Method for Analyzing Bit Error Rates (BERs) of Nonlinear Circuits and Systems for High-Performance Signaling

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***Abstract*— Bit error rate (BER) is an important figure of merit to evaluate the performance of a communication system. Analyzing the BER of a linear-time-invariant system has been extensively studied. However, analyzing the BER of nonlinear circuits and systems is challenging because it cannot rely on linear time invariant (LTI) principles, while an exhaustive nonlinear simulation is computationally prohibitive. To find BER, an exhaustive approach requires nonlinear simulations of 2^m bit patterns for a channel of m-bit memory, where m can be larger than 100 in today's high-speed and high-performance design. In this work, we develop a fast and accurate method to analyze the BER of large-scale nonlinear circuits. Only $O(k)$ nonlinear simulations are required, with k much less than 2^m and independent of 2^m. While performing $O(k)$ nonlinear simulations only, we find a method to determine the probability density function of the nonlinear responses, from which accurate BER results can be obtained. An error assessment method is also developed to evaluate the true error of the nonlinear signaling analysis without the need for knowing the entire nonlinear responses of the channel. Simulations of large-scale real-world nonlinear circuits have demonstrated the accuracy, efficiency, and capacity of the proposed method. A BER as low as 10^{-56} is accurately predicted.**

***Index Terms*— Bit error rate (BER), crosstalk, eye-diagram, intersymbol interference (ISI), nonlinear circuits, nonlinear signal integrity, nonlinear time invariant (non-LTI), signaling analysis, worst case eye.**

I. Introduction

NONLINEARITY is critical in today's circuits and system design for high-performance signaling. It calls for fast and accurate methods to account for it in signaling analysis. Like a linear channel for signaling [1], [2]; in a nonlinear channel, the signal received at the current bit can be affected by previous bits transmitted on the same channel due to intersymbol interference (ISI). The received signal can also be affected by other channels due to crosstalk effects. However, the performance of the nonlinear channel cannot be predicted by linear time-invariant (LTI)-based principles. Meanwhile, a brute-force nonlinear simulation method, which performs an exhaustive simulation of all possible input bit patterns, is not feasible for realistic signaling analysis either. Consider a nonlinear channel whose channel memory is 100, the brute-force method would require 2^{100} (1.27×10^{30}) nonlinear simulations of 2^{100} input bit patterns, each of which has 100 bits. The bit error rate (BER) is commonly used to assess a channel's performance [3]–[7]. A low BER at the level of 10^{-18} to 10^{-12} is, in general, required to ensure the quality of a communication system [4], [5]. Such a BER analysis is computationally prohibitive for a nonlinear channel if a brute-force approach is used since nonlinear simulations must be performed on at least 10^{12} to 10^{18} bits or an even larger number of bits for statistic accuracy.

Various methods have been developed to estimate the BER of a signaling system. Methods such as [6], [7] obtain the probability density function (PDF) of a channel response by convolution, from which BER can be found by an integration of the PDF. These methods are efficient requiring only a single bit response of the channel. However, they are inaccurate for analyzing BER of nonlinear circuits since they are based on LTI principles. Volterra model-based methods [8], [9] decompose a nonlinear channel into several linear ones to tackle the problem. The multiple edge response (MER) method [10]–[12] calculates the system response using MERs based on linear superposition. The method in [13] estimates the BER by using 2^m waveforms obtained from the mth order MER method, and a convolution approach for statistical analysis. In [14], the convolution process is divided into a nonlinear region, a transition region, and a linear region, where a different order of MER is applied to avoid the expensive 2^m simulations when m is large. Although these approaches have greatly improved the accuracy and efficiency for nonlinear signaling analysis, the range of validity can still be limited due to the use of linear principles. The Monte Carlo method [15]–[17] has been employed to simulate channel responses to a large number of random bits, from which BER is estimated. However, the number of nonlinear simulations that can be performed in feasible run time is still much smaller than that required for a low BER analysis. The multicanonical Monte Carlo method [18] can reduce the number of nonlinear simulations by using a nonuniform sampling strategy. To obtain a low BER, a common practice is to extrapolate the results of the Monte Carlo method by using the extreme value distribution (EVD) method [19]–[22] or Dual-Dirac model [23].

