Common Information of Random Linear Network Coding Over A 1-Hop Broadcast Packet Erasure Channel

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Motivation — The COPE Principle

- The COPE protocol — 2-hop relay networks [Katti *et al.* 06]

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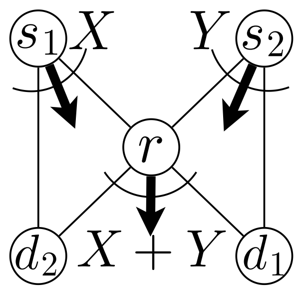

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  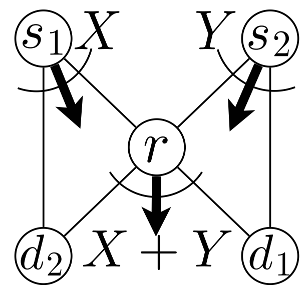

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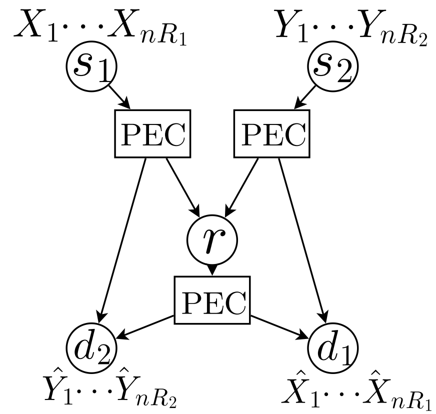
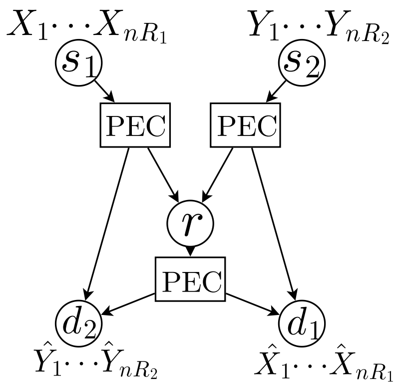
- The capacity can be defined over the corresponding PEC network.



The Capacity Region for $M = 2$



- Assume round-based schemes, and $p_{r;d_1} \geq p_{r;d_2} > 0$.

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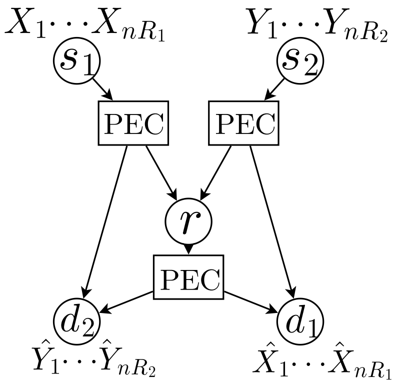

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- The capacity region for $M = 2$, [W, Asilomar 09]:

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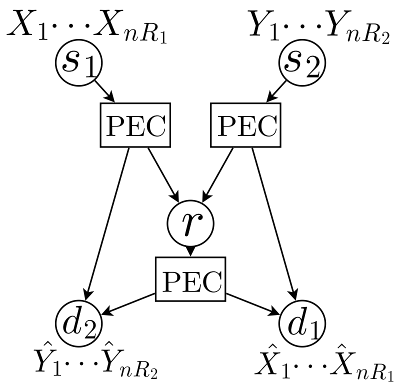

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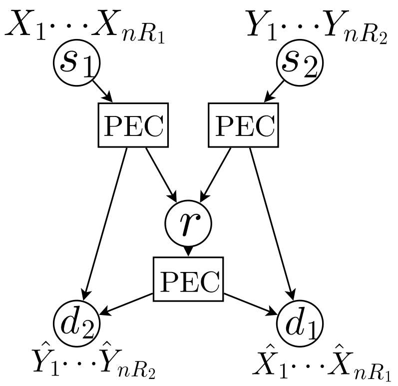

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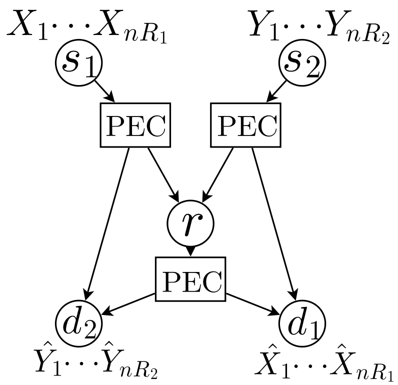

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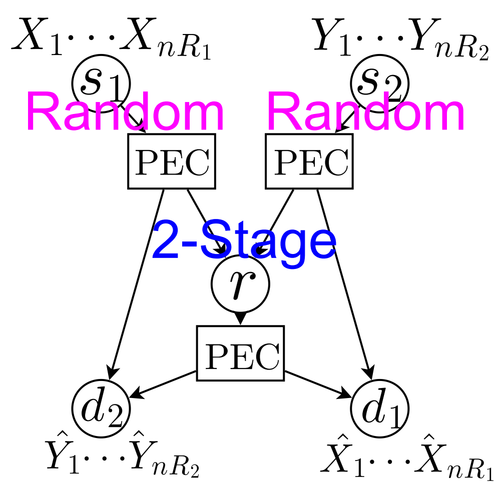

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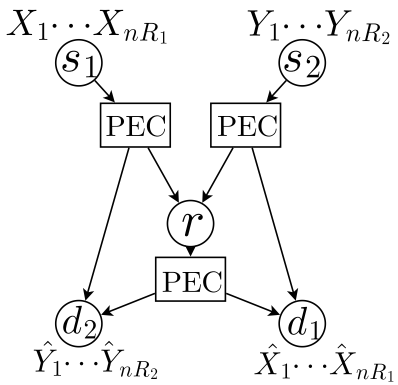

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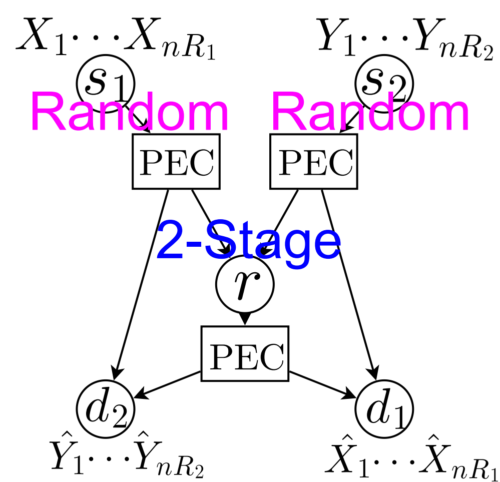

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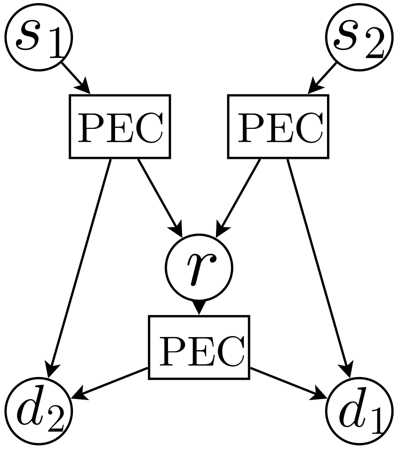

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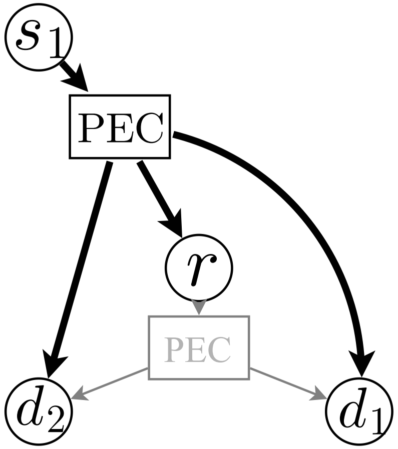

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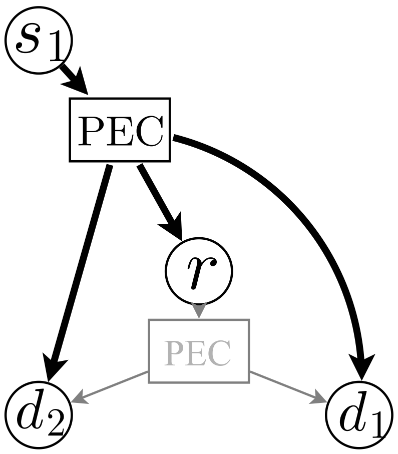

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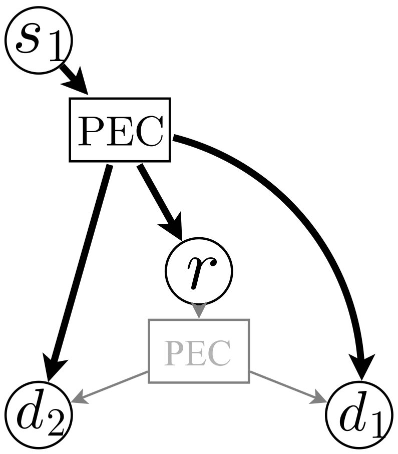

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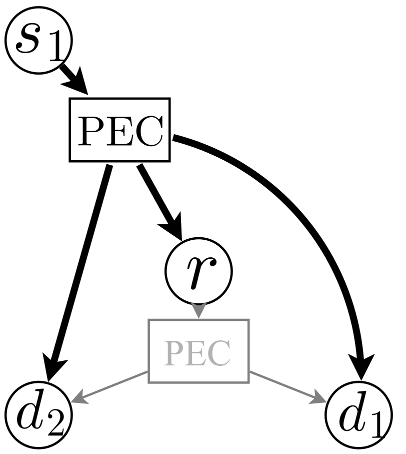

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Can we combine the benefits of Network Coding & Opportunistic Routing?

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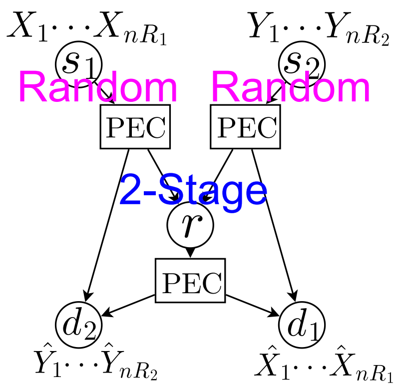

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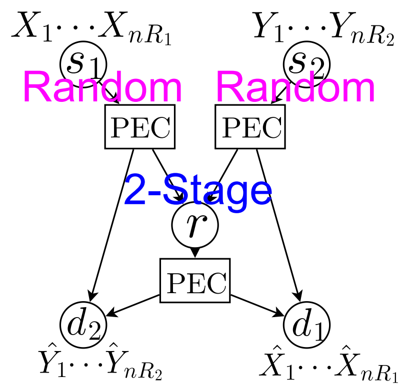

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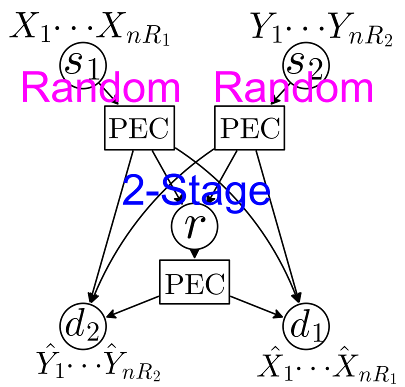

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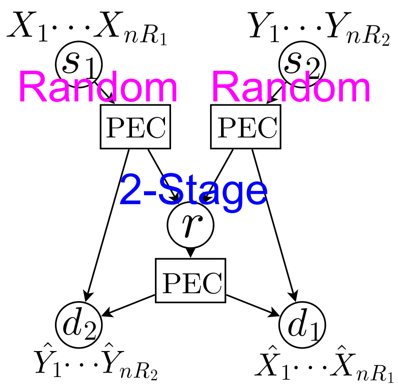

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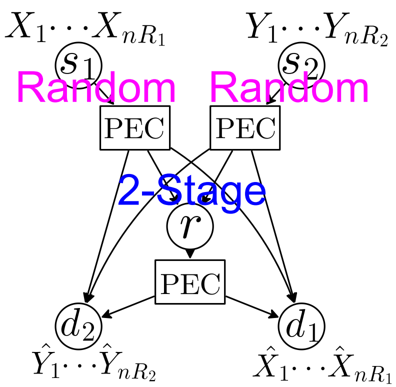

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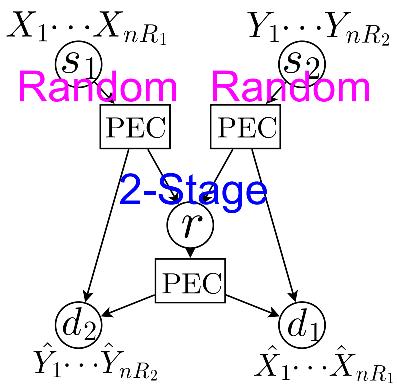

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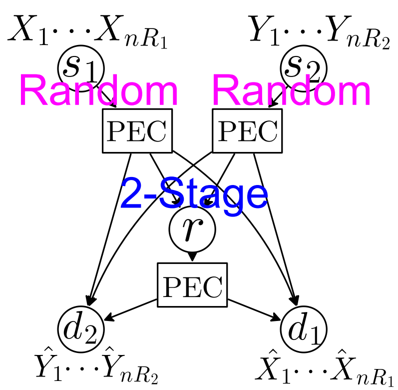

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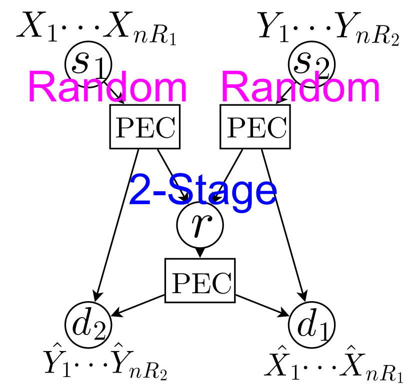

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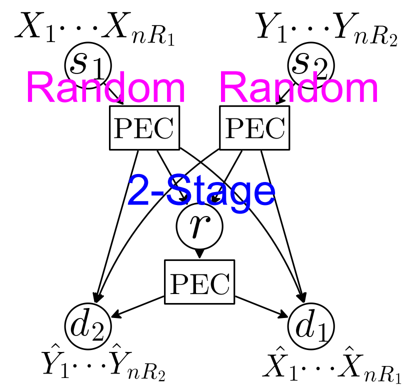

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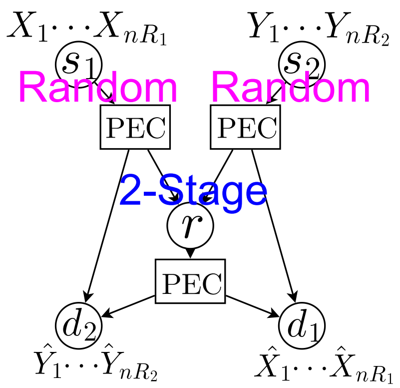

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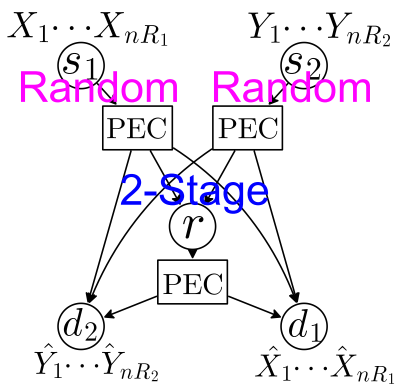

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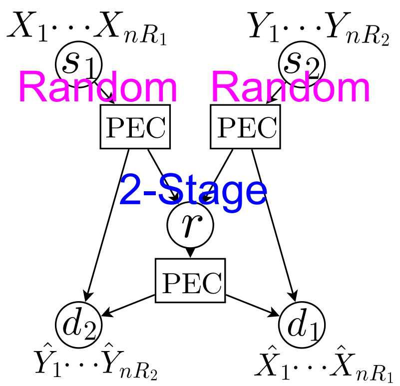

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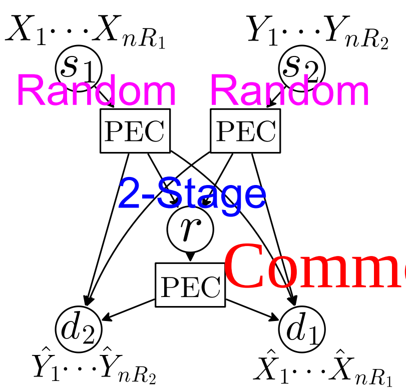

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With this motivation, this work studies the Common Info. of Random Linear Network Coding.

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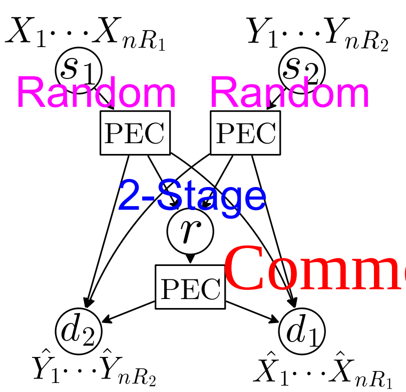

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Settings and Definitions

- Random Linear Network Coding (RLNC)
  - N packets: $\mathbf{W} \triangleq (W_1, \cdots, W_N) \in (\mathsf{GF}(q))^N$.
  - Each time t, source sends $Y_t = \mathbf{v}_t \mathbf{W}^{\mathrm{T}}$ via an erasure CH.
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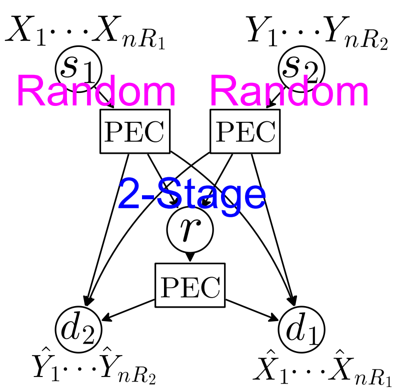
- Other notations: $\mathcal{T}_S \triangleq |\{\forall t : S \subseteq \mathcal{R}_t|$: The amount of time when all d_k in S receive the pkt; $A \oplus B = \mathsf{span}(\mathbf{v} : \mathbf{v} \in A \cup B)$.

