Surface-based Finite Element Method for Large-scale 3D Circuit Modeling

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Abstract

This paper presents a novel surface-based finite element method for full-wave modeling of large-scale 3D physical circuits. In contrast to traditional finite element methods that involve 3D volumetric unknowns, this method reduces the unknowns one needs to solve to those on 2D surface elements of interest only. It preserves the advantages of the finite element method in circuit application such as the flexibility in modeling irregular geometry, the capability in handling arbitrary inhomogeneity, and the handling of sparse matrices. Meanwhile, it eliminates the disadvantages long associated with traditional finite element methods such as large memory requirement and high CPU cost resulted from 3D volumetric discretization. Experimental and numerical results are given to demonstrate its accuracy and computational efficiency.

1. Introduction

High-performance microprocessor design imposes many challenges to circuit modeling. Among them, problem size is often the No. 1 challenge. Taking the full-chip power grid as an example, its modeling involves 7+ metal layers, ~ 10,000μm x 10,000μm chip area, millions of vias, and hundreds of thousands of power rails. Such a large-scale global interconnect system is traditionally regarded to be computationally prohibitive. To address this problem, divide-and-conquer approaches and many other engineering solutions have been proposed. However, the approximations made in these methods often led to inaccurate or even erroneous results especially when global interactions have to be characterized. As a result, it is of paramount importance to study and develop a high-capacity, numerically rigorous CAD solution for large-scale problems such as the design of full-chip-level interconnect.

In addition to a high-capacity solution, a full-wave solution is also of great importance to high-performance microprocessor design [1]. A full-wave electromagnetic solution tackles Maxwell's equations, which has been traditionally studied by researchers in computational electromagnetics. Computational electromagnetics, since its advent in the 1960s, has evolved into its prime to date [2]. Numerous fast algorithms have been developed. Among them, partial-differential-equation (PDE) based methods have exhibited certain advantages over integral-equation based approaches especially in circuit modeling. The reasons are two folds: first, it has greater flexibility in handling strong inhomogeneity; second, it generates sparse matrices. Although it cannot impose a radiation condition as naturally as its integral-equation based counterparts, complicated absorbing boundary condition is generally not required in circuit modeling as on-die circuits are usually designed for operations that do not have significant radiations. The only drawback is that PDE-based solvers generally cannot avoid 3D volumetric discretization, and hence generate a large number of unknowns. To overcome this disadvantage, in this paper, we propose a surface-based finite element method (SFEM) to significantly reduce the unknowns of a 3D finite-element solution to unknowns on 2D surfaces of interest only. By doing so, we drastically improve the efficiency and capacity of 3D finite element methods, while retaining all its generality.

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In section 2, we will elaborate the proposed surface-based finite element method. In section 3, two numerical case studies will be given to demonstrate the accuracy and efficiency of the proposed SFEM method, and followed by conclusions in Section 4.

2. Formulation

To rigorously analyze 3D digital, mixed-signal or RF circuits, we formulate a numerical algorithm to solve Maxwell's equations therein. As we know, the solution of Maxwell's equations is the E field or H field at the discretized points inside the computational domain. However, the design parameters of interest are generally circuit parameters such as SPICE netlist and Z-, Y- or S-parameters at the terminals/ports of interest. Therefore, instead of constructing an electromagnetic solution at discretized points inside the circuit structure, we propose to formulate a circuit abstraction of the original Maxwell's system. This abstraction results in a system that only involves fields that contribute to the final circuit parameter extraction [3]. Certainly, the fewer the field unknowns involved in the circuit parameter extraction are, the smaller the abstracted system will be, and hence the more efficient the simulation will result. The surface-based finite-element method proposed here is such an efficient and unique circuit abstraction of electromagnetic systems.

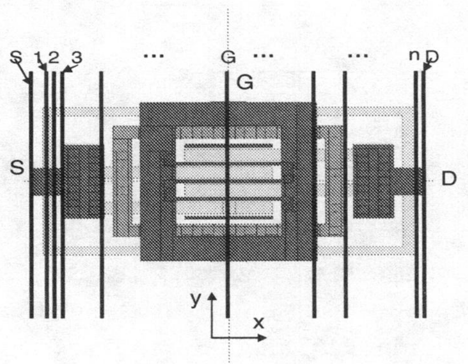

Fig. 2. Illustration of surface-based finite element scheme.