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But the accuracy of extrapolation is unpredictable. The Bayesian optimization (BO) has also been used to analyze the worst case eye of high-speed channels [24], [25].

In [26] and [27], a rank-revealing based method is developed for nonlinear signaling analysis. This method keeps the original nonlinear problem as it is without linearizing it, and meanwhile it only requires a small number of nonlinear simulations to characterize the channel performance. The underlying idea is to represent the nonlinear responses of a channel by a response matrix, and find a low-rank representation of the response matrix. The row dimension of the matrix (r) is the number of samples in a single bit width, and the column dimension is $2^m - 1$, where m is the channel memory. Although the rank of the matrix is bounded by the row dimension for large m, the rank pursued in the work of [27] is the number of distinct columns of the response matrix for a prescribed accuracy. In this way, when the low-rank model is found, other columns of the response matrix do not need to be simulated. A fast algorithm is also developed in [27] to obtain the low-rank model efficiently in $O(k^2 r)$ time complexity, where k is the rank that is small and independent of 2^m. However, no method for BER analysis is provided in [26] and [27]. Since only a small set of nonlinear simulations is performed, it is not clear how to accurately predict BER of a nonlinear channel using the method of [26], [27]. Furthermore, the error control of the method in [26] and [27] is related to the low-rank model, instead of physical parameters such as worst case eye, eye height (EH), eye width (EW), and so on. The true error of the resultant eye diagram is unknown and not directly controlled.

The aforementioned problems are solved in this work. First, an efficient and accurate method for BER analysis is developed for general nonlinear circuits and systems. Retaining the advantages of [27], the method only requires $O(k)$ nonlinear simulations, with k much less than 2^m and independent of 2^m. A typical number of k is in the hundreds. Meanwhile, the method is capable of finding the PDF of the nonlinear responses using the $O(k)$ simulations only without performing the rest of the $2^m - k$ simulations. Second, a new error assessment method is developed to assess the true error of worst case eye without the need of knowing the whole nonlinear responses. While assessing the error, the method further improves the accuracy of the nonlinear signaling analysis. The proposed work has been applied to real-world nonlinear circuits having a large channel memory and many crosstalk links. Its accuracy, efficiency, and capacity have been demonstrated. In [28], we have focused on the fast algorithm for rank revealing (RR), and discussed very little the method for analyzing BER. In this article, we focus on the BER analysis, describe the method for analyzing BER in detail, and also develop a new method for error assessment. In addition, we have performed an extensive number of numerical experiments to validate the performance of the proposed method for large-scale nonlinear signaling analysis.

The rest of the article is organized as follows. In Section II, we introduce the preliminary knowledge of this work including the LTI-based statistical method for BER analysis and the rank-revealing method of [26], [27] for nonlinear signaling analysis, and describe the problem encountered in BER analysis of nonlinear circuits. In Section III, we present the proposed method for analyzing BER. In Section IV, we detail a derivative check (DC) method for assessing true error and meanwhile controlling accuracy. In Section V, we analyze the accuracy and computational cost of the proposed method. The proposed method is then applied to analyze many real-world signaling problems provided by Intel, which is detailed in Section VI. In Section VII, we draw our conclusions.

II. Preliminaries and Problem Statement

In this section, we introduce the preliminaries of the proposed work, and describe the problems encountered in the BER analysis of nonlinear circuits.

A. LTI-Based Method for Analyzing BER

For an LTI system, the statistical method developed in [6] is fast and accurate for analyzing BER. Consider a channel whose memory is m bits, the method only requires a single bit response to generate the PDF of the entire channel response via a convolution procedure as follows:

$$P(v,t) = f_{-m+1}(v,t) \circledast \cdots \circledast f_{-1}(v,t) \circledast f_0(v,t) \quad (1)$$

where

$$f_i(v,t) = 0.5\delta(v) + 0.5\delta(v - y_i(t)) \quad (2)$$

and t denotes time, v denotes the value of the received signal, subscripts $i = 0, 1, \ldots, -m+1$ denote the indices of the input m bits with b_0 being the current bit and others as the preceding bits, y_i is the single bit response received when the ith bit is 1 and the other bits are 0, and $\circledast$ denotes a convolution. The above expression (1) can be recursively evaluated using (2) at the ith step to convolve with $P(v,t)$ obtained at the previous step, starting at $i = 0$ and finishing at $i = -m+1$. The whole computational cost is negligible, since (2) has only two nonzero values.