- Our goal: Quantifying rank $\left(\bigcap_{k=1}^{K}\Omega_k\right)$ for all combinations of K, N, and $\{\mathcal{R}_t : \forall t\}$ assuming large GF(q).



The total # of to-be-sent symbols:

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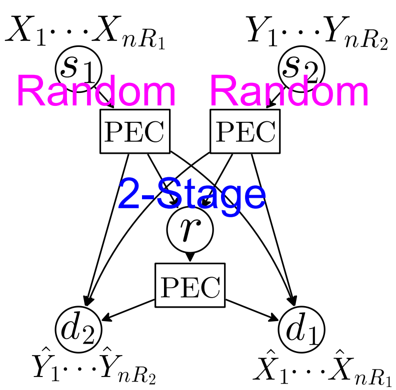

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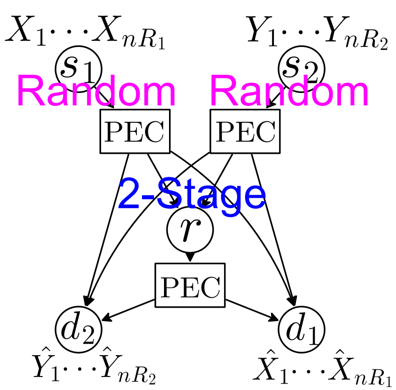

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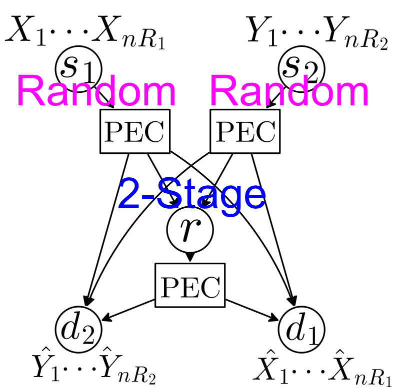

The total # of to-be-sent symbols:

$$\text{rank}\left(\Omega_r^{[s_1]}\right) - \text{rank}\left(\Omega_r^{[s_1]} \cap \Omega_{d_1}^{[s_1]}\right)$$

The overheard info.:

$$\text{rank}\left(\Omega_r^{[s_1]} \cap \Omega_{d_2}^{[s_1]}\right) -$$

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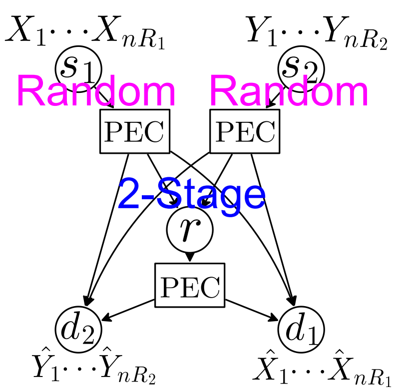

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The Main Challenge

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$$\begin{aligned}\text{rank}\,(\Omega_1 \cap \Omega_2) &= \text{rank}\,(\Omega_1) + \text{rank}\,(\Omega_2) - \text{rank}\,(\Omega_1 \oplus \Omega_2) \\ &= \min(\mathcal{T}_1, N) + \min(\mathcal{T}_2, N) - \min(\mathcal{T}_1 + \mathcal{T}_2 - \mathcal{T}_{\{1,2\}}, N)\end{aligned}$$

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- Our first thought was when $K = 3$, we should have

$$\text{rank}\left(\bigcap_{k=1}^{3}\Omega_k\right) = \sum_{k=1}^{K}\text{rank}\left(\Omega_k\right) - \sum_{i<j}\text{rank}\left(\Omega_i \oplus \Omega_j\right) + \text{rank}\left(\Omega_1 \oplus \Omega_2 \oplus \Omega_3\right)$$

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DOES NOT HOLD!!

An example is provided in the paper.

Main Result

- A *partition* of $\{1, \cdots, K\}$ is a collection of disjoint subsets $\{S_m\} \triangleq \{S_1, S_2, \cdots, S_M\}$ such that $\bigcup_{m=1}^{M} S_m = \{1, \cdots, K\}$.

Theorem 1 *Define* $(\cdot)^+ \triangleq \max(\cdot, 0)$. *For any receiving sets* $\{\mathcal{R}_t : \forall t\}$, *with sufficiently large* $\mathsf{GF}(q)$ *we have*

$$\mathsf{rank}\left(\bigcap_{k=1}^{K} \Omega_k\right) = \max\left\{N - \sum_{m=1}^{M} (N - \mathcal{T}_{S_m})^+ : \forall \textit{ partition } \{S_m\}\right\}.$$

Proof of A Simplified Example

- We prove a degenerate case in this presentation. Assume

$$N \geq \max_{k \in [K]} \mathcal{T}_k$$

$$\forall S \subseteq [K], \left(\sum_{k \in S} \mathcal{T}_k \right) - (|S| - 1)N \geq \mathcal{T}_S.$$

- We will prove that $\mathsf{rank}\left(\bigcap_{k=1}^{K} \Omega_k\right) = \sum_{k=1}^{K} \mathcal{T}_k - (K-1)N.$

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An illustrative example w. $K = 3$:

- $N = 5$ dimensional vector space.
- 7 vectors with reception status 001, 010, 011, 100, 101, 110, 111.
- $\mathcal{T}_1 = \mathcal{T}_2 = \mathcal{T}_3 = 4$, $\mathcal{T}_{\{1,2\}} = \mathcal{T}_{\{1,3\}} = \mathcal{T}_{\{2,3\}} = 2$, and $\mathcal{T}_{\{1,2,3\}} = 1$.

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An illustrative example w. $K = 3$: We will prove $\text{rank}\left(\bigcap_{k=1}^{3} \Omega_k\right) = 4 + 4 + 4 - (3-1) \cdot 5 = 2.$

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- 7 vectors with reception status 001, 010, 011, 100, 101, 110, 111.
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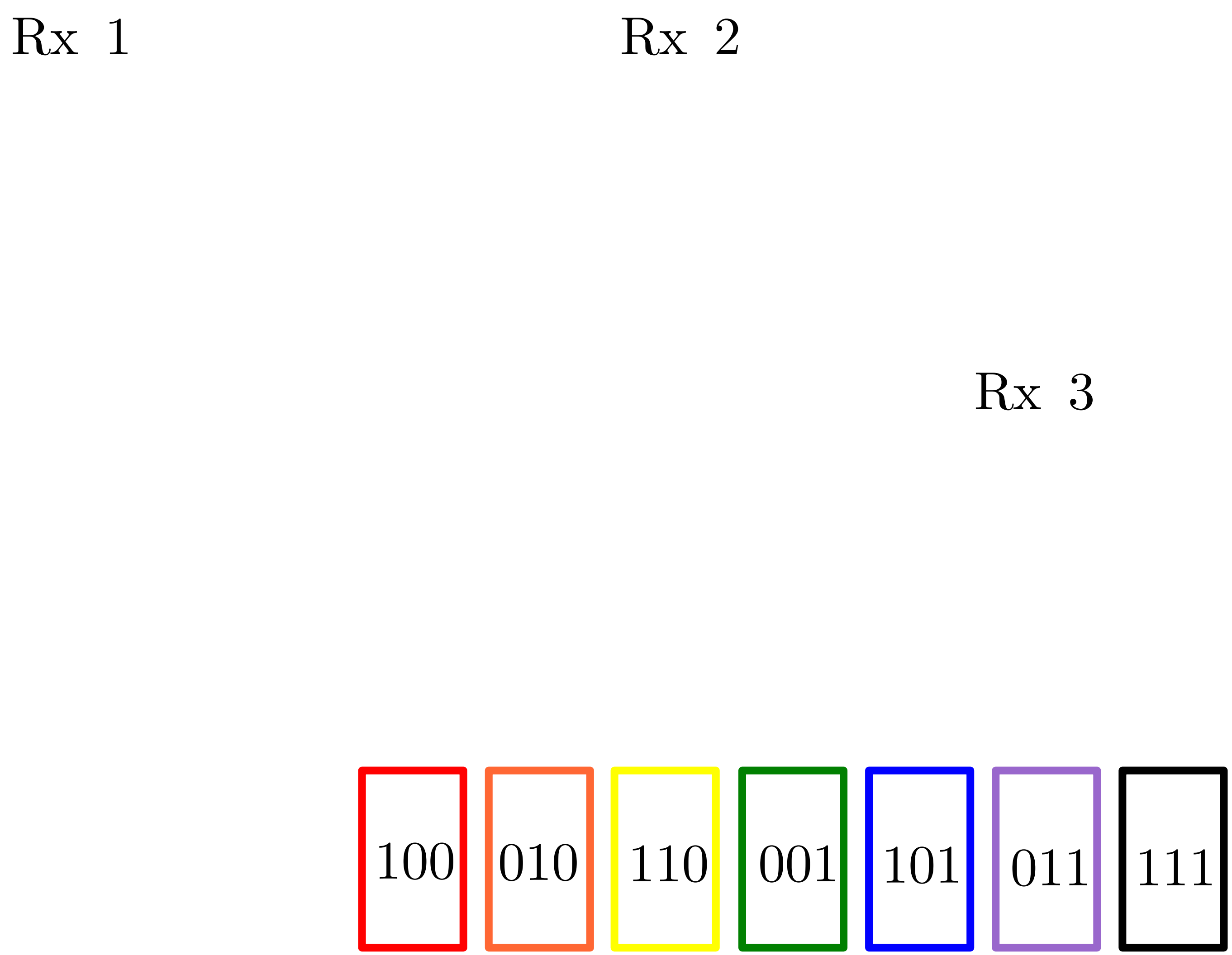
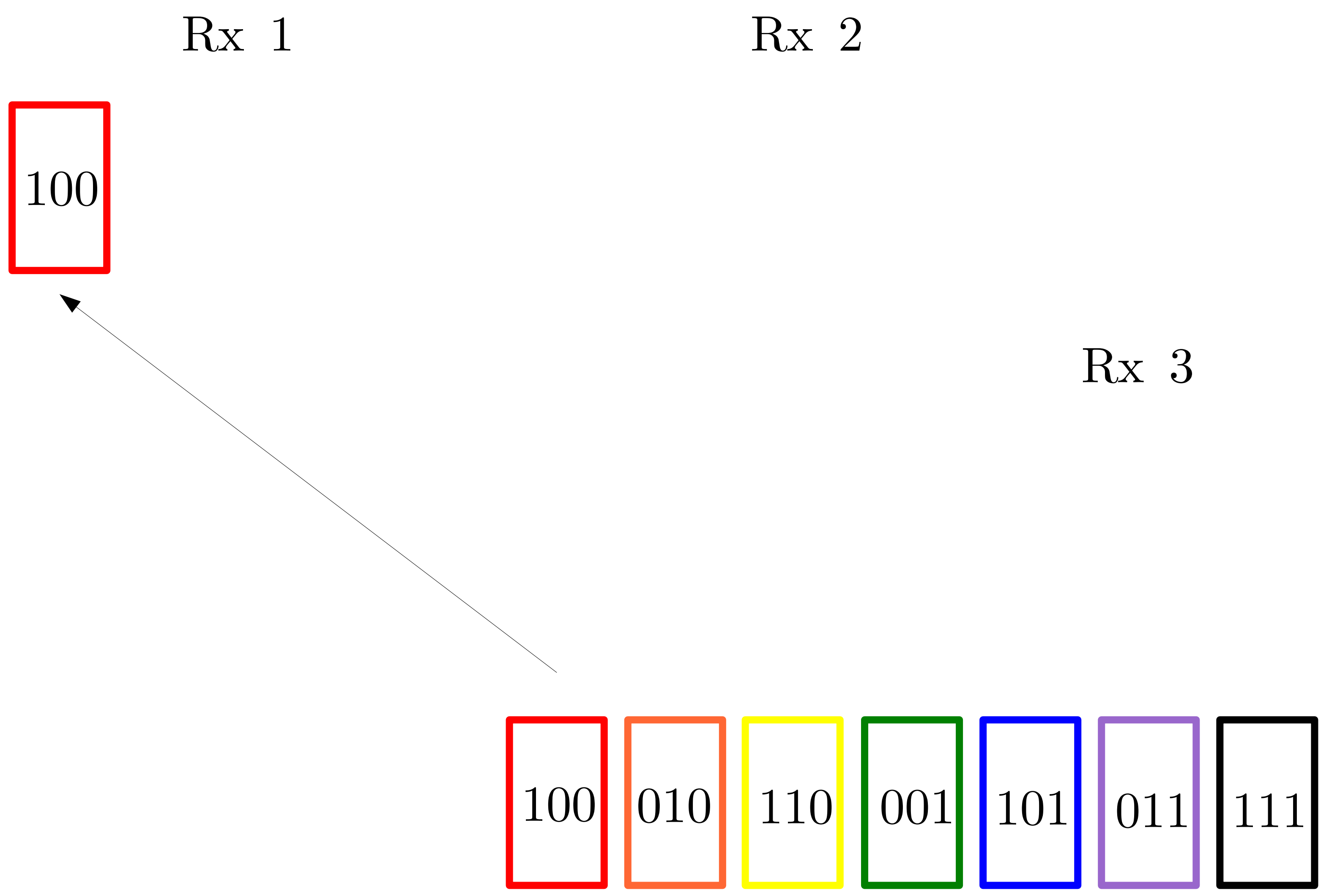
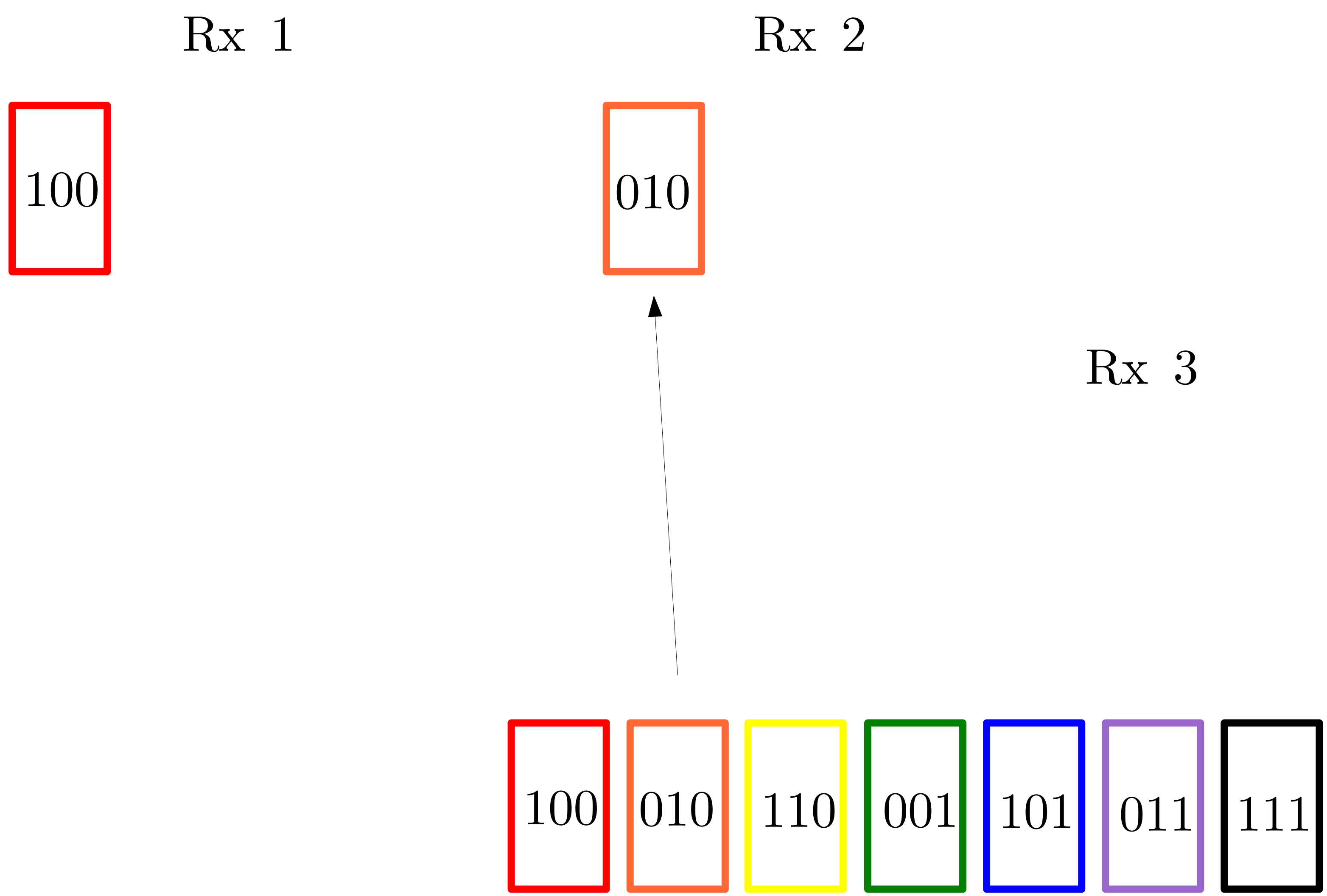
Proof (Cont'd)