In the aforementioned method, we first develop a circuit parameter extraction technique that only involves field unknowns on surfaces of interest. We then slice the computational domain into multi-surfaces. Take a RF CMOS metal system as an example. Fig. 2 shows its top view. The circuit parameters of interest are the SPICE netlists between source and drain, gate and source, and gate and drain. To obtain them, first, we use a series of surfaces to slice the computational volumetric domain as shown by black solid lines in Fig. 2. These surfaces lie in the y-z plane. Y-z-plane surfaces are chosen here instead of x-y, x-z or other orientations to minimize the dimension of the resultant cross section, and hence to yield the smallest number of unknowns. Two classes of surfaces are generated. One is called the essential surfaces. These essential surfaces are generated at the locations where the final circuit parameters are extracted. For example, surfaces S, G, and D belong to essential surfaces because they are needed to extract source, gate, and drain circuit parameters from field solutions. The other class is called the back-fill surfaces. These back-fill surfaces are generated to partition the unknowns. For example, in Fig. 2, surfaces 1, 2, 3, …, and n are the back-fill surfaces.

Second, we perform discretization. We discretize the computational domain either in tetrahedral (for irregular structures) or prism elements (if the mesh can be extruded). Because of the pre-defined surfaces, the mesh automatically partitions at these surfaces.

Next, we form a surface-based finite-element scheme. In this scheme, all of the volumetric unknowns are eliminated. For instance, the volumetric unknowns between surfaces 1 and 2 can be eliminated by using the procedure illustrated in Fig. 3. The left figure in Fig. 3 represents the original sub-matrix **A** formed by the unknowns residing between surfaces 1 and 2, and those on surfaces 1 and 2. N_{1s} denotes unknowns on surface 1 and N_{2s} denotes those on surface 2. N_{12v} represents the volumetric unknowns between surfaces 1 and 2. Here, we are able to eliminate N_{12v} because the surface and volumetric unknowns are correlated. And hence, the original sub-matrix **A** is reduced to matrix $\mathbf{A_r}$, which only involves unknowns residing on surfaces 1 and 2. The computational complexity of this matrix reduction is proportional to the number of volumetric unknowns between surfaces 1 and 2 that have been eliminated. This explains why back-fill surfaces are introduced although they do not contribute to the final circuit extraction: When problem size is large, one can use back-fill surfaces to partition unknowns to efficiently perform the reduction in a much smaller region.

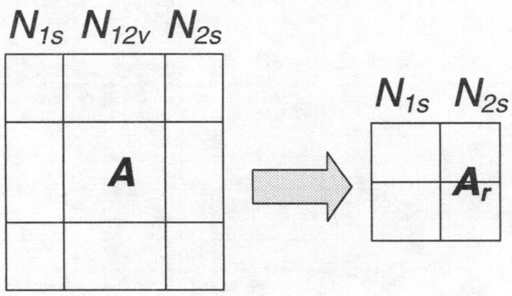

Fig. 3. Procedure of eliminating volume unknowns.

Repeating the above procedure, the volumetric unknowns between surfaces 2 and 3, 3 and 4, - - -, and n-1 and n can be eliminated. With all the volumetric unknowns eliminated, the final matrix system only involves the unknowns on surfaces, and hence rendering a surface-based finite-element method. Compared to conventional volumetric finite element method, it constitutes a significant improvement as it drastically reduces the number of unknowns. Since backfill surfaces are irrelevant to the final circuit extraction, they can also be eliminated. With all of the backfill surfaces eliminated, the final matrix system only involves unknowns residing on three essential surfaces: S, G, and D, which can be readily and efficiently solved.

3. Numerical Results

We first simulated a 3D test-chip bus structure that was fabricated using conventional Si processing techniques [4]. The test structure involves thousands of conductors in M1 and M3. In Fig. 4, we compare measured and simulated S-parameters in both magnitude and phase. Clearly the agreement is very good between the de-embedded data and the data simulated by the proposed method. Fig. 4 also shows the mismatch between static-based extraction and measurements at multi-GHz frequencies, which is attributed to the decoupling of E and H.