After $P(v,t)$ is found, the BER can be readily obtained by the following integration:

$$\text{BER}(v_{\text{th}}, t) = \int_{v_{\text{th}}}^{+\infty} P(v, t|b_0 = 0) + \int_{-\infty}^{v_{\text{th}}} P(v, t|b_0 = 1) \quad (3)$$

where v_{th} is the threshold voltage to determine the received v is 0 or 1, and $P(v,t|b_0 = 0)$ and $P(v,t|b_0 = 1)$ are the PDFs of the responses whose input bit b_0 is 0 and 1, respectively. The lowest nonzero value of BER is $1/2^m$ for a channel of m-bit memory, i.e., having one wrong bit out of all possibilities.

The PDF computed in (1) utilizes the principle of convolution, which is not valid when applied in nonlinear settings. Hence, we cannot directly use the method of [6] to obtain BER of nonlinear channels.

B. RR Method for Nonlinear Signaling Analysis

In [27], a fast and accurate method is developed for nonlinear signaling analysis. This method does not utilize any linear principles, and hence is suitable for analyzing general nonlinear channels. Here, we present an overview of the method and explain why it is not sufficient yet for

BER analysis, although it can accurately predict nonlinear eye diagrams and obtain worst case eye metrics.

Consider a nonlinear channel: one can send a number of random bits through the channel, and plot the responses received in a single bit width to obtain an eye diagram. The number of wrong bits received over the total number of bits sent yields the BER of the channel. However, for the BER and the eye diagram to be accurate, one has to use a large number of random bits especially for a high-speed channel, making an exhaustive channel simulation not feasible when the channel is nonlinear. In [27], it shows that if the channel memory is m bits, sending an exhaustive number of random bits to test the channel performance is equivalent to studying the channel response to 2^m kinds of input bit sequences. It introduces a response matrix $\mathbf{E}$ to store the 2^m responses. The row dimension of the matrix, r, is simply the number of samplings per bit width, and the column dimension c is $2^m - 1$ excluding the response to zero input. Each column of $\mathbf{E}$ denotes one response due to a single input bit sequence, with column j denoting the signal received at the current bit due to input bit pattern j (in decimal number). For example, for $m = 3$, there are seven columns in $\mathbf{E}$, which correspond to channel responses due to input bit patterns 001, 010, 011, 100, 101, 110, and 111, respectively.

The algorithm in [27] seeks the following low-rank representation of $\mathbf{E}$:

$$\tilde{\mathbf{E}} = \mathbf{E}_{:,\hat{J}}\left(\mathbf{E}_{\hat{I},\hat{J}}\right)^{-1}\left[\mathbf{E}_{\hat{I},\hat{J}} \quad \mathbf{E}_{\hat{I},J\backslash\hat{J}}\right] \tag{4}$$

where J, $\hat{J}$, and $\hat{I}$ denote the whole set of column indices, the set of selected column indices, and the set of selected row indices, respectively. The $\mathbf{E}_{:,\hat{J}}$ denotes the selected columns of $\mathbf{E}$, whose column indexes are contained in $\hat{J}$; the $\mathbf{E}_{\hat{I},J\backslash\hat{J}}$ represents the selected rows of $\mathbf{E}$, whose row indexes are contained in $\hat{I}$, and its column set does not include the selected columns in $\hat{J}$. The $\mathbf{E}_{\hat{I},\hat{J}}$ denotes the matrix formed in the selected rows ($\hat{I}$) and the selected columns ($\hat{J}$) of $\mathbf{E}$, while $\mathbf{E}^{-1}_{\hat{I},\hat{J}}$ is its inverse. The number of selected columns/rows is the rank of $\tilde{\mathbf{E}}$, denoted by k. The rank for the algorithm in [27] refers to the number of distinctive waveforms in the eye diagram for a prescribed accuracy. The column space of $\mathbf{E}_{\hat{I},J\backslash\hat{J}}$ is included in the space of $\mathbf{E}_{\hat{I},\hat{J}}$, and it does not need to be computed since its columns overlap with the k distinct waveforms. Finding a factorization like (4) can be computationally prohibitive requiring knowing the entire $\mathbf{E}$, in addition to computations of cubic time complexity for carrying out the low-rank representation. This computational challenge is overcome in [27], and one only needs to perform k nonlinear simulations, i.e., finding k columns of $\mathbf{E}$, and the algorithm that determines which k columns to simulate only requires $O(k^2r)$ operations in time and costing $O(kr)$ in memory.