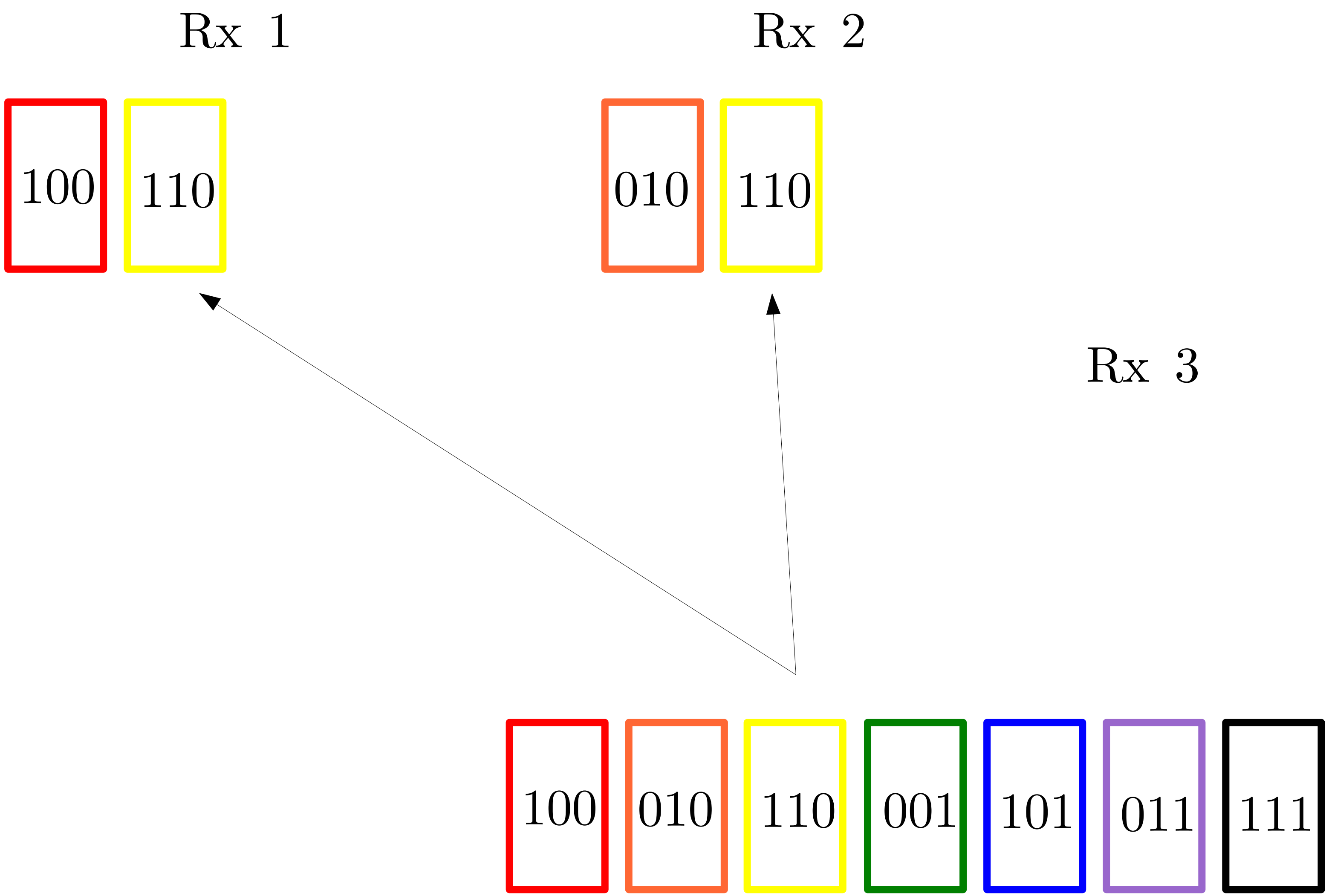
Proof (Cont'd)



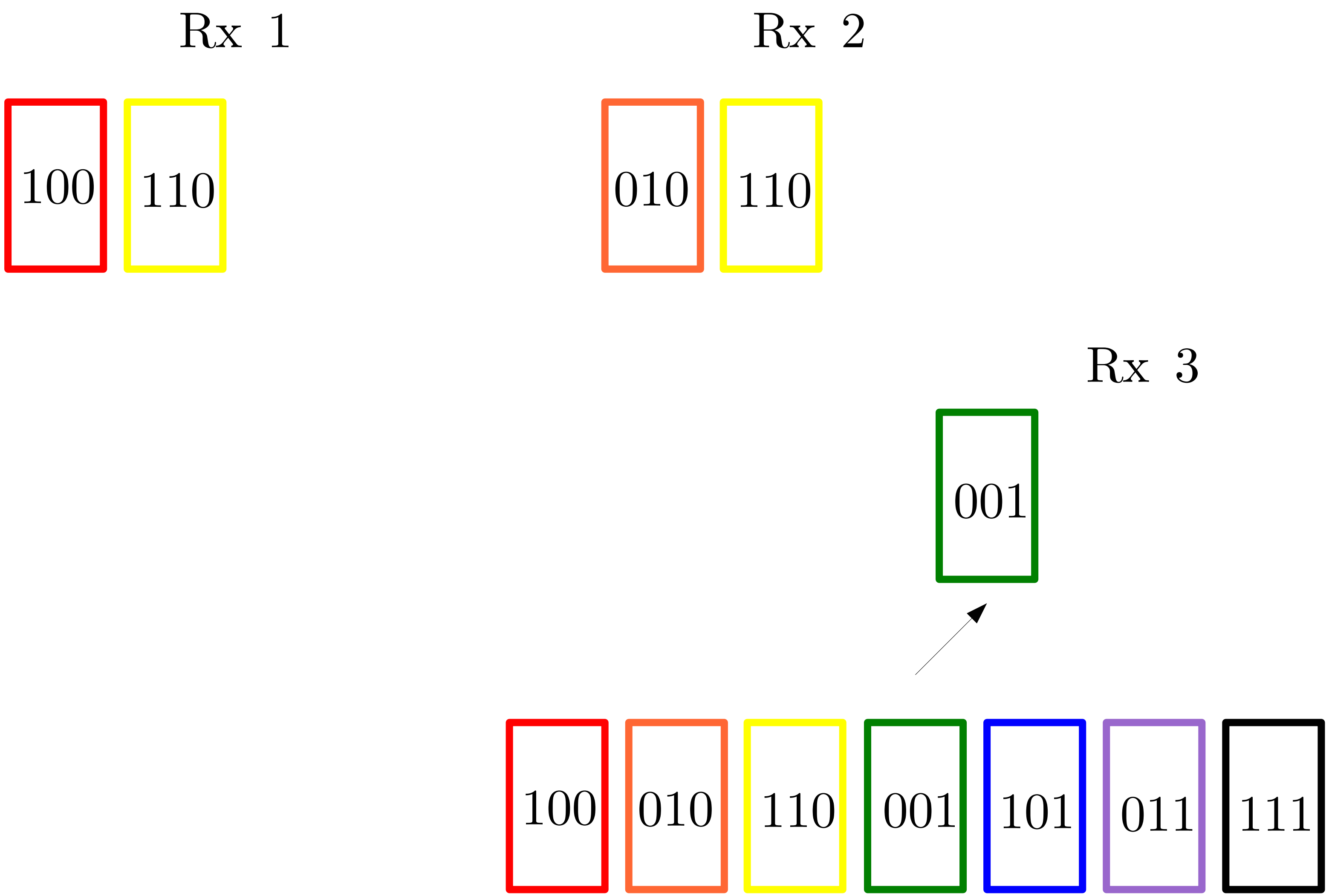
Proof (Cont'd)



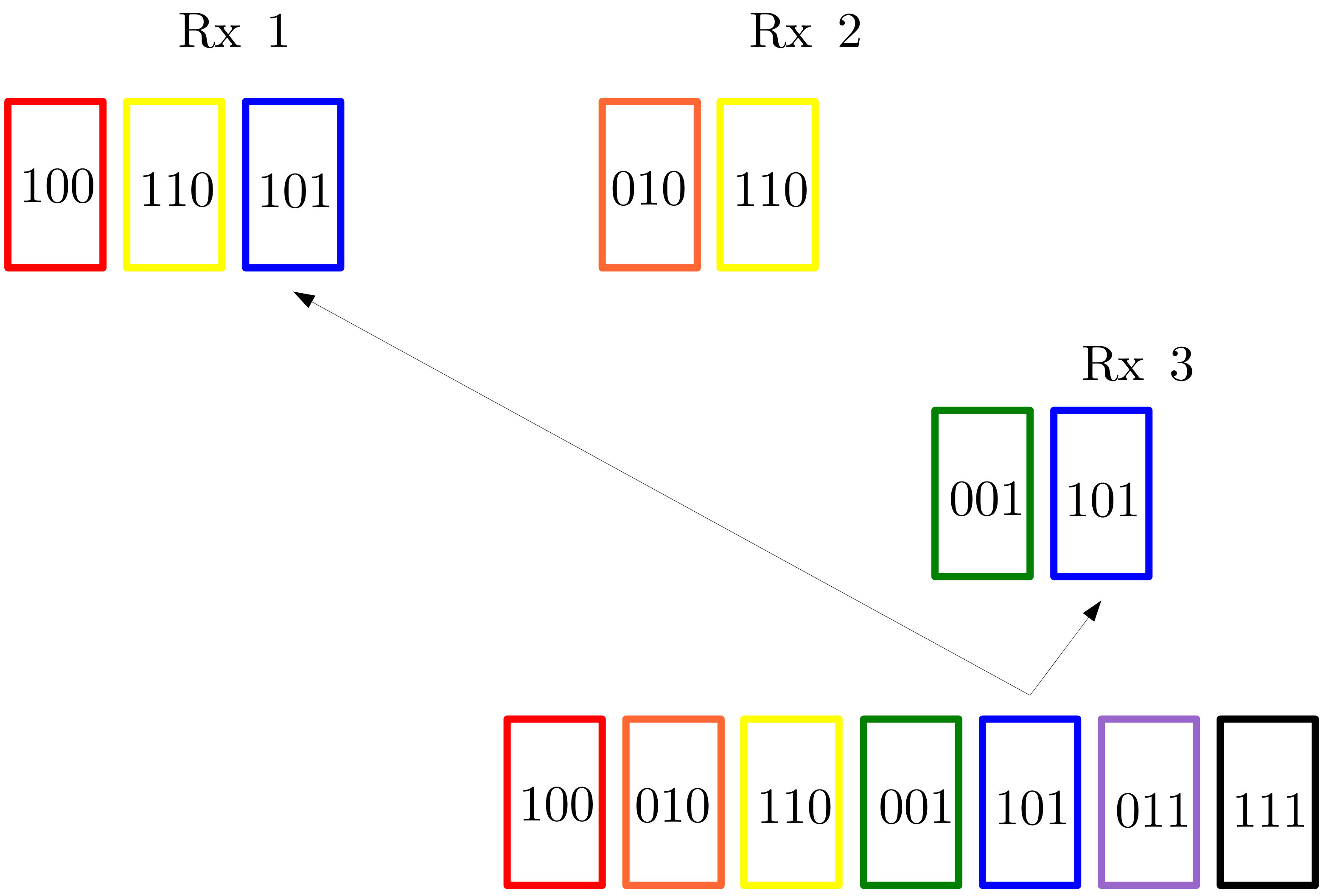
Proof (Cont'd)



Proof (Cont'd)

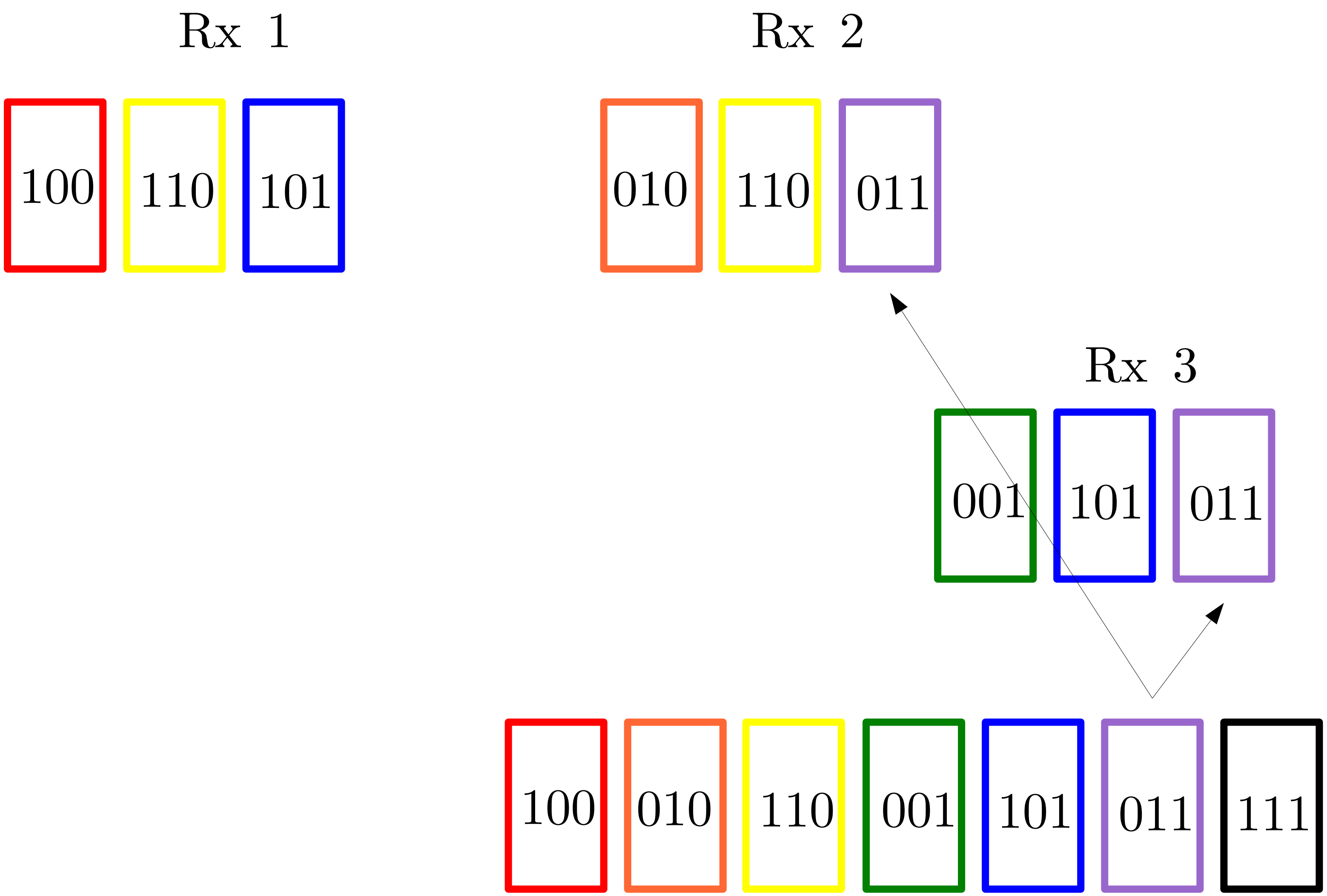


Proof (Cont'd)



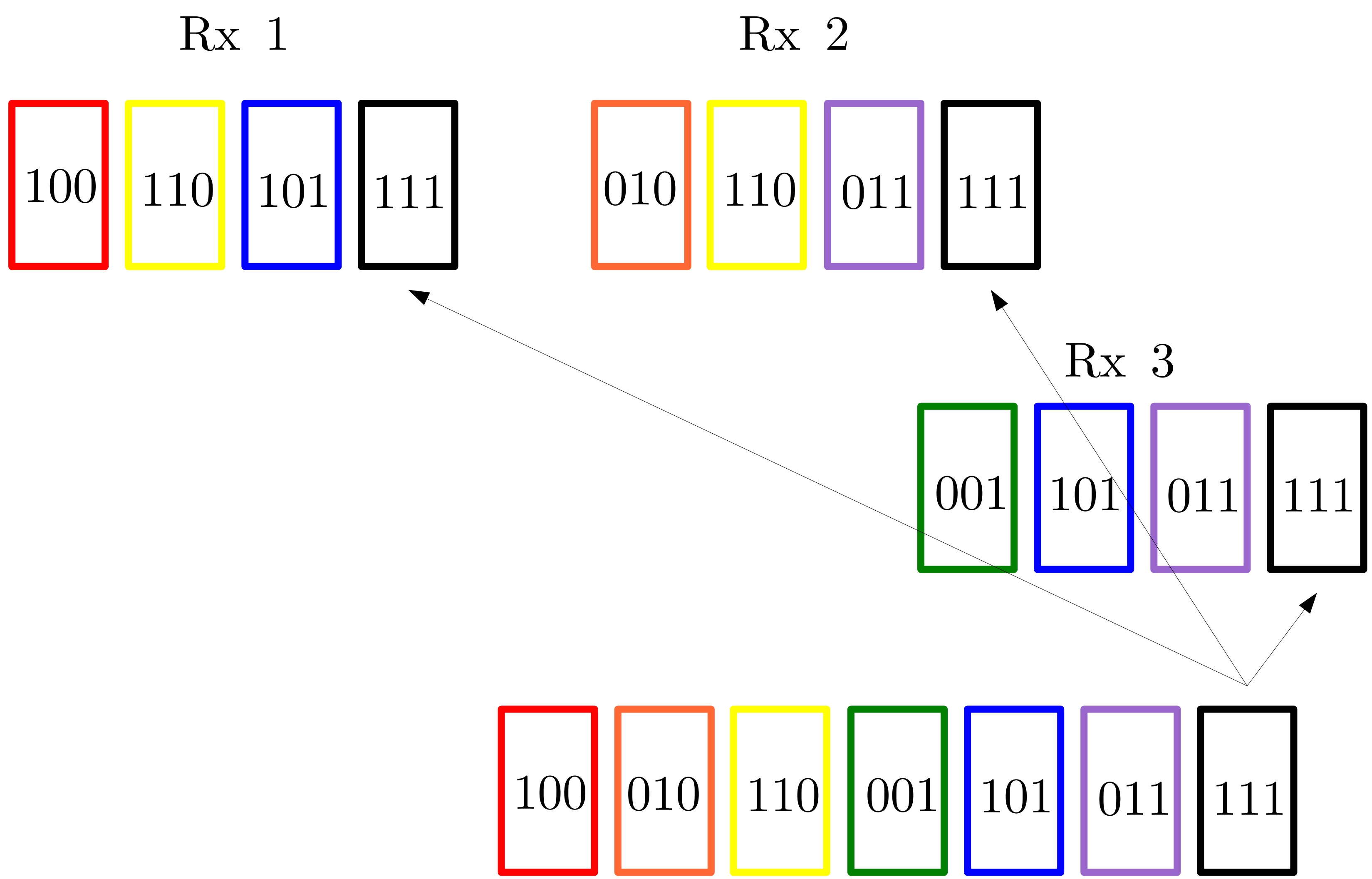
Rx 1
100
110
101
Rx 2
010
110
Rx 3
001
101
100
010
110
001
101
011
111

