state. In spite of these differences, the three structures show fairly similar modal properties.

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# ASYMPTOTIC WAVEFORM EVALUATION FOR SCATTERING BY A DISPERSIVE DIELECTRIC OBJECT

## Dan Jiao1 and Jian-Ming Jin1

<sup>1</sup> Center for Computational Electromagnetics Department of Electrical and Computer Engineering University of Illinois Urbana, Illinois 61801-2991

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**ABSTRACT:** The asymptotic waveform evaluation (AWE) method is applied to the moment-method solution of scattering by a dispersive dielectric object, resulting in an efficient technique for the calculation of the radar cross section over a specified frequency band. © 2000 John Wiley & Sons, Inc. Microwave Opt Technol Lett 24: 232–234, 2000.

**Key words:** asymptotic waveform evaluation; electromagnetic scattering; moment method

#### 1. INTRODUCTION

A formulation that is widely used for scattering by a dielectric object is the so-called PMCHW [1], named after Poggio, Miller, Chang, Harrington, and Wu, who originally developed the formulation. In this formulation, the electric-field integral equation (EFIE) for the field inside the dielectric object is combined with the EFIE for the field outside the object to form a combined equation. Similarly, the magnetic-field integral equation (MFIE) for the field inside the object is combined with the MFIE for the field outside the object to form another combined equation. These two equations are then solved by the moment method. This formulation is found to be free of interior resonances, and yields accurate and stable solutions. This letter describes the application of the asymptotic waveform evaluation (AWE) method [2] to the PMCHW-based moment method to accelerate the calculation of the scattering by a dispersive dielectric object, whose permittivity is characterized by the Debye model.

#### 2. FORMULATION

Consider an arbitrarily shaped homogeneous dielectric scatterer characterized by permittivity  $\epsilon_2$  and permeability  $\mu_2$ , and immersed in an infinite and homogeneous medium having permittivity  $\epsilon_1$  and permeability  $\mu_1$ . Using either the equivalence principle or the vector Green's theorem, one can formulate a set of four integral equations to calculate the electric and magnetic fields **E** and **H** in terms of equivalent electric and magnetic currents **J** and **M** on the surface of the

scatterer. The equation to calculate the electric field is known as the EFIE, and there are two such equations: one is for the field inside the object (EFIE-I), and the other is for the field outside the object (EFIE-O). The equation to calculate the magnetic field is known as the MFIE, and there are also two such equations: one is for the field inside the object (MFIE-I), and the other is for the field outside the object (MFIE-O). A simple combination of EFIE-I and EFIE-O yields an integral equation:

$$[Z_1L_1(\mathbf{J}) + Z_2L_2(\mathbf{J}) - \mathbf{K}_1(\mathbf{M}) - \mathbf{K}_2(\mathbf{M}) = \mathbf{E}^i]_{tan} \quad (1)$$

and a similar combination of MFIE-I and MFIE-O results in another integral equation:

$$[Z_1\mathbf{K}_1(\mathbf{J}) + Z_2\mathbf{K}_2(\mathbf{J}) + \mathbf{L}_1(\mathbf{M}) + \mathbf{L}_2(\mathbf{M}) = Z_1\mathbf{H}^i]_{tan}$$
 (2)

where  $Z_i = \sqrt{\mu_i/\epsilon_i}$ , and  $\mathbf{L}_i$  and  $\mathbf{M}_i$  are integral operators defined by

$$\mathbf{L}_{i}(\mathbf{X}) = jk_{i} \int_{S} \left[ \mathbf{X}(\mathbf{r}') + \frac{1}{k_{i}^{2}} \nabla \nabla' \cdot \mathbf{X}(\mathbf{r}') \right] G_{i}(\mathbf{r}, \mathbf{r}') \, dS' \quad (3)$$

$$\mathbf{K}_{i}(\mathbf{X}) = \int_{S} \mathbf{X}(\mathbf{r}') \times \nabla G_{i}(\mathbf{r}, \mathbf{r}') \, dS'$$
 (4)

in which  $k_i = \omega \sqrt{\mu_i \epsilon_i}$ , S denotes the surface of the scatterer, and  $G_i(\mathbf{r}, \mathbf{r}')$  is the scalar Green's function given by

$$G_i(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_i|\mathbf{r} - \mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|}.$$
 (5)

Equations (1) and (2) are known as the PMCHW formulation [1]. When  $\mathbf{r} = \mathbf{r}'$ , the integral in (4) is interpreted in the principal value sense.

For a dispersive dielectric object,  $\epsilon_2$  and  $\mu_2$  can be a function of frequency. For simplicity, we assume that  $\mu_2$  is a constant, and only  $\epsilon_2$  varies with frequency. The complex permittivity of a dielectric can be described by the Debye

model [3]:

$$\epsilon_2(\omega) = \epsilon_2'(\omega) - j\epsilon_2''(\omega) = \epsilon_{2\infty}' + \frac{\epsilon_{2s}' - \epsilon_{2\infty}'}{1 + j\omega\tau_e}$$
 (6)

where  $\epsilon'_{2s}$  denotes the static dielectric constant,  $\epsilon'_{2\infty}$  is the optical dielectric constant, and  $\tau_e$  is a relaxation time constant related to the original relaxation time constant  $\tau$  by

$$\tau_e = \tau \frac{\epsilon'_{2s} + 2\epsilon_0}{\epsilon'_{2x} + 2\epsilon_0} \tag{7}$$

with  $\epsilon_0$  denoting the permittivity of free space.