However, since only k nonlinear responses are simulated, it seems not feasible to use the aforementioned method to predict BER, as BER requires the information of how many times each response appears among the 2^m possibilities at each time instant. Assuming each of the k distinct responses has an equal probability to appear in the output of the channel is error-prone. Even for a linear channel, the output signal generally has a PDF that is not uniform across all output values of the channel.

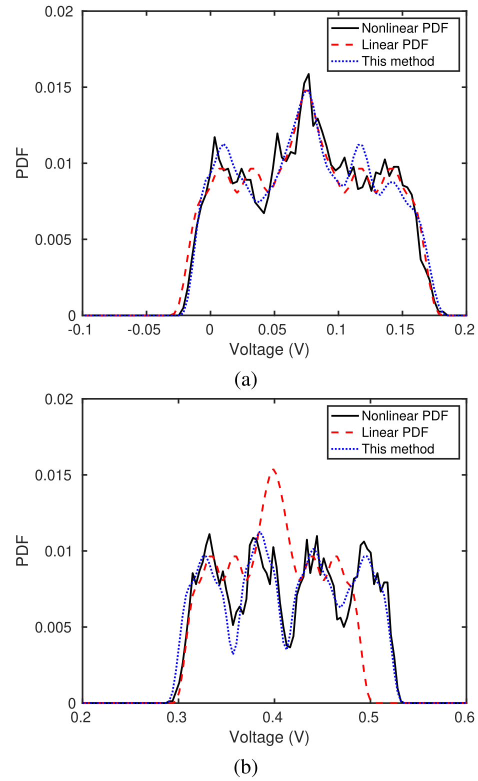


Fig. 1. Nonlinear PDF and its corresponding linear PDF of a nonlinear circuit (LPDDR5 channel). (a) Case I. (b) Case II.

III. Proposed Method for Nonlinear BER Analysis

To obtain the BER of a nonlinear channel, we need to find the PDF of the nonlinear responses. In the context of the method of [27], we need to find out the frequency in which each of the k nonlinear responses appears out of the 2^m responses. Let n_i ($i = 1, 2, \ldots, k$) be the number of times an ith type response appears among the 2^m responses, we have

$$n_1 + n_2 + \cdots + n_k = 2^m \tag{5}$$

and the BER at time instant t can be computed as

$$\text{BER}(t) = \frac{e_1(t)n_1 + e_2(t)n_2 + \cdots + e_k(t)n_k}{n_1 + n_2 + \cdots + n_k} \tag{6}$$

in which $e_i(t)$ is 1 or 0 depending on whether the received ith type signal at time t is wrong or not. Intuitively, one may simply shift the linear PDF, obtained by treating the nonlinear channel as a linear one, to obtain the nonlinear PDF, hoping that the shape of the PDF is preserved. However, we find that this relationship does not always hold true. Sometimes, the nonlinear PDF can be well approximated by a simple shift of the linear PDF as can be seen from Fig. 1(a). Sometimes, the nonlinear PDF is very different as can be seen from Fig. 1(b).

Although there is no clear relationship between the nonlinear PDF and the linear one for the whole channel responses, we find that if we separate the whole 2^m types of responses into *clusters*, there is a clear relationship between the nonlinear and linear PDF for each cluster, as both of them are governed by a Gaussian distribution. This important finding can be used to develop a fast and accurate cluster-based method to obtain the nonlinear PDF. The details of this method are given as follows.