Proof (Cont'd)



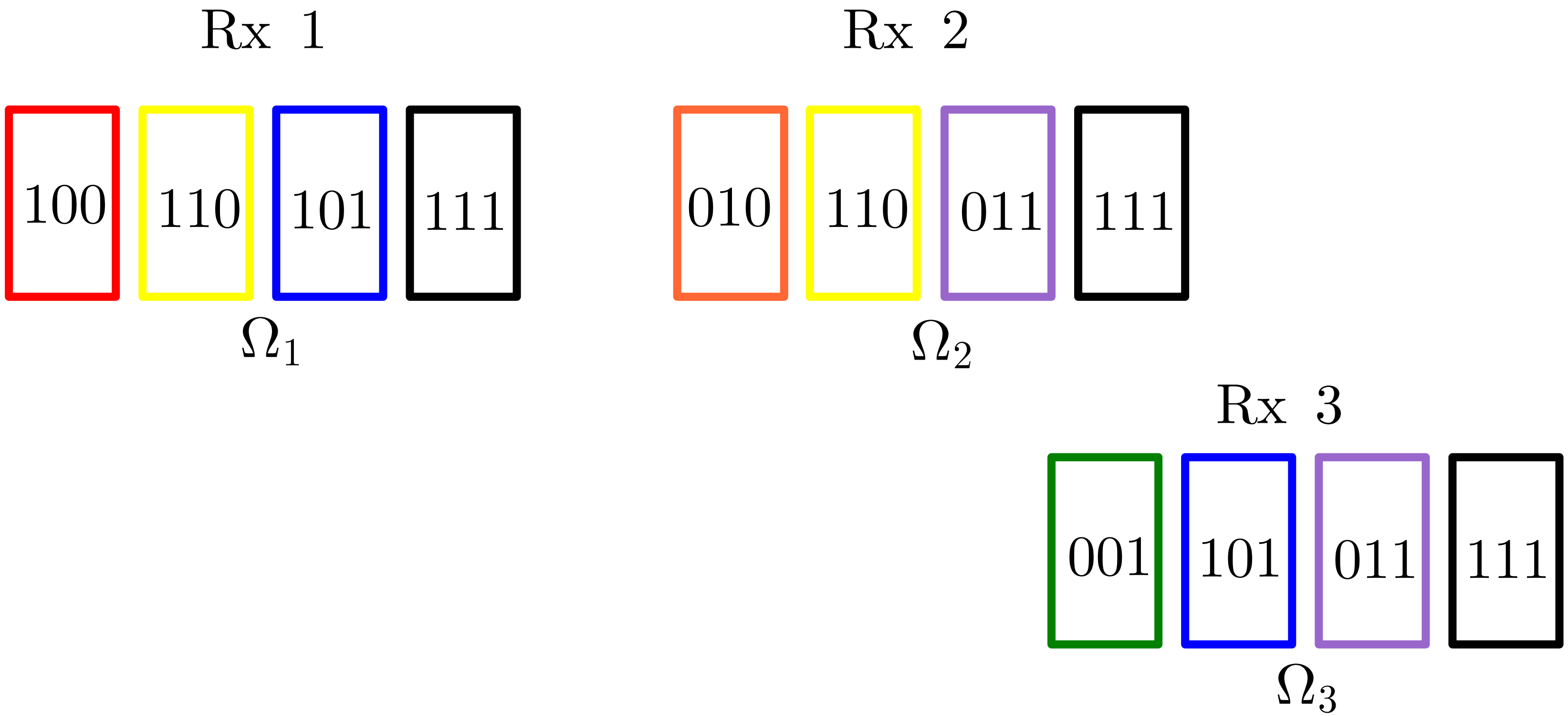
Rx 1
100
110
101
Rx 2
010
110
011
Rx 3
001
101
011
100
010
110
001
101
011
111

Proof (Cont'd)

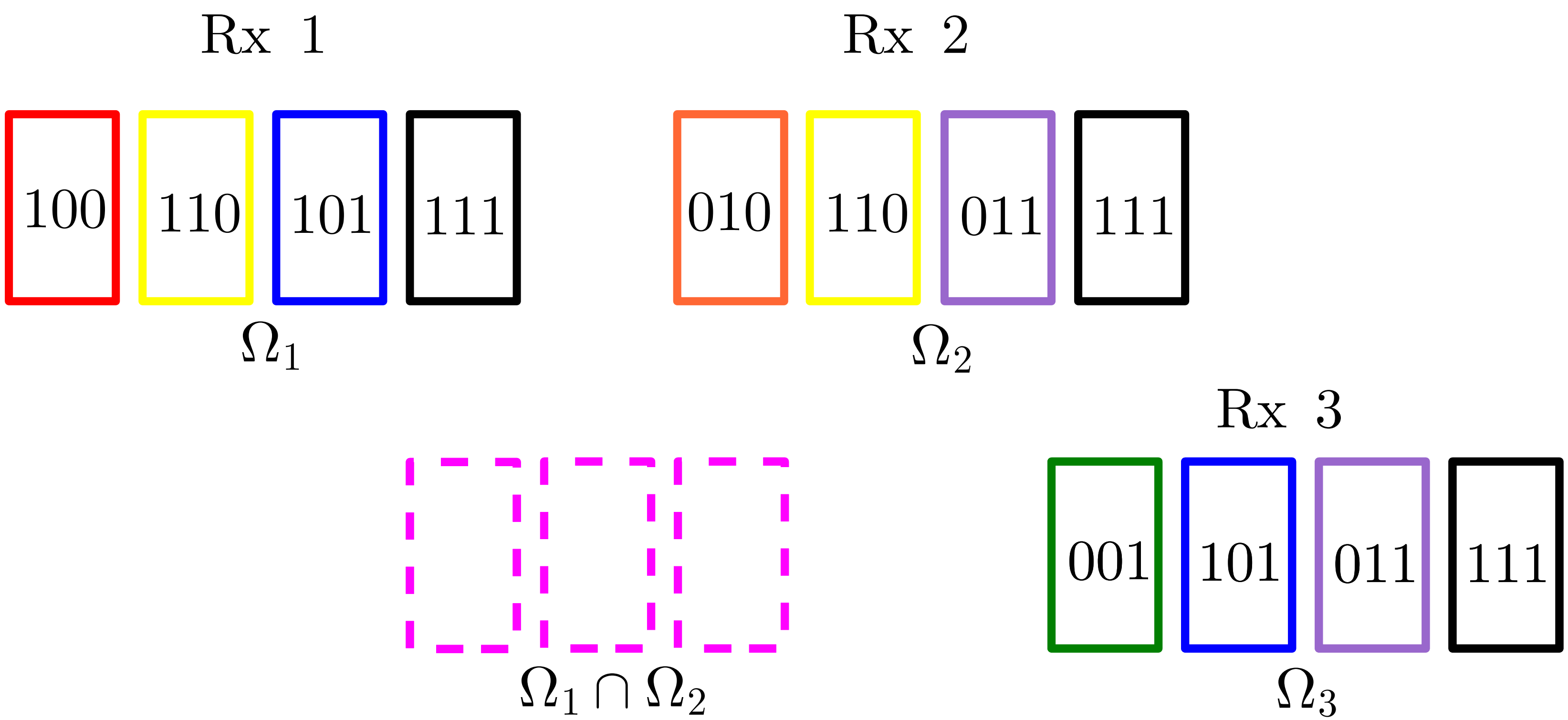


Rx 1
100
110
101
111
Rx 2
010
110
011
111
Rx 3
001
101
011
111
100
010
110
001
101
011
111

Proof (Cont'd)

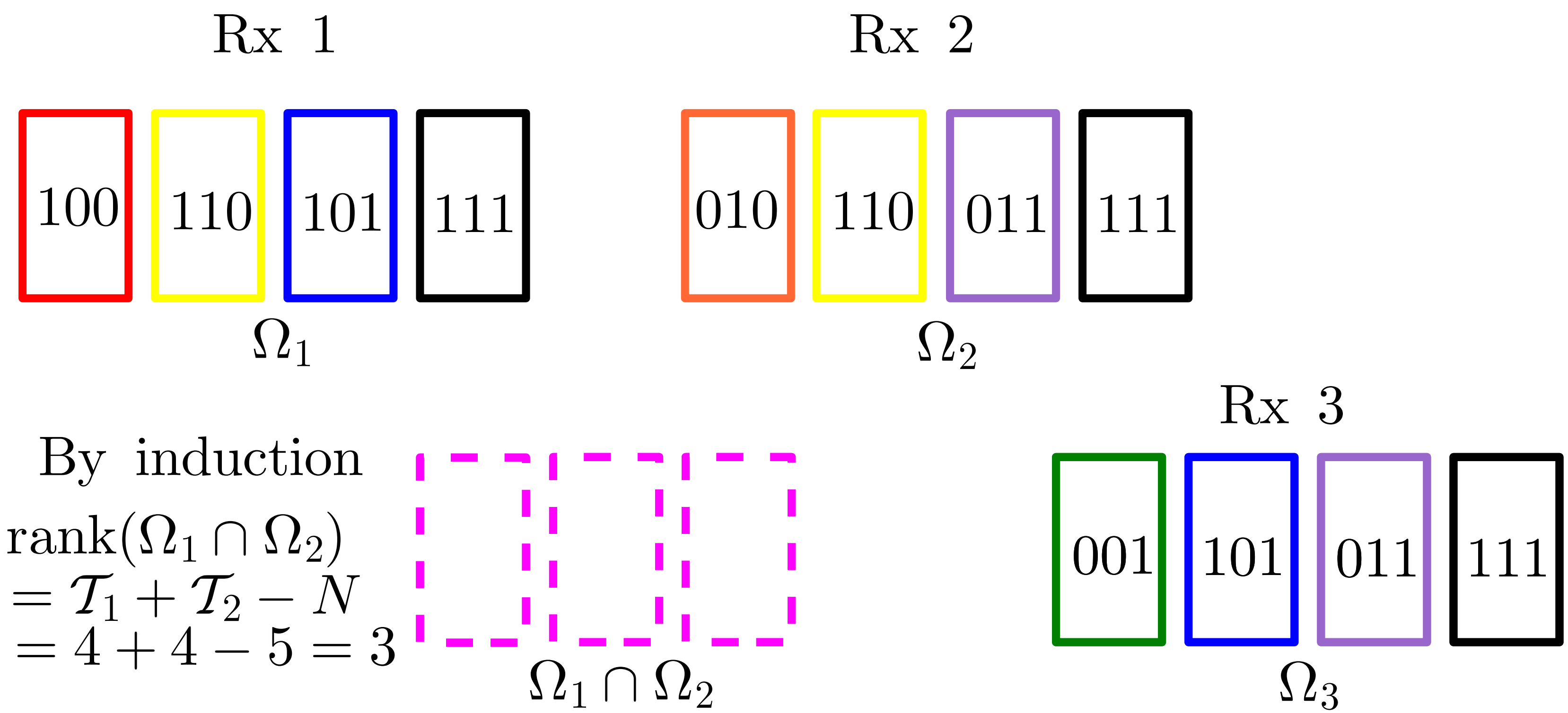

Rx 1
100
110
101
111
Ω1
Rx 2
010
110
011
111
Ω2
Rx 3
001
101
011
111
Ω3

Proof (Cont'd)

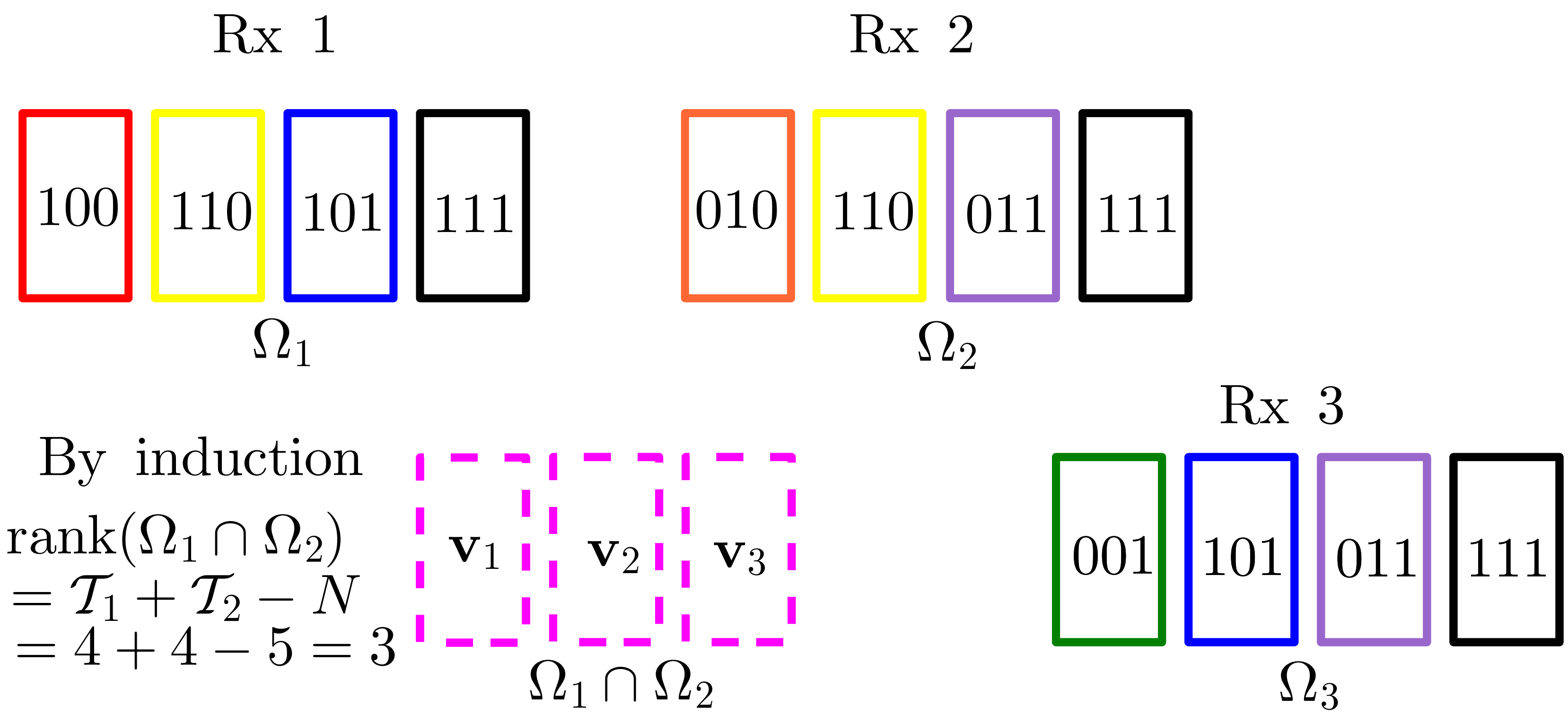


Rx 1
100
110
101
111
Ω_1
Rx 2
010
110
011
111
Ω_2
Rx 3
001
101
011
111
Ω_3
$\Omega_1 \cap \Omega_2$

Proof (Cont'd)


Rx 1
100
110
101
111
Ω_1
Rx 2
010
110
011
111
Ω_2
By induction
$\text{rank}(\Omega_1 \cap \Omega_2)$
$= \mathcal{T}_1 + \mathcal{T}_2 - N$
$= 4 + 4 - 5 = 3$
$\Omega_1 \cap \Omega_2$
Rx 3
001
101
011
111
Ω_3

Proof (Cont'd)


Rx 1
100
110
101
111
Ω_1
Rx 2
010
110
011
111
Ω_2
Rx 3
001
101
011
111
Ω_3
By induction
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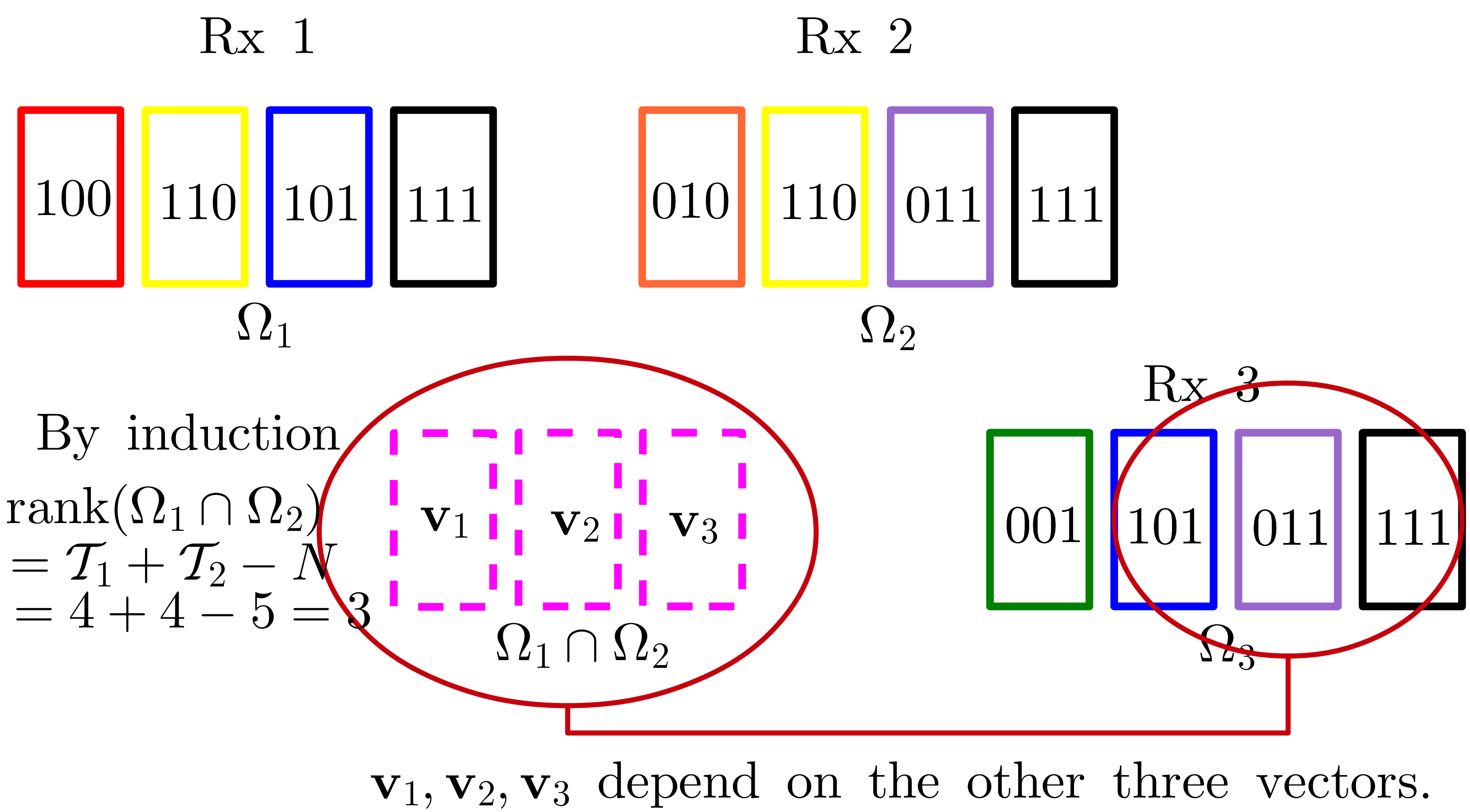
$= \mathcal{T}_1 + \mathcal{T}_2 - N$
$= 4 + 4 - 5 = 3$
$\mathbf{v}_1$
$\mathbf{v}_2$
$\mathbf{v}_3$
$\Omega_1 \cap \Omega_2$

