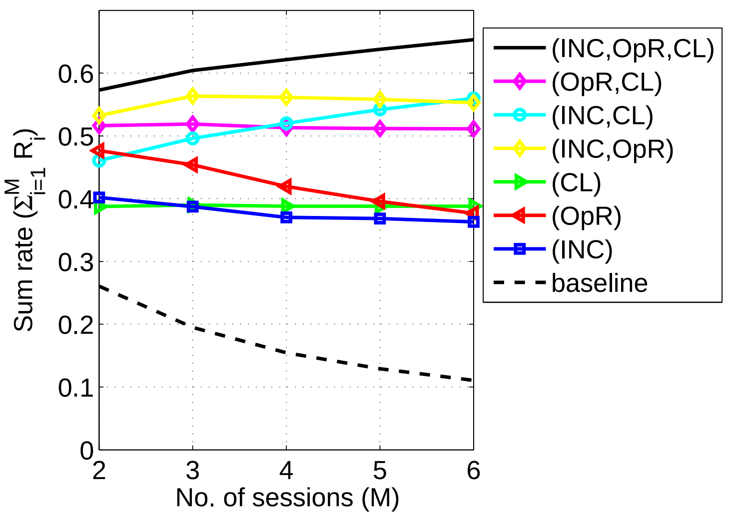




Conclusion + Future Directions

- Exact capacity characterization for $M = 3$.
- Numerically tight outer/inner bounds for $M \geq 4$.
- The promised throughput of the COPE principle.





COPE		Our Setting
Listening/Overhearing	=	Memoryless Broadcast Packet Erasure Channels
XOR coding	<	Arbitrary ENC/DEC functions, GF(2^b)
Opportunistic	<	Sufficiently large n
Learning neighbors' states	<	Knowledge about the exact overhearing patterns
Periodic feedback	>	One-time feedback before tx of the relay Remark 1: Feedback increases the capacity [Ozarow et al. 84], [Xue et al. 08], [Georgiadis et al. 09]. Remark 2: Improvement is small when the number of receivers is small [W, Allerton 10].

- The code-alignment-based achievability scheme sheds insights on how to improve the COPE protocol.