A. Definition of Clusters

For a channel of memory m, the nonlinear response y received at the current bit b_0 is a function of both time t, and its preceding $m-1$ bits, $b_{-1}, b_{-2}, \ldots, b_{-m+1}$, as well as the bit following it, b_1. Thus

$$y(t, \mathbf{b}) = y(t, b_{-m+1}, \ldots, b_{-1}, b_0, b_1). \tag{7}$$

The b_1 is incorporated to close the eye diagram at the end of a single bit width because the response due to b_1 can have a rising edge that falls into the bit width of b_0. The effect of each bit on the current bit is different. Since the ISI decreases as the bit is further away from the current bit in time, typically, the bit b_{-m+1} has the smallest effect, whereas b_0 is the most significant. When crosstalk is involved, there is also a decay in the signal strength received at the current bit depending on the proximity of the channels. We term those bits that have a significant effect on the current bit as *significant bits*, and the others as *insignificant bits*. The significance of a bit b_i is determined by the maximum response received at the current bit when b_i is 1 while the other bits are 0, and hence

$$\text{sig}(i) = \mathbf{max}(|y(t, 0, \ldots, b_i = 1, \ldots, 0)| - V_{\text{ref}}) \tag{8}$$

where V_{ref} is the reference voltage when there is no channel input, and $\text{sig}(i)$ denotes the significance value of bit b_i. The following criterion is then used to quantitatively determine the set of insignificant bits:

$$\frac{\text{sig}(i)}{\mathbf{max}(\text{sig})} \leq \epsilon \tag{9}$$

in which $\mathbf{max}(\text{sig})$ denotes the maximum value of all significance values. We can select the value of epsilon according to the accuracy requirement. A choice of $\epsilon = 10\%$ is sufficient for use, which typically results in the last three bits, b_{-1}, b_0, b_1, being the significant bits. As an example, the significance values of the 13 input bits for a nonlinear low power double data rate fifth (LPDDR5) channel are plotted in Fig. 2. It shows that the two sets of bits can be well partitioned at the 10th bit (corresponding to b_{-2}), which is ten times smaller than the maximum significance value.

The *cluster* is then defined as the set of input bit patterns that share the same significant bits in common. For example, if the number of significant bits is 3 and they are the last 3 bits, then the bit patterns whose last three bits are 000 make a cluster; those whose last 3 bits are 001 make another cluster, and thus the input bit patterns make eight clusters in total. The channel responses in the same cluster share a dominant feature in common, and hence, their differences become small. Take the nonlinear LPDDR5 channel as an example, although the channel responses shown in the eye-diagram are quite different as can be seen from Fig. 3(a), the channel responses in each of the eight clusters are similar, as can be seen from Fig. 3(b).

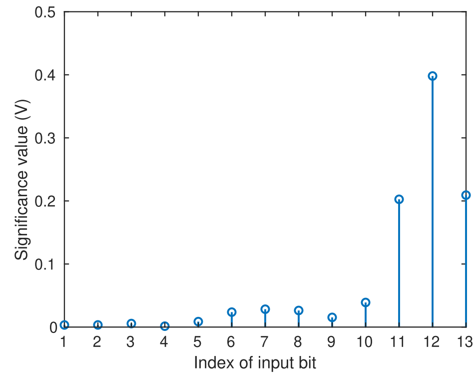


Fig. 2. Significance value of each bit in a typical nonlinear channel.

B. PDF of a Cluster

Assume the last m_s bits to be the significant bits. The whole channel responses are hence, classified into 2^{m_s} clusters. The nonlinear channel response y is a function of both significant bits $\mathbf{b}_s$ and insignificant ones $\mathbf{b}_{\text{ns}}$, thus

$$y(t, \mathbf{b}) = y(t, \mathbf{b}_s, \mathbf{b}_{\text{ns}}) \tag{10}$$

where

$$\mathbf{b}_s = \left[b_{-m_s+2}, \ldots, b_0, b_1\right] \tag{11}$$

$$\mathbf{b}_{\text{ns}} = \left[b_{-m+1}, \ldots, b_{-m_s}, b_{-m_s+1}\right] \tag{12}$$

and $\mathbf{b}$ includes both $\mathbf{b}_s$ and $\mathbf{b}_{\text{ns}}$. For a cluster whose m_s significant bits are $\bar{\mathbf{b}}_s$, there are 2^{m-m_s} responses that can be written as follows:

$$y\left(t, \bar{\mathbf{b}}_s, \mathbf{b}_{\text{ns}}\right) = y\left(t, \bar{\mathbf{b}}_s, \mathbf{0}\right) + \left[y\left(t, \bar{\mathbf{b}}_s, \mathbf{b}_{\text{ns}}\right) - y\left(t, \bar{\mathbf{b}}_s, \mathbf{0}\right)\right] \tag{13}$$