Proof (Cont'd)


Rx 1
100
110
101
111
Ω_1
Rx 2
010
110
011
111
Ω_2
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$\mathbf{v}_2$
$\mathbf{v}_3$
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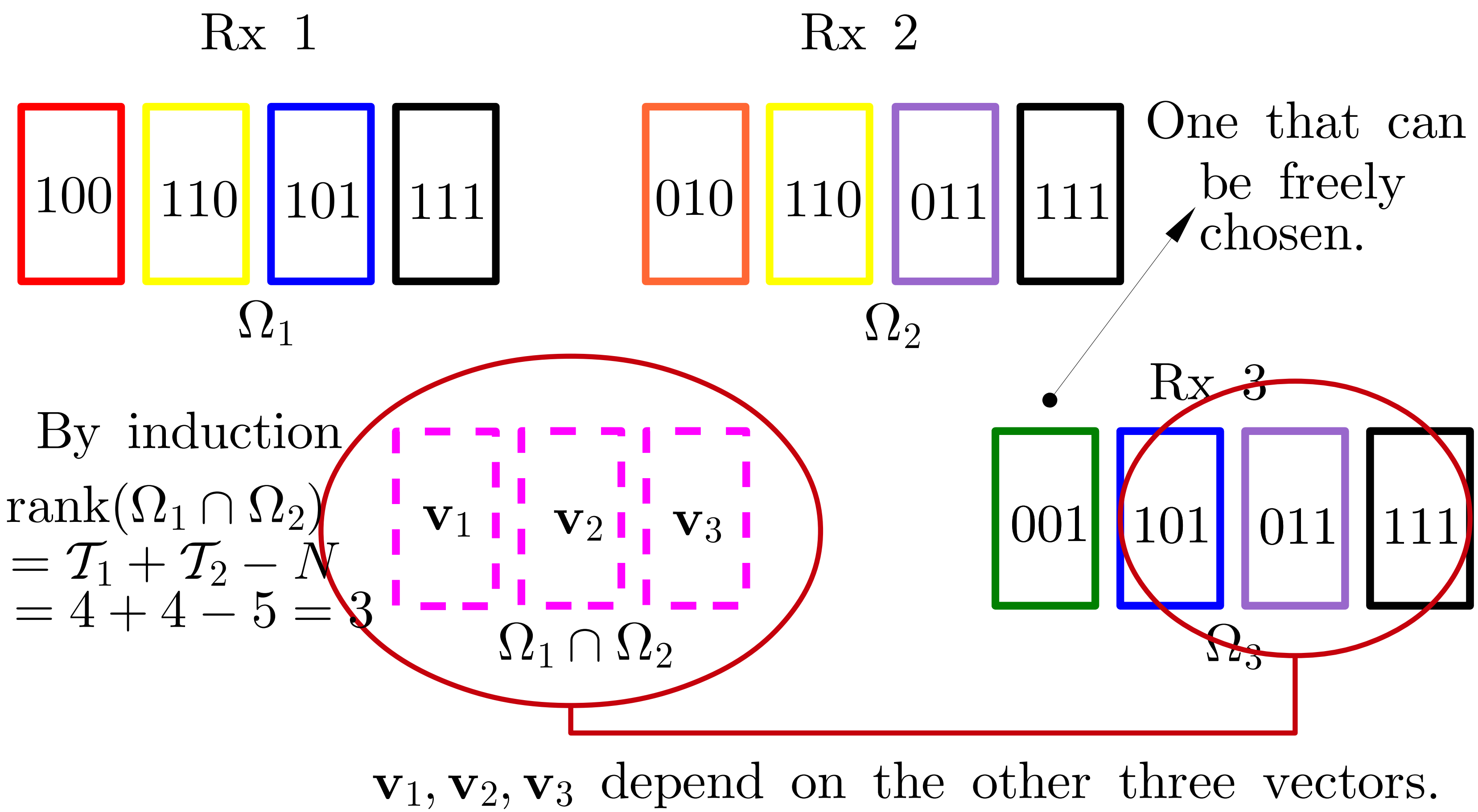
Rx 3
001
101
011
111
Ω_3
$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ depend on the other three vectors.

Proof (Cont'd)



Rx 1
100
110
101
111
Ω_1
Rx 2
010
110
011
111
Ω_2
One that can be freely chosen.
Rx 3
001
101
011
111
Ω_3
By induction
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$\mathbf{v}_2$
$\mathbf{v}_3$
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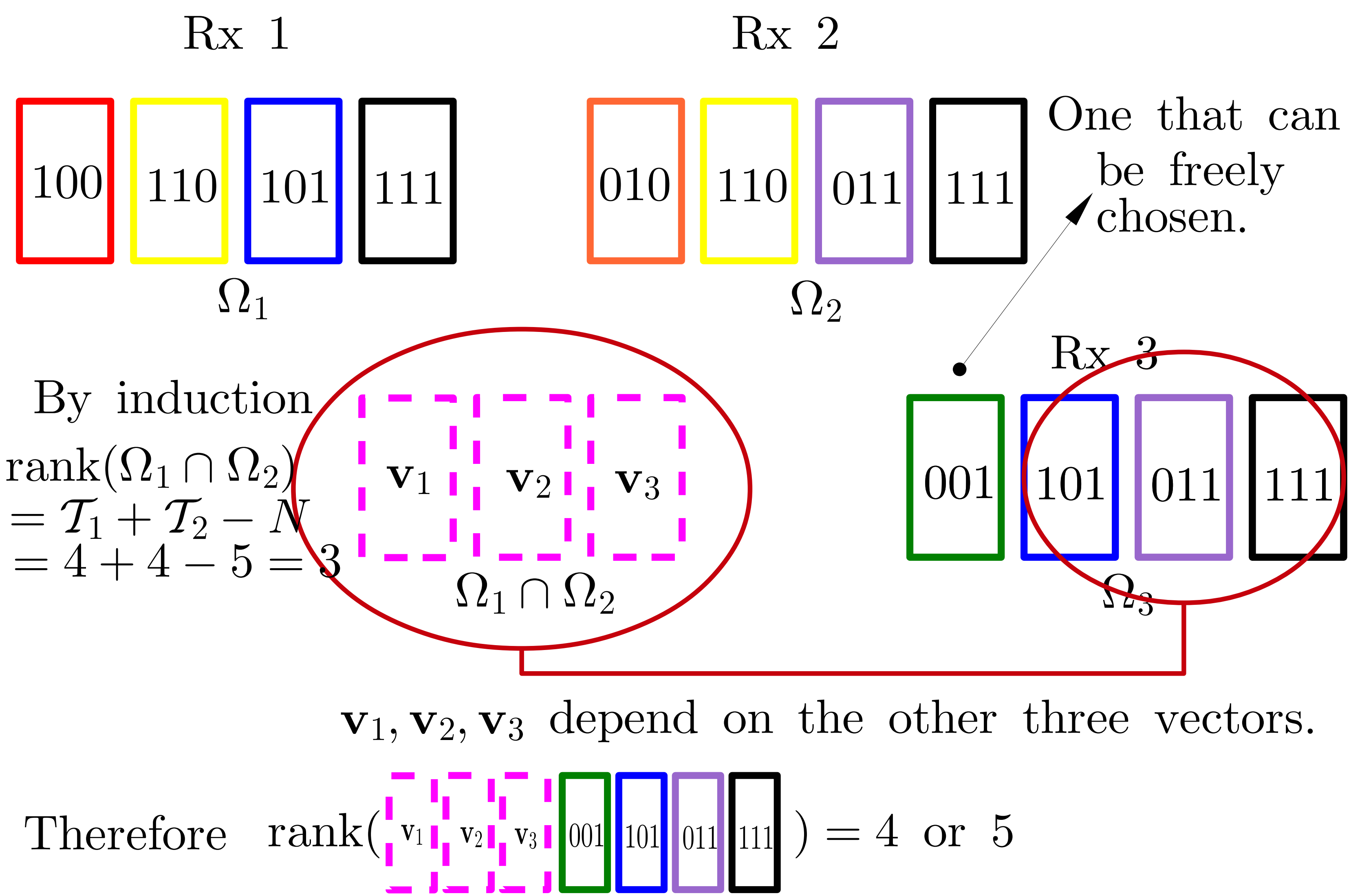
Proof (Cont'd)


Rx 1
100
110
101
111
Ω_1
Rx 2
010
110
011
111
Ω_2
One that can be freely chosen.
Rx 3
001
101
011
111
Ω_3
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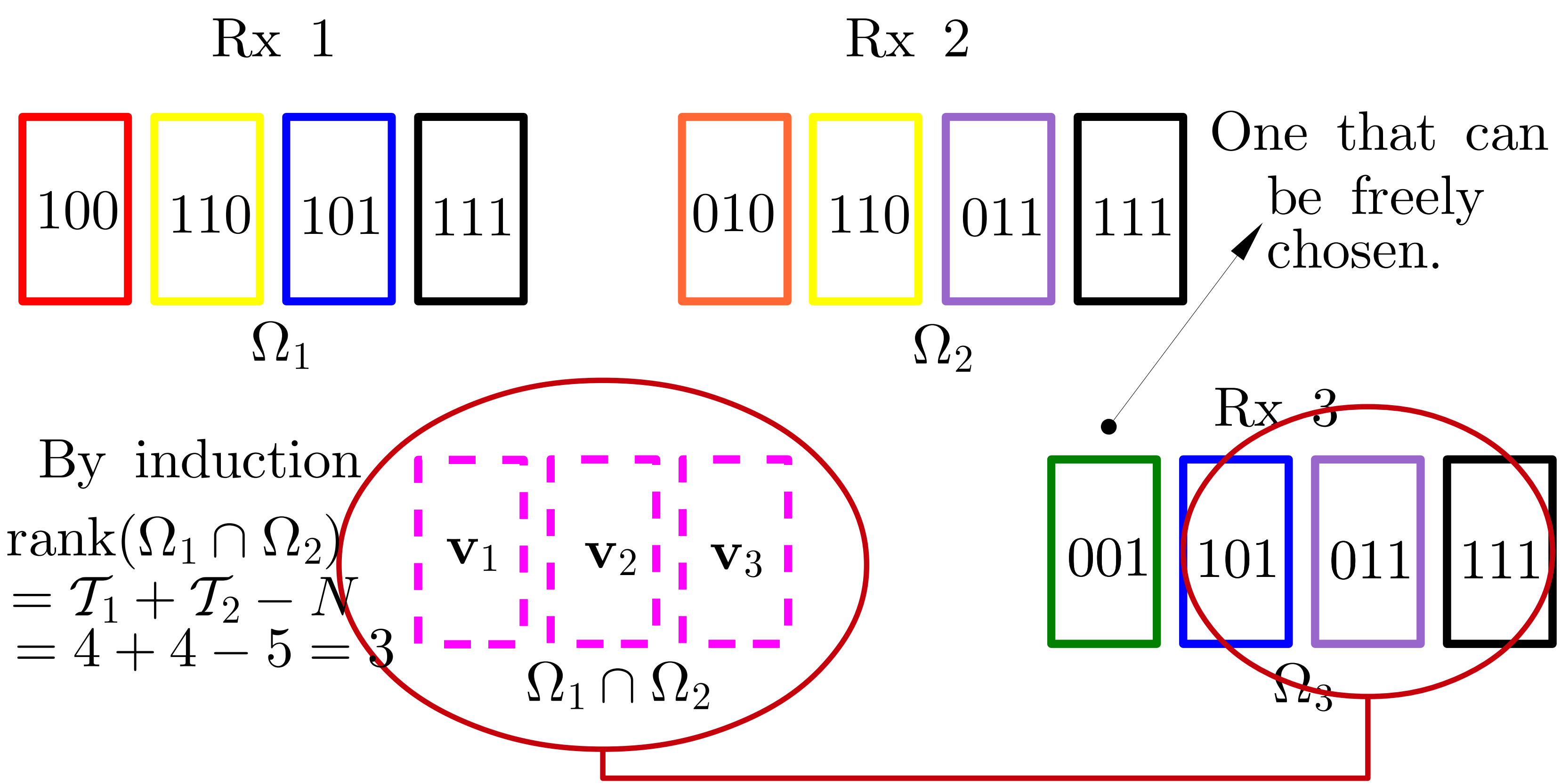
$\mathbf{v}_1$
$\mathbf{v}_2$
$\mathbf{v}_3$
$\Omega_1 \cap \Omega_2$
$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ depend on the other three vectors.
Therefore rank($\mathbf{v}_1$ $\mathbf{v}_2$ $\mathbf{v}_3$ 001 101 011 111) $= 4$ or 5

Proof (Cont'd)

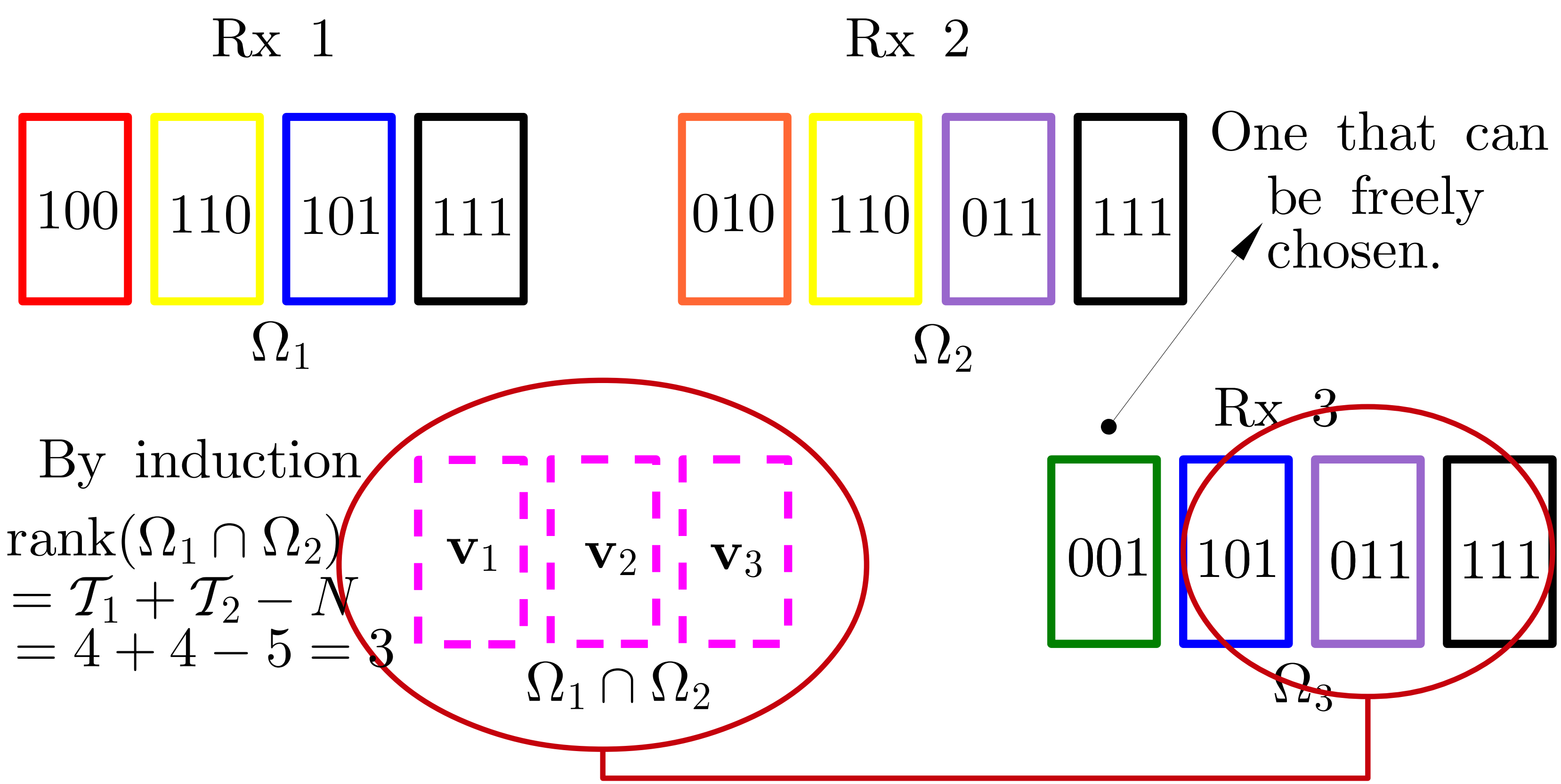


$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ depend on the other three vectors.