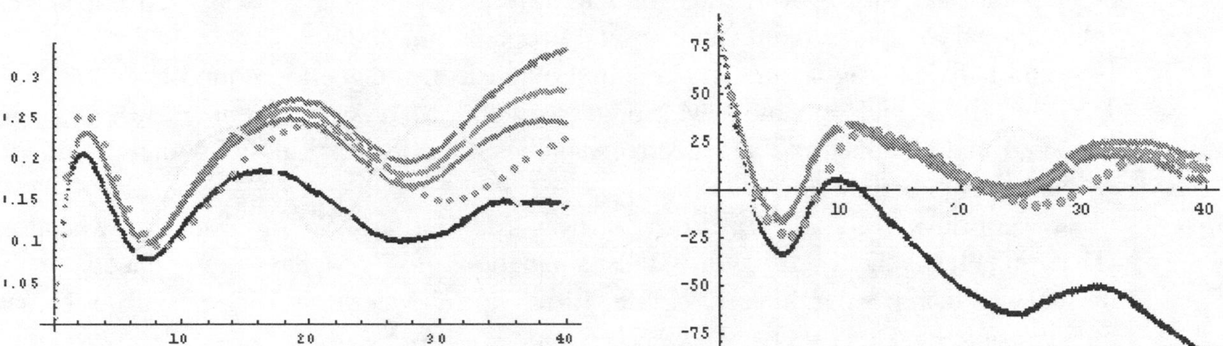

Figure 4. Modeling of a 3D on-chip bus structure. (a). Crosstalk magnitude. (b) Crosstalk phase. (Red: de-embedded; Magenta: This method; Black: as measured; Blue: De-embedded +/- error; Light blue: Static model.)

The second example considers a high-density package decoupling capacitor structure. An AVX 0612 0.47 μF 2-terminal capacitor was measured from 30KHz to 3GHz by using an HP 8753D Vector Network Analyzer (VNA). The proposed method was used to simulate this structure. Fig. 5 shows the calculated input impedance of the decap against measurements. Clearly, excellent agreement can be seen between the modeled impedance and the de-embedded one. In modeling this structure, the proposed method only uses 1-2 minutes, whereas a standard finite-element solution takes greater than 24 hours [5] due to the presence of large number of power/ground planes, high permittivity, and large aspect ratio.

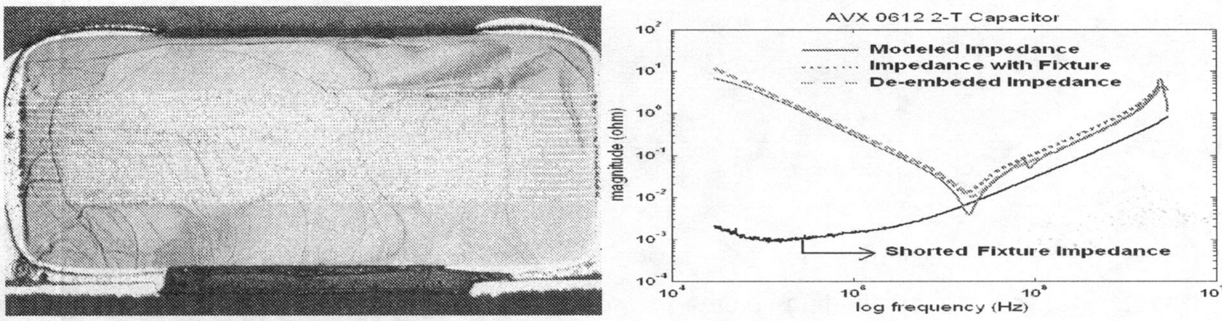

Figure 5. An AVX 0612 0.47 μF 2-terminal capacitor. (a) Geometry. (b) Input impedance.

4. Conclusion

This paper presents a surface-based finite element method for large-scale 3D circuit modeling. It rigorously formulates a surface FEM solution that only needs to solve surface unknowns residing on circuit terminals of interest, and hence dramatically improves the efficiency and capacity of 3D finite-element solution while retaining all the generalities and advantages of traditional FEM methods. Compared to surface-based integral methods, the surface-based finite element method not only deals with the reduced set of surface unknowns, but also avoids dense matrix kernel. Furthermore, the method is very suitable for parallelization. In addition, the concept developed in this method can be generalized to any other PDE-based solvers, and hence establishing a new class of electromagnetic solvers: surface-based PDE methods.

References:

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