Equations (1) and (2) can be solved numerically using the moment method. In this work, the Rao-Wilton-Glisson function [4] is used both as an expansion and testing function. The solution is shown to be accurate and stable at interior resonances [5]. The resulting matrix equation can be symbolically written as

$$A(\omega)x(\omega) = y(\omega) \tag{8}$$

where A is a square matrix, x is an unknown vector consisting of the discretized equivalent electric and magnetic currents, and y is a known vector associated with the incident electric and magnetic fields  $(\mathbf{E}^i, \mathbf{H}^i)$ . Since the matrix A depends on frequency, it must be generated and solved repeatedly at each frequency in order to obtain a solution over a frequency band, which can be time consuming.

To accelerate the solution of (8) over a frequency band, we employ the AWE method. In accordance with this method [2],  $x(\omega)$  is expanded into a rational function:

$$x(\omega) = \frac{\sum_{i=0}^{L} a_i (\omega - \omega_0)^i}{1 + \sum_{i=1}^{M} b_i (\omega - \omega_0)^i}$$
(9)

where  $\omega_0$  denotes the expansion point. The expansion coefficients  $a_i$  and  $b_j$  can be determined from the inverse and the L+M derivatives of A and the L+M derivatives of y at frequency  $\omega_0$ . In the case where one expansion point is not

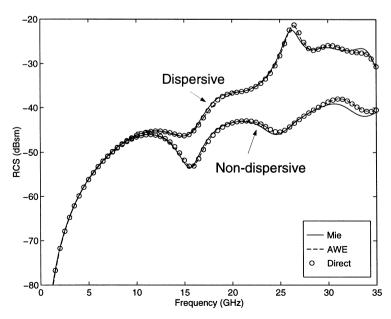


Figure 1 Backscatter RCS versus frequency of a dielectric sphere having a radius of 0.5 cm

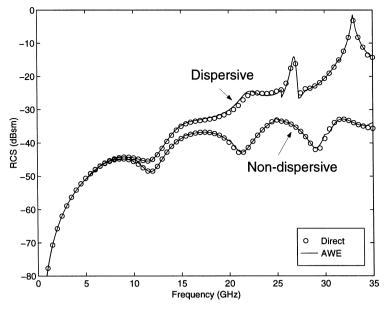


Figure 2 Backscatter RCS versus frequency of a dielectric cube having a side length of 1 cm

sufficient to cover the desired frequency band, one can use multipole expansion points, which can be selected automatically using a simple binary search algorithm [6].

#### 3. NUMERICAL EXAMPLES

Consider a dielectric sphere of radius 0.5 cm immersed in free space. For the numerical solution, the surface of the sphere is modeled by 648 triangular patches, resulting in 972 edges, and thus 1944 unknowns. The permittivity of the sphere is characterized by  $\epsilon'_{rs} = 2.56$ ,  $\epsilon'_{r\infty} = 1.0$ , and  $\tau =$ 1.59 ps/rad. As a result, as the frequency varies from 0.5 to 35 GHz, the real part of the relative permittivity varies from 2.56 to 2.22, and the imaginary part varies from -0.012 to -0.65. The backscatter radar cross section (RCS) of the sphere is shown in Figure 1, where the backscatter RCS of a nondispersive sphere of the same size and having a relative permittivity 2.56 is also given. Three solutions are displayed in the figure. One is the exact Mie series solution, the second is the solution obtained by solving (8) directly at each frequency, and the third is the solution obtained using the AWE method. With a frequency step of 0.5 GHz, it takes the direct method 24611 s to obtain the solution on a digital personal workstation (500 MHz Alpha 21164 processor). In contrast, the AWE method produces the solution with 0.01 GHz increments in 2206 s on the same computer. Figure 2 shows similar results for a 1 cm  $\times$  1 cm  $\times$  1 cm dielectric cube with normal incidence.

## 4. CONCLUSIONS

This letter described the application of the AWE method to the moment-method solution of the PMCHW equations for scattering by a dispersive dielectric object. It was shown that the use of AWE can speed up the calculation by more than an order of magnitude.

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# SLOT-LOADED RECTANGULAR MICROSTRIP ANTENNA FOR DUAL-FREQUENCY OPERATION

# Jui-Han Lu1

<sup>1</sup> Department of Electronic Communication Engineering National Kaohsiung Institute of Marine Technology Kaohsiung, Taiwan 811, R.O.C.

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ABSTRACT: A new design of a slot-loaded rectangular microstrip antenna for single-feed dual-frequency operation is proposed and experimentally studied. The slots loaded in the patch are a pair of bent slots centered in the patch and oriented along the resonant direction, with the spacing between the two slots being about half the patch's radiating-edge length. By varying the bent angle and horizontal-section length of the slots, the frequency ratio between the two operating frequencies is tunable in a range from about 1.28 to 1.79. The two frequencies are also of the same polarization planes and similar broadside radiation patterns.

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