Therefore $\text{rank}($ v_1 v_2 v_3 001 101 011 111 $) = 4$ or 5

If being 5, $\text{rank}(\Omega_1 \cap \Omega_2 \cap \Omega_3) = 3 + 4 - \text{rank}($ v_1 v_2 v_3 001 101 011 111 $) = 2$

Proof (Cont'd)



$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ depend on the other three vectors.

Question: How to prove the rank is 5, when RLNC is used?

Therefore $\text{rank}(\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3\ 001\ 101\ 011\ 111) = 4$ or 5

If being 5, $\text{rank}(\Omega_1 \cap \Omega_2 \cap \Omega_3) = 3 + 4 - \text{rank}(\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3\ 001\ 101\ 011\ 111) = 2$

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1 Rx 2

$$\text{rank}(\ 100\ 110\ 101\ 111 \quad 010\ 110\ 011\ 111\ ) = 5$$

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1 — Rx 2

$$\text{rank}(\underbrace{100\ 110\ 101\ 111}_{\text{Rx 1}}\ \underbrace{010\ 110\ 011\ 111}_{\text{Rx 2}}) = 5 \longrightarrow \text{Fully Spanned Cond.}$$

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1 | Rx 2

rank(100 110 101 111 | 010 110 011 111) = 5 ⟶ Fully Spanned Cond.

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

The rank problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned Cond.

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

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rank(100 110 101 111 | 010 110 011 111) = 5 $\longrightarrow$ Fully Spanned Cond.

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

The rank problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned Cond.

[New]: If we can find *deterministically* 10 vectors: $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, plus 7, s.t.

Rx 1 | Rx 2

rank(100 110 101 111 | 010 110 011 111) = 5 Fully Spanned

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1: [100] [110] [101] [111]; Rx 2: [010] [110] [011] [111]

$$\text{rank}(\,[100]\,[110]\,[101]\,[111]\quad[010]\,[110]\,[011]\,[111]\,) = 5 \longrightarrow \text{Fully Spanned Cond.}$$

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

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[New]: If we can find *deterministically* 10 vectors: $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, plus 7, s.t.

Rx 1: [100] [110] [101] [111]; Rx 2: [010] [110] [011] [111]

$$\text{rank}(\,[100]\,[110]\,[101]\,[111]\quad[010]\,[110]\,[011]\,[111]\,) = 5$$

Fully Spanned

$$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \text{span}(\,[100]\,[110]\,[101]\,[111]\,) \cap \text{span}(\,[010]\,[110]\,[011]\,[111]\,)$$

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1 | Rx 2

rank(100 110 101 111 | 010 110 011 111) $= 5$ $\longrightarrow$ Fully Spanned Cond.

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

The rank problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned Cond.

[New]: If we can find *deterministically* 10 vectors: $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, plus 7, s.t.

Rx 1 | Rx 2

rank(100 110 101 111 | 010 110 011 111) $= 5$ Fully Spanned

Rx 1 | Rx 2

$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in$ span(100 110 101 111) $\cap$ span(010 110 011 111) $\longrightarrow$ Fully Intersected Cond.

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

$\text{rank}(\underbrace{100\ 110\ 101\ 111}_{\text{Rx 1}}\ \ \underbrace{010\ 110\ 011\ 111}_{\text{Rx 2}}) = 5$ $\longrightarrow$ Fully Spanned Cond.

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

The rank problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned Cond.

[New]: If we can find *deterministically* 10 vectors: $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, plus 7, s.t.

$\text{rank}(\underbrace{100\ 110\ 101\ 111}_{\text{Rx 1}}\ \ \underbrace{010\ 110\ 011\ 111}_{\text{Rx 2}}) = 5$ Fully Spanned

$\text{rank}(\underbrace{\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3}_{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3}\ \ \underbrace{001\ 101\ 011\ 111}_{\text{Rx 3}}) = 5$ Fully Spanned

$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \text{span}(\underbrace{100\ 110\ 101\ 111}_{\text{Rx 1}}) \cap \text{span}(\underbrace{010\ 110\ 011\ 111}_{\text{Rx 2}})$ $\longrightarrow$ Fully Intersected Cond.

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1 Rx 2

$$\text{rank}(\,[100]\,[110]\,[101]\,[111]\quad[010]\,[110]\,[011]\,[111]\,) = 5 \longrightarrow$$ Fully Spanned Cond.

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

The rank problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned Cond.

[New]: If we can find *deterministically* 10 vectors: $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, plus 7, s.t.

Rx 1 Rx 2

$$\text{rank}(\,[100]\,[110]\,[101]\,[111]\quad[010]\,[110]\,[011]\,[111]\,) = 5$$ Fully Spanned

$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ Rx 3

$$\text{rank}(\,[\mathbf{v}_1]\,[\mathbf{v}_2]\,[\mathbf{v}_3]\quad[001]\,[101]\,[011]\,[111]\,) = 5$$ Fully Spanned

Rx 1 Rx 2 Rx 3

$$\text{rank}(\,[100]\,[110]\,[101]\,[111]\,) = \text{rank}(\,[010]\,[110]\,[011]\,[111]\,) = \text{rank}(\,[001]\,[101]\,[011]\,[111]\,) = 4$$

Fully Spanned

$$\text{rank}(\,[\mathbf{v}_1]\,[\mathbf{v}_2]\,[\mathbf{v}_3]\,) = 3$$

Rx 1 Rx 2

$$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \text{span}(\,[100]\,[110]\,[101]\,[111]\,) \cap \text{span}(\,[010]\,[110]\,[011]\,[111]\,)$$

$\longrightarrow$ Fully Intersected Cond.

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1 | Rx 2

$\text{rank}(\,[100]\,[110]\,[101]\,[111]\quad[010]\,[110]\,[011]\,[111]\,) = 5$ $\longrightarrow$ Fully Spanned Cond.

then for *almost all random choices* of the 7 vectors, $\text{rank}(\Omega_1 \oplus \Omega_2) = 5$.

The rank problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned Cond.

[New]: If we can find *deterministically* 10 vectors: $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, plus 7, s.t.

Rx 1 | Rx 2

$\text{rank}(\,[100]\,[110]\,[101]\,[111]\quad[010]\,[110]\,[011]\,[111]\,) = 5$ Fully Spanned

$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ | Rx 3

$\text{rank}(\,[\mathbf{v}_1]\,[\mathbf{v}_2]\,[\mathbf{v}_3]\quad[001]\,[101]\,[011]\,[111]\,) = 5$ Fully Spanned

Rx 1 | Rx 2

$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \text{span}(\,[100]\,[110]\,[101]\,[111]\,) \cap \text{span}(\,[010]\,[110]\,[011]\,[111]\,)$

Rx 1 | Rx 2 | Rx 3 | Fully Spanned

$\text{rank}(\,[100]\,[110]\,[101]\,[111]\,) = \text{rank}(\,[010]\,[110]\,[011]\,[111]\,) = \text{rank}(\,[001]\,[101]\,[011]\,[111]\,) = 4$ $\quad \text{rank}(\,[\mathbf{v}_1]\,[\mathbf{v}_2]\,[\mathbf{v}_3]\,) = 3$

$\longrightarrow$ Fully Intersected Cond.

then for *almost all rand. choices* of the 7 vectors, $\text{rank}((\Omega_1 \cap \Omega_2) \oplus \Omega_3) = 5$.

Proof (Cont'd)

[Koetter *et al.* 03, Ho *et al.* 06]: If we can find *deterministically* 7 vectors s.t.

Rx 1 — Rx 2

rank(100 110 101 111 | 010 110 011 111) $= 5$ $\longrightarrow$ Fully Spanned Cond.

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rank($\mathbf{v}_1$ $\mathbf{v}_2$ $\mathbf{v}_3$ | 001 101 011 111) $= 5$ Fully Spanned

$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in$ span(100 110 101 111) (Rx 1) $\cap$ span(010 110 011 111) (Rx 2)

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Fully Spanned: rank(v_1 v_2 v_3) $= 3$

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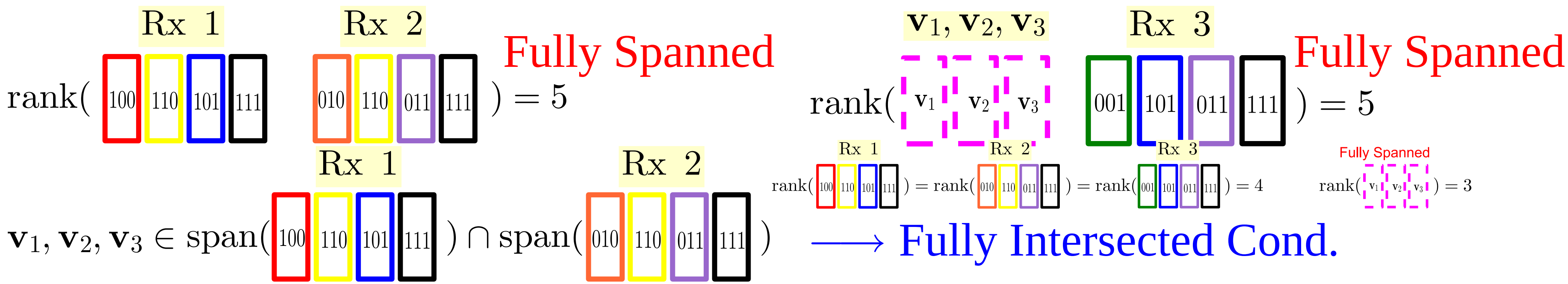
The Common Info. problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned and Fully Intersected Conds.

Proof (Cont'd)

It does not matter how we choose $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, plus 7,
and in what order we construct the vectors.

Any deterministic construction will suffice!

[New]: If we can find *deterministically* 10 vectors: $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, plus 7, s.t.



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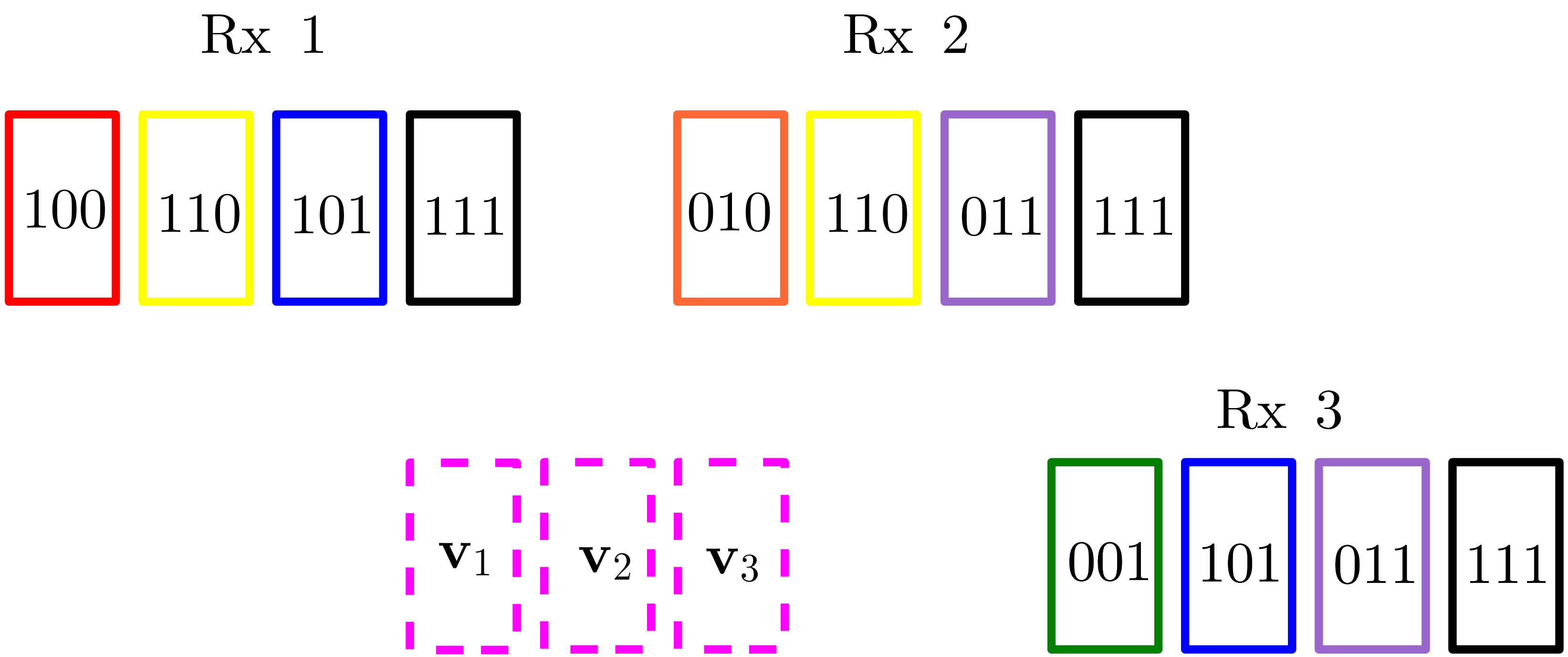
A Deterministic Assignment



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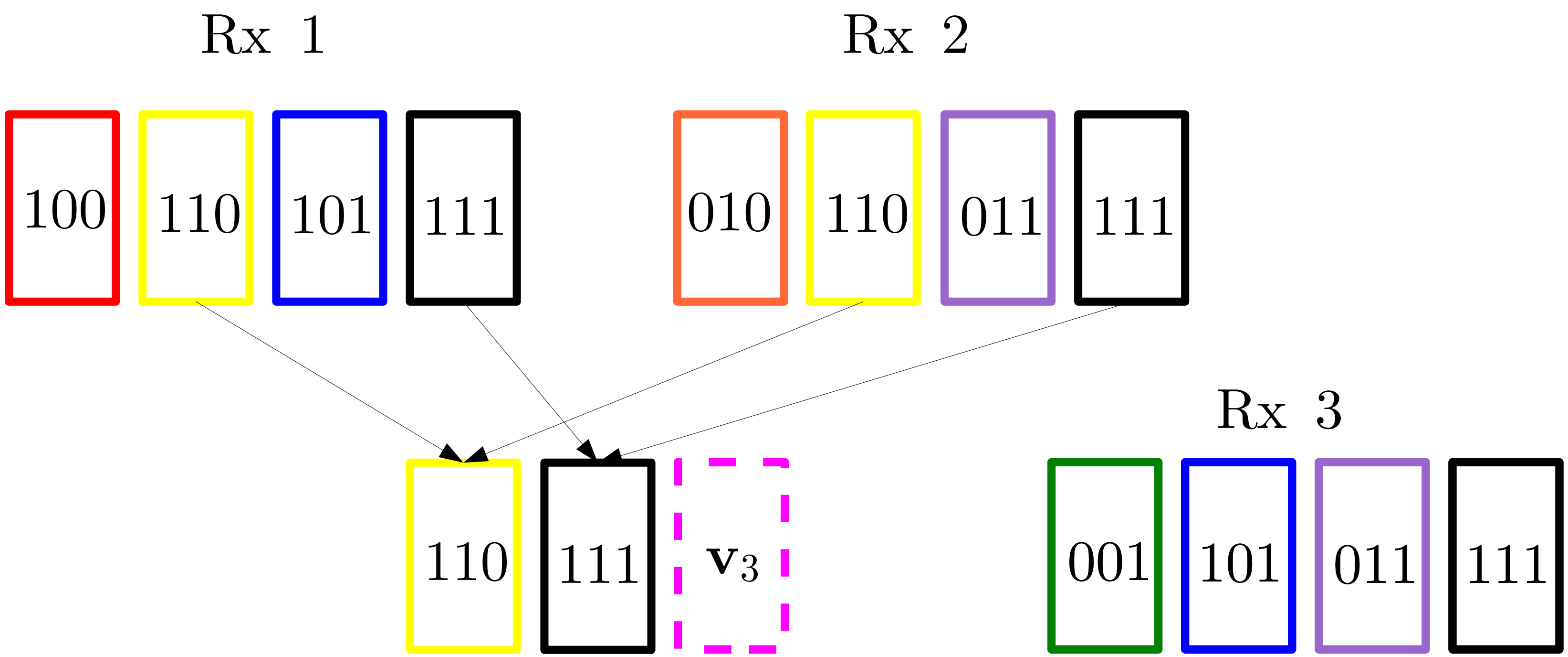

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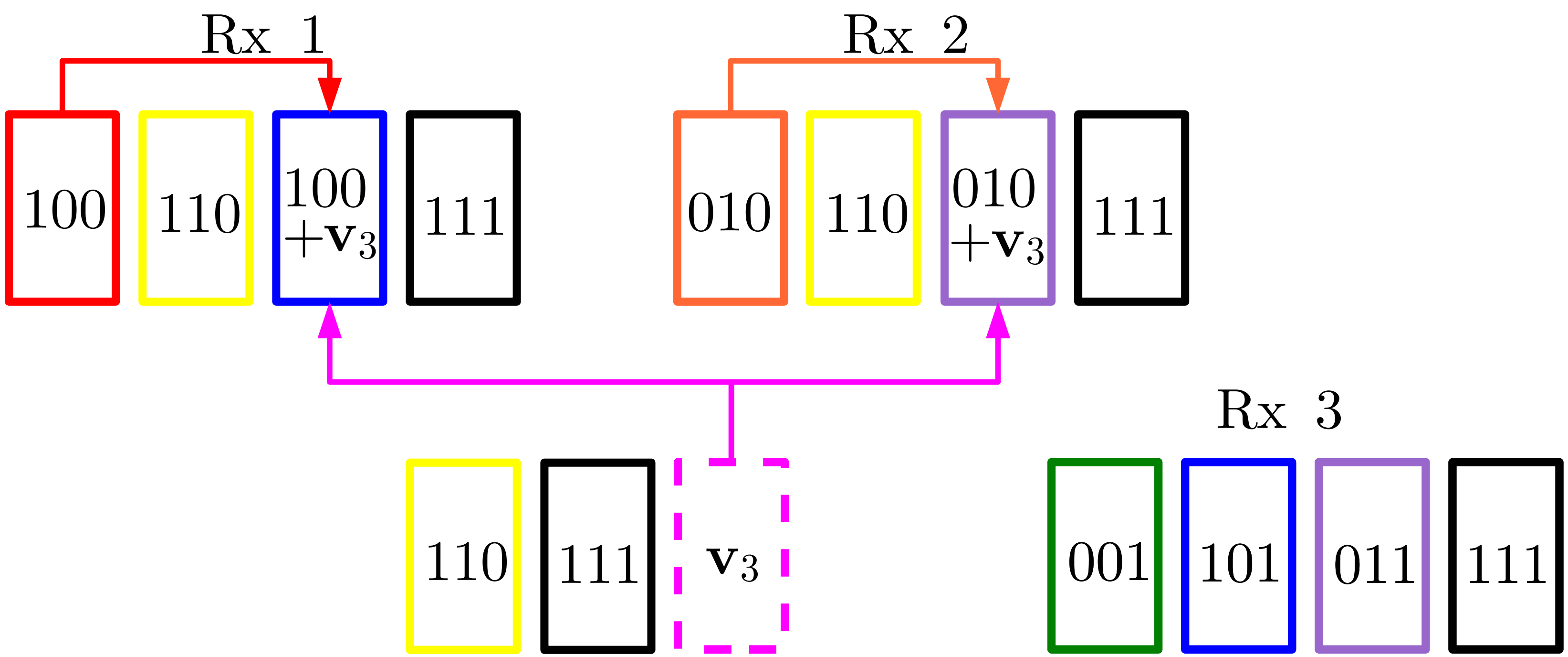

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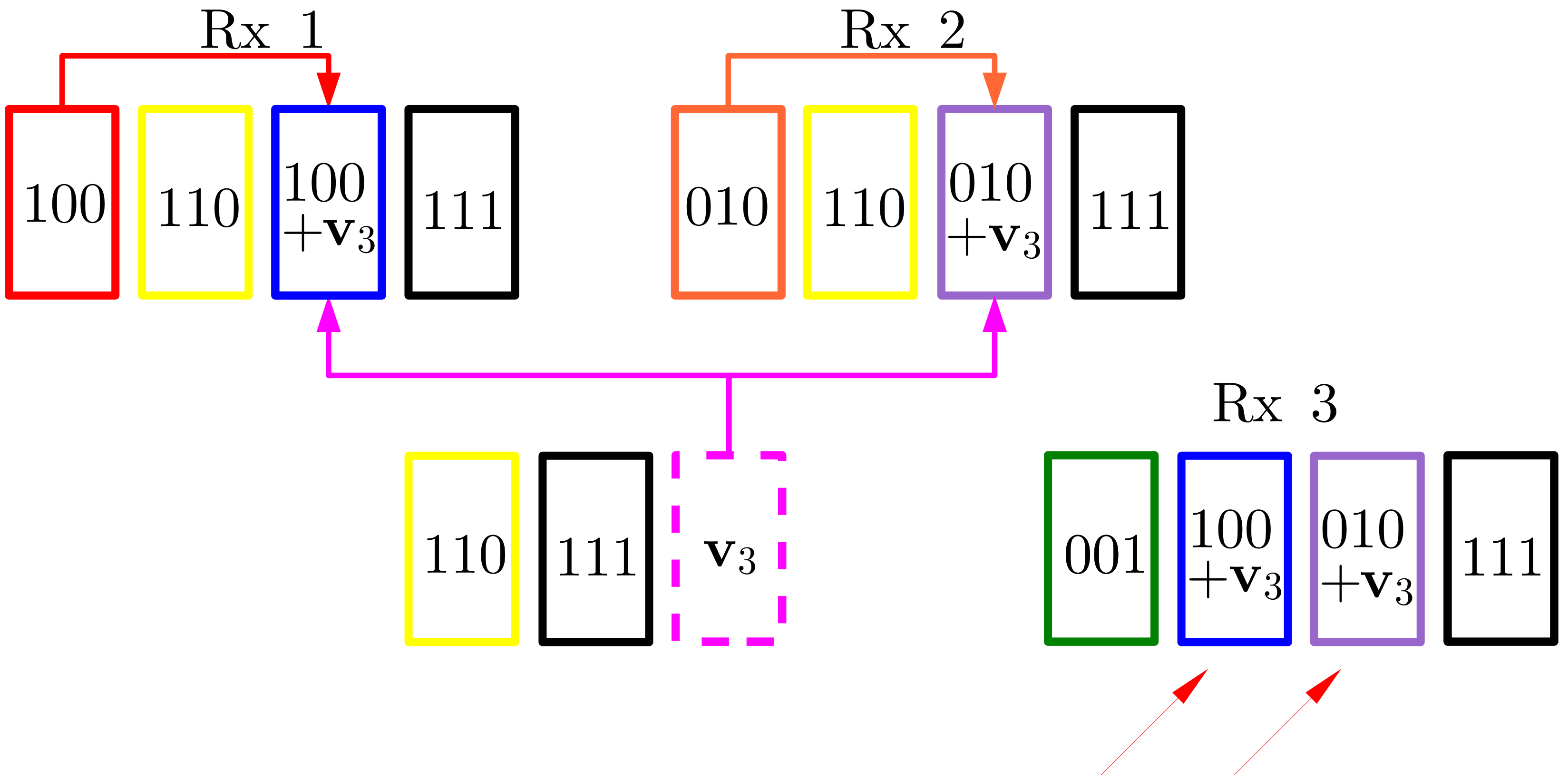

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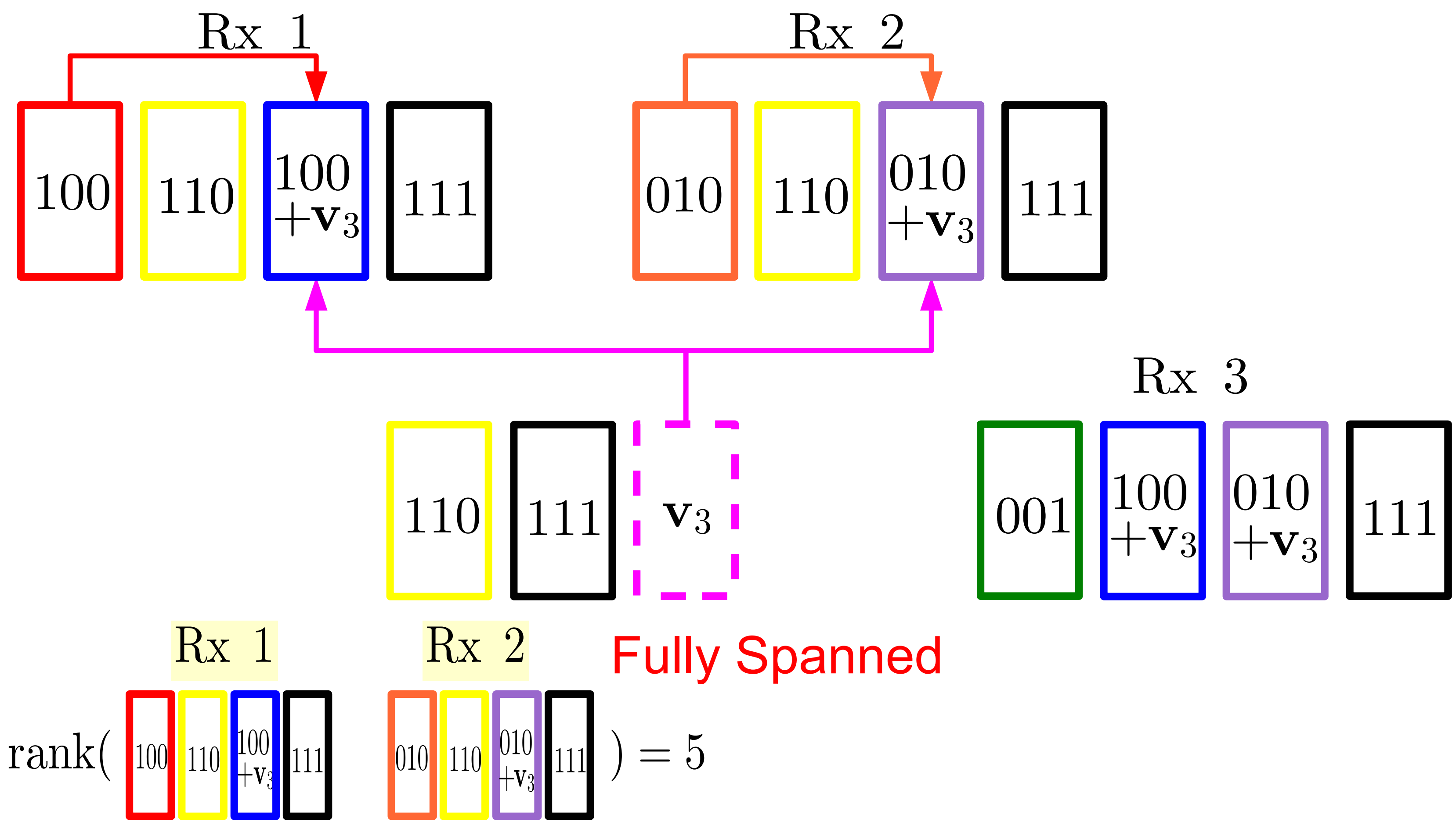

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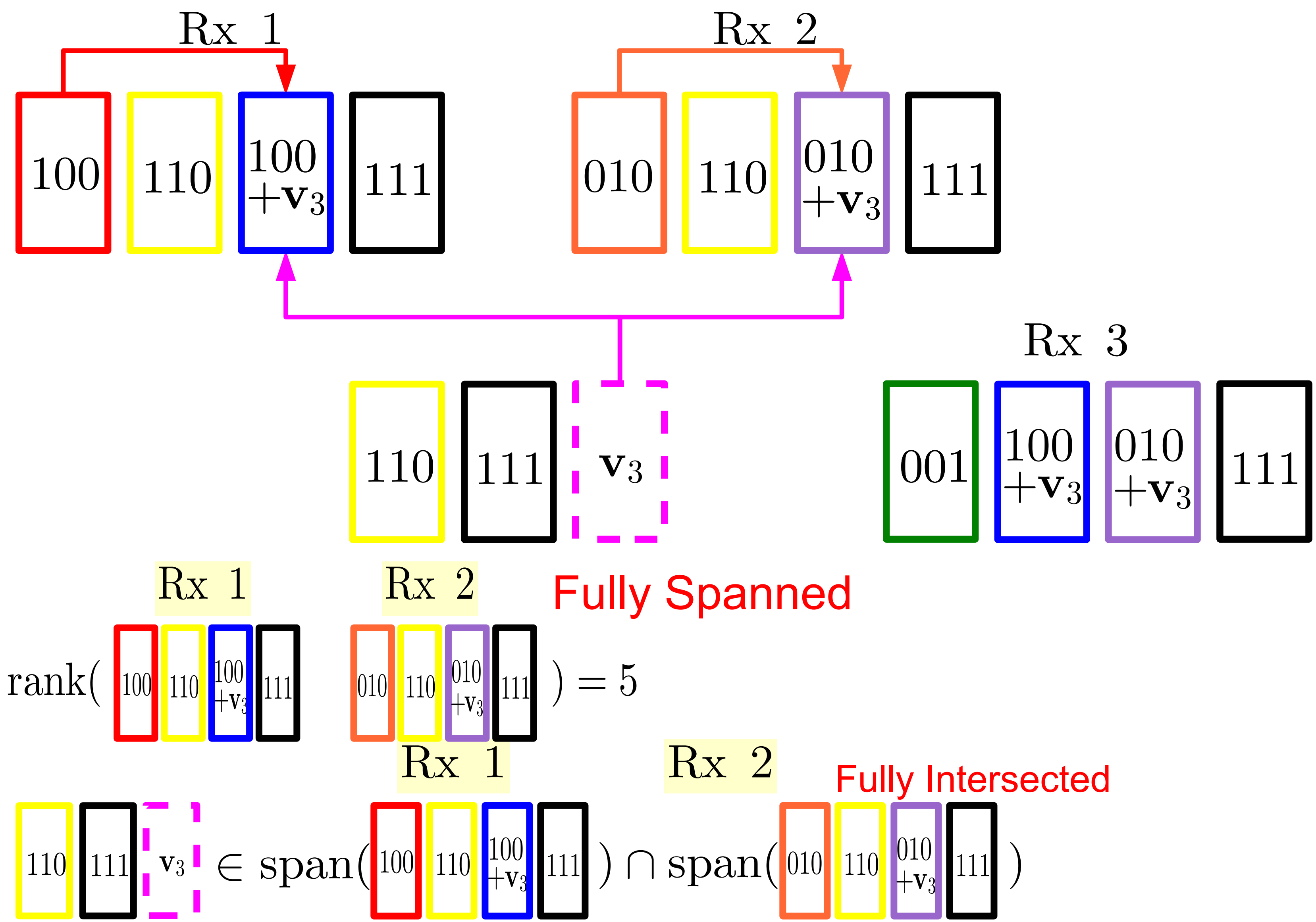

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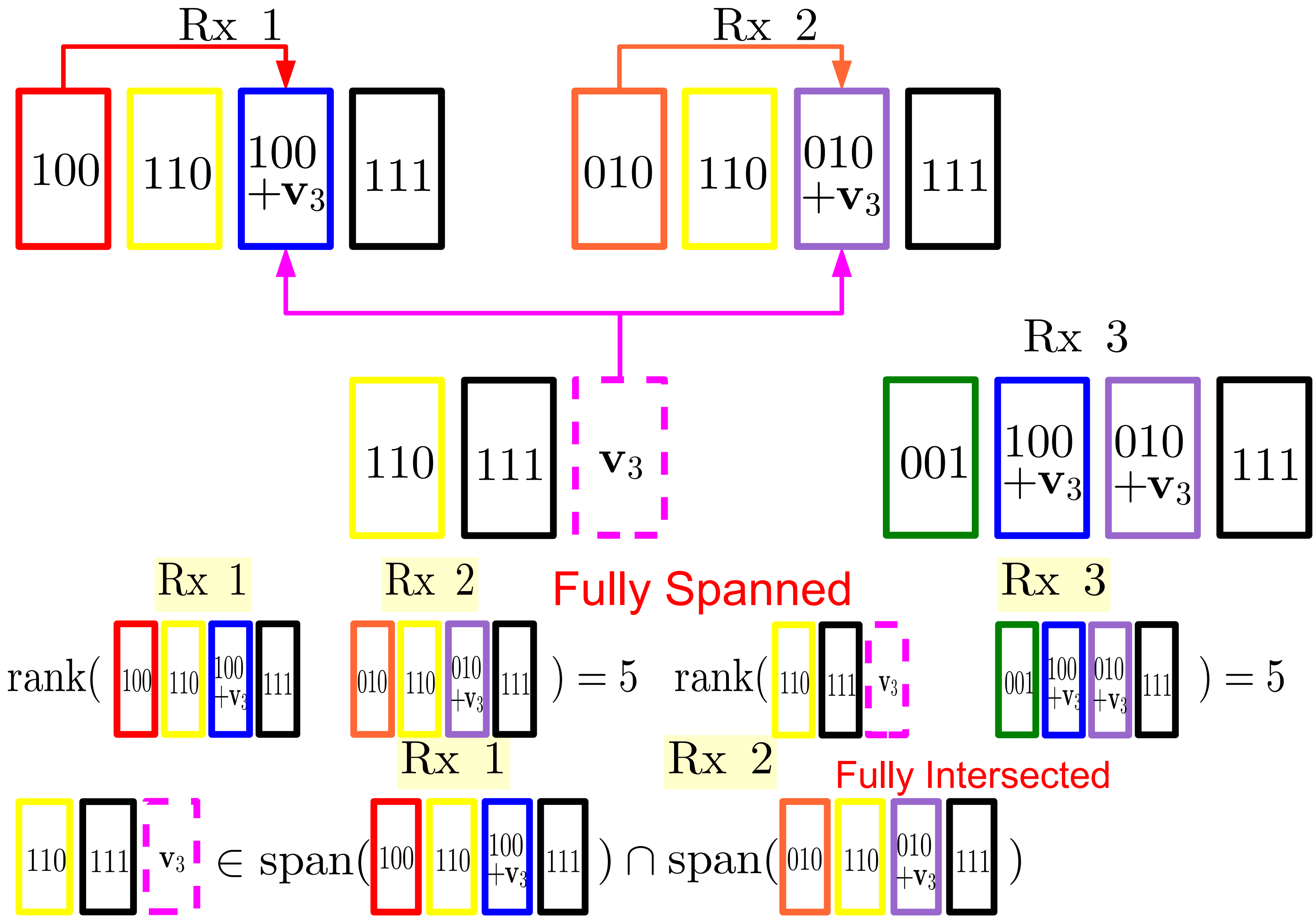

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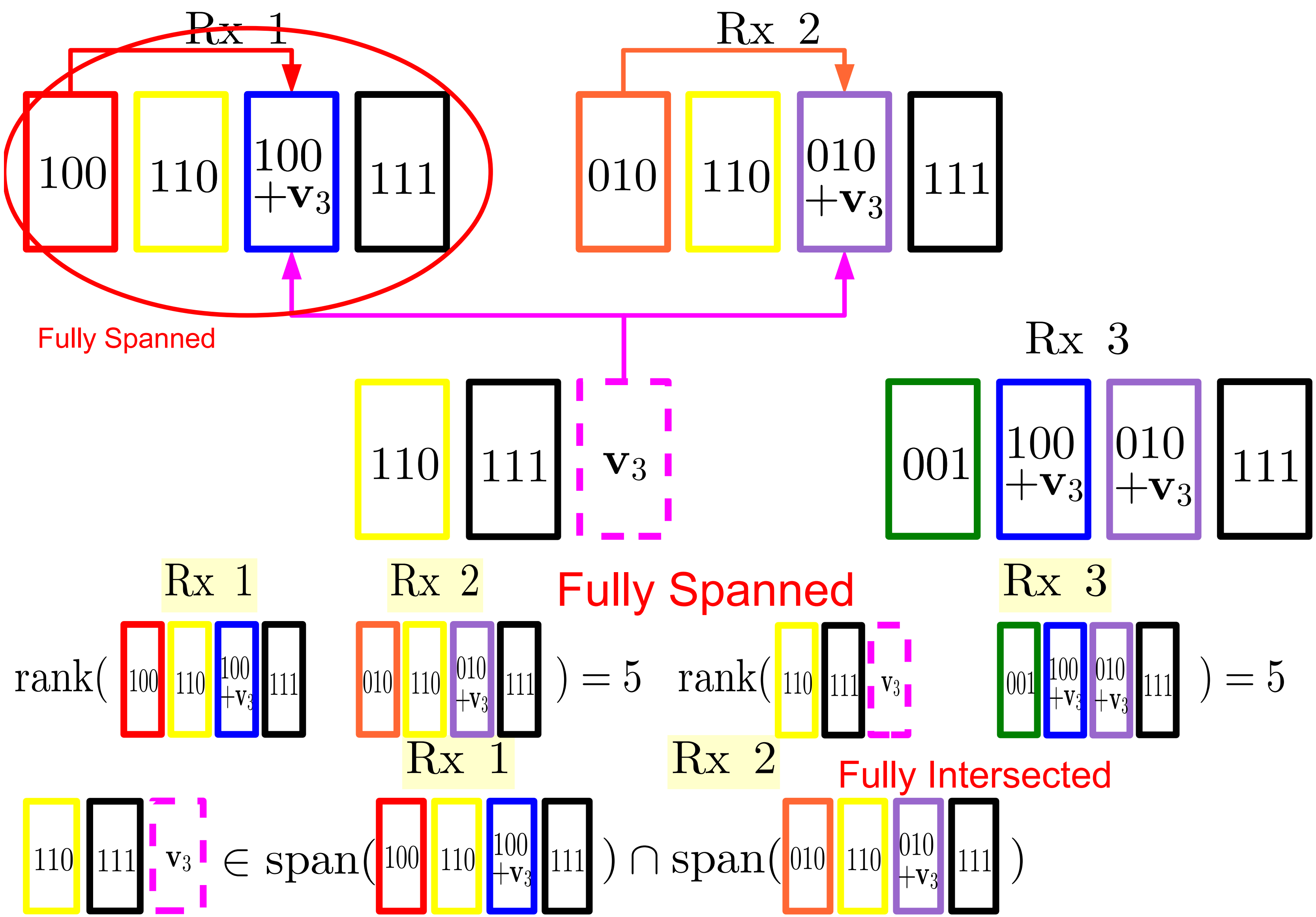

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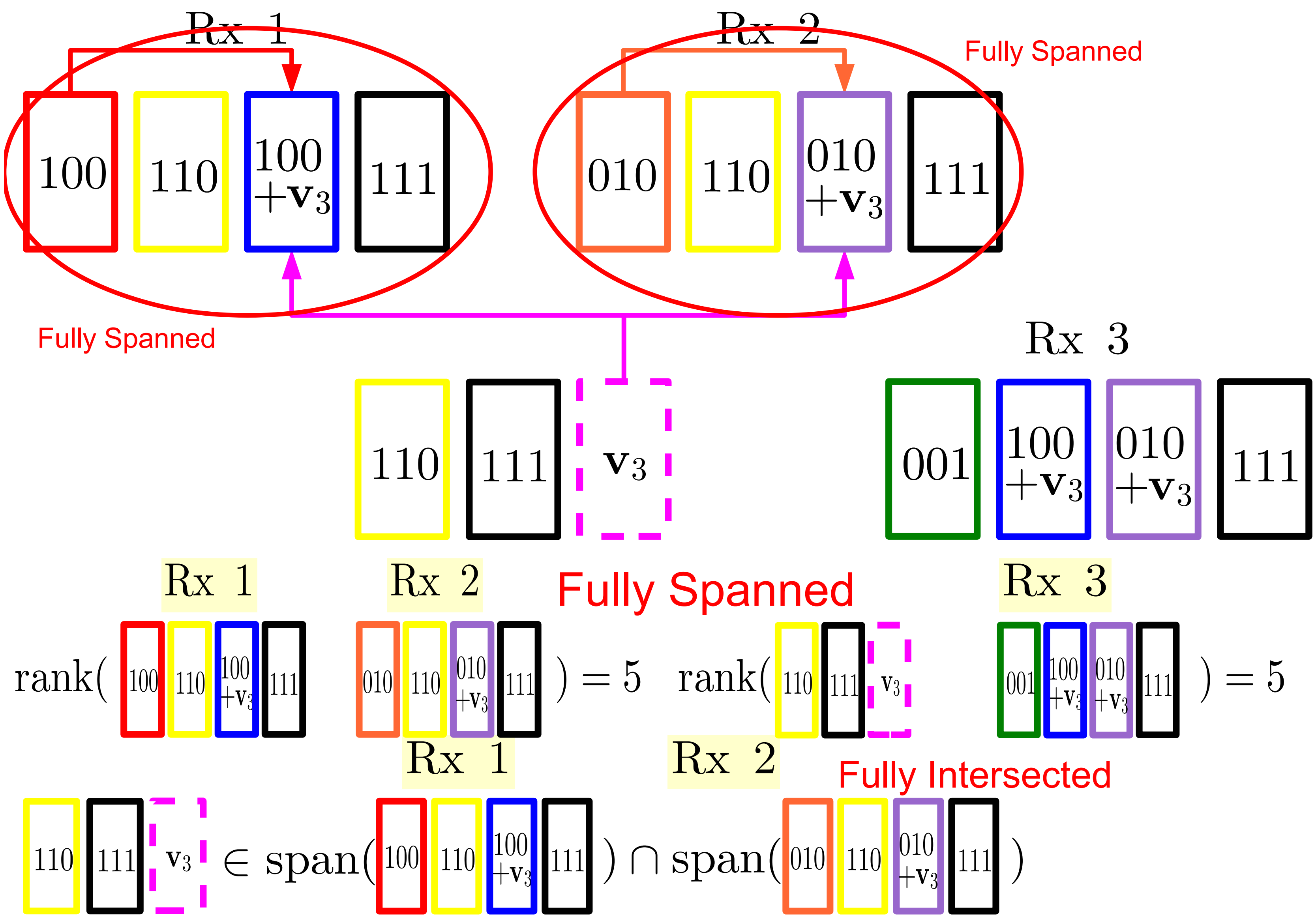

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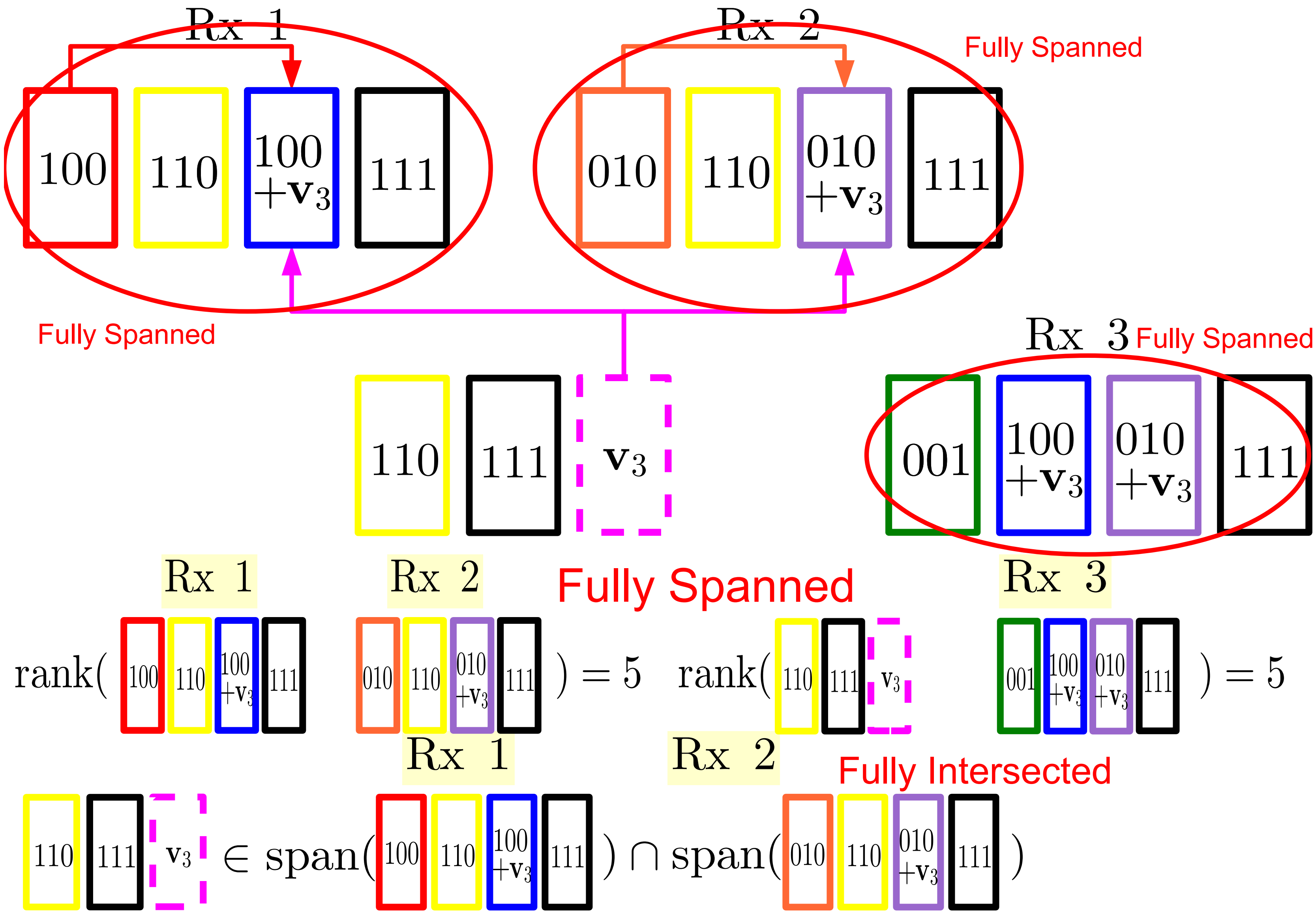

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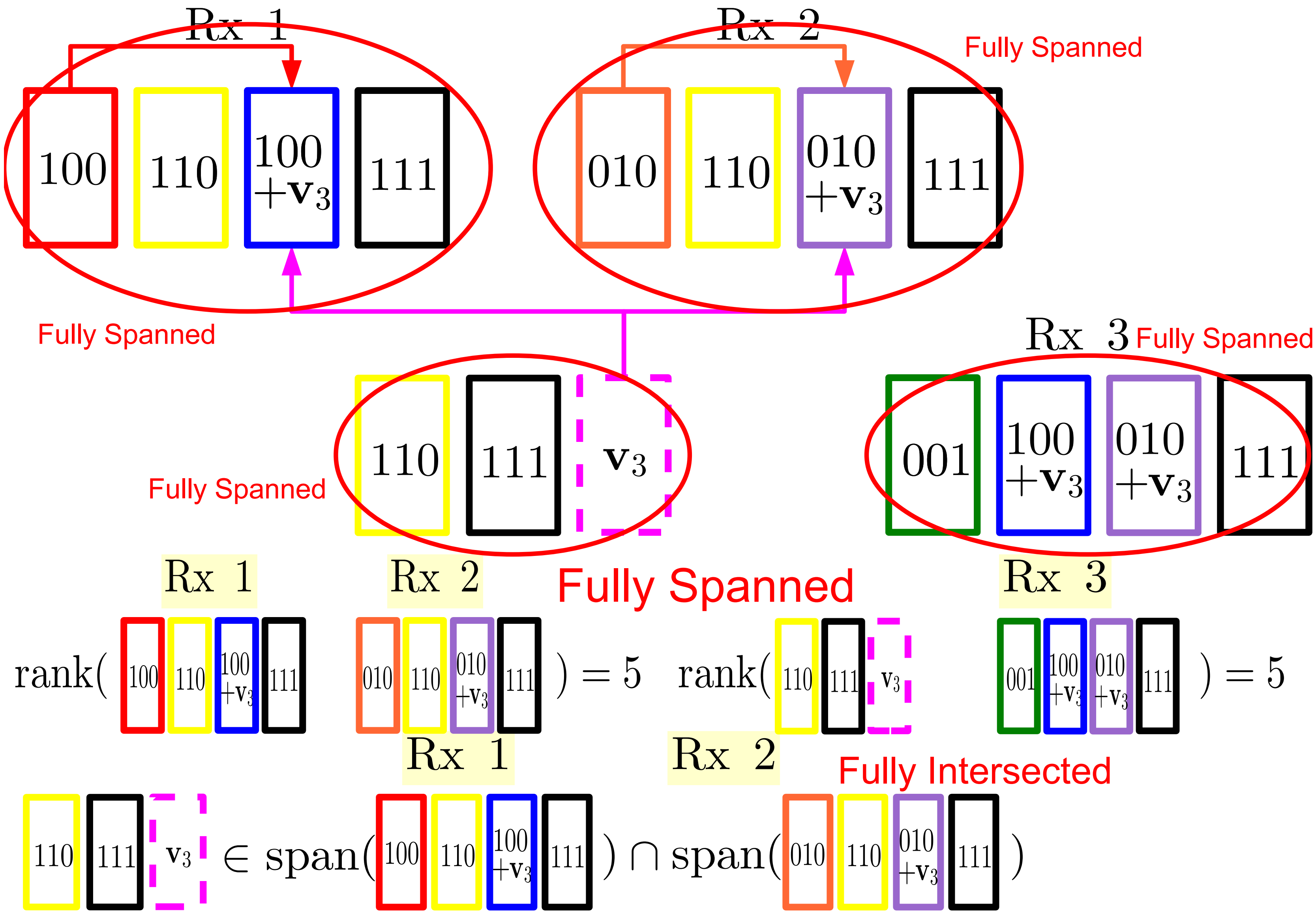

Intuition of the FS and FI Conds.

The Common Info. problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned and Fully Intersected Conds.

- Let $\mathbf{x}$ denote the input coding vectors and the local mixing kernels.
- Then in RLNC, each message along an edge has the form of

$$M = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_N(\mathbf{x}))$$

where $f_i(\mathbf{x})$ are polynomials of $\mathbf{x}$. [Koetter *et al.* 03].

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- When constructing the basis vectors of $\bigcap_k \Omega_k$, we need to solve linear equations (i.e., being in all marginal spaces), which results in the form
$$\mathbf{v} = \left(\frac{f_1(\mathbf{x})}{g_1(\mathbf{x})}, \frac{f_2(\mathbf{x})}{g_2(\mathbf{x})}, \cdots, \frac{f_N(\mathbf{x})}{g_N(\mathbf{x})}\right).$$

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- [Fully Intersected] associates a deterministic $\mathbf{x}_0$ assignment with the corresponding fractional expressions; [Fully Spanned] guarantees both $g_n(\mathbf{x})$ and $\det([\mathbf{v}_i])$ are non-zero.

Conclusion & Future Works

The Common Info. problem of RLNC is reduced to finding a deterministic assignment satisfying the Fully Spanned and Fully Intersected Conds.

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- Main Results

$$\mathrm{rank}\left(\bigcap_{k=1}^{K}\Omega_k\right) = \max\left\{N - \sum_{m=1}^{M}\left(N - \mathcal{T}_{S_m}\right)^+ : \forall \text{ partition } \{S_m\}\right\}.$$

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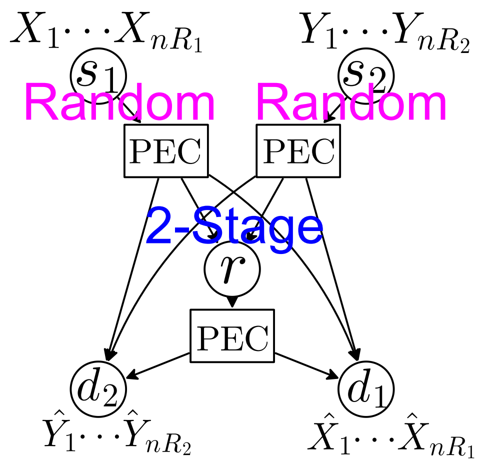

The total # of to-be-sent symbols:

$$\operatorname{rank}\left(\Omega_r^{[s_1]}\right) - \operatorname{rank}\left(\Omega_r^{[s_1]} \cap \Omega_{d_1}^{[s_1]}\right)$$

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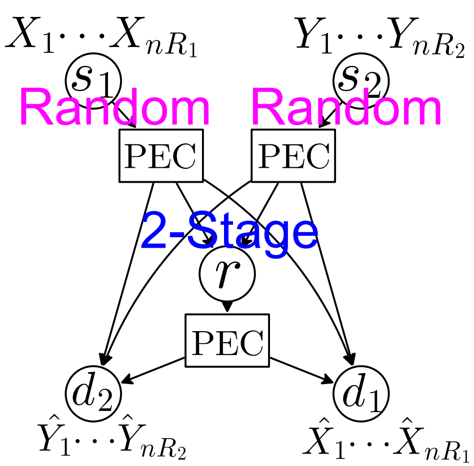

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- Future works: (i) Arbitrary combinations: Ex: $(\Omega_1 \oplus \Omega_2) \cap \Omega_3$. (ii) Common Information of RLNC over multi-hop networks.