where the first term is dominant since it is determined by the significant bits. The second term can be expressed as a weighted sum of insignificant bits b_i as

$$y\left(t, \bar{\mathbf{b}}_s, \mathbf{b}_{\text{ns}}\right) - y\left(t, \bar{\mathbf{b}}_s, \mathbf{0}\right) = \sum_{i=-m+1}^{-m_s+1} w_i b_i \tag{14}$$

in which weights w_i are variables and can be time and bit pattern dependent due to nonlinear effects. Although w_i are unknown without nonlinear simulations, the above term is a sum of random variables having a similar dynamic range since it arises from insignificant bits. Based on the following central limit theorem (CLT) [29], the PDF of (13) can be well approximated by a bounded Gaussian distribution.

1) Central Limit Theorem: If S_n is the sum of n mutually independent random variables with mean μ and variance σ, then the PDF of S_n is well-approximated by a normal density (Gaussian distribution)

$$f_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/\left(2\sigma^2\right)}. \tag{15}$$

Therefore, for each cluster of the nonlinear channel responses, the PDF is governed by a Gaussian distribution, although the entire nonlinear PDF is not a Gaussian. To validate this, in Fig. 4, we plot the PDFs of all eight clusters for the same nonlinear example used to plot Fig. 1(b). As can be seen, the nonlinear PDF and the linear one for each cluster are similar, both having a Gaussian distribution, although

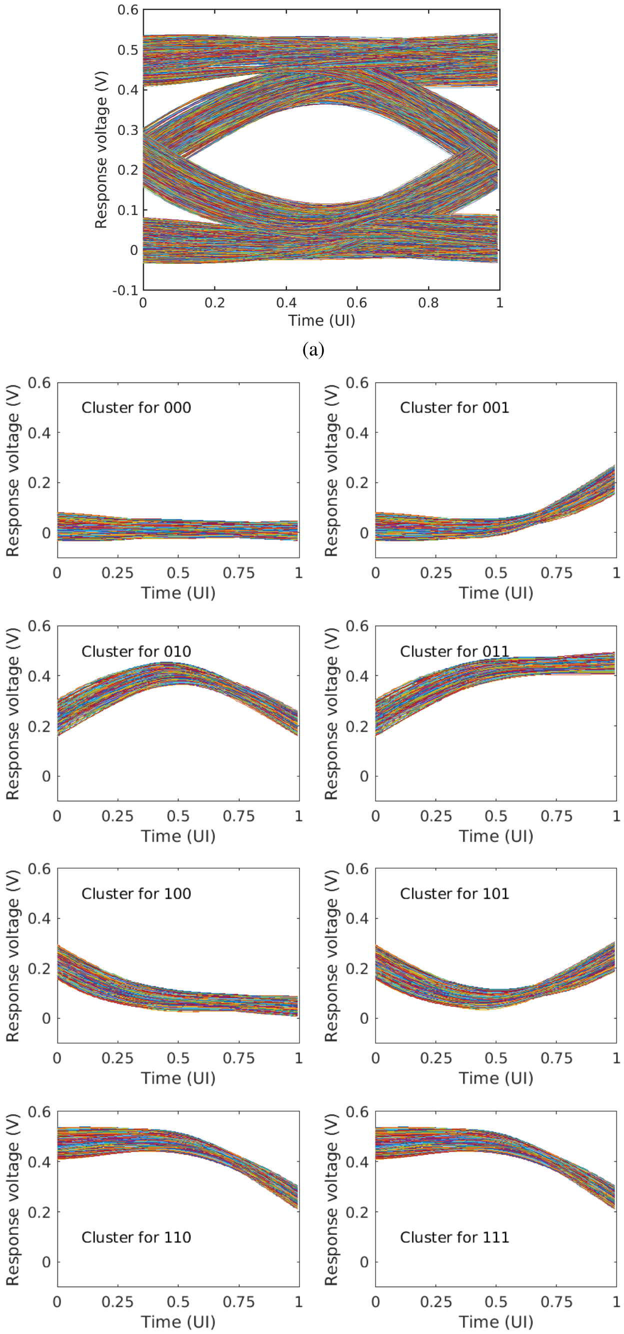


Fig. 3. Typical eye-diagram and its clusters of a nonlinear circuit. (a) Eye-diagram. (b) Eight clusters of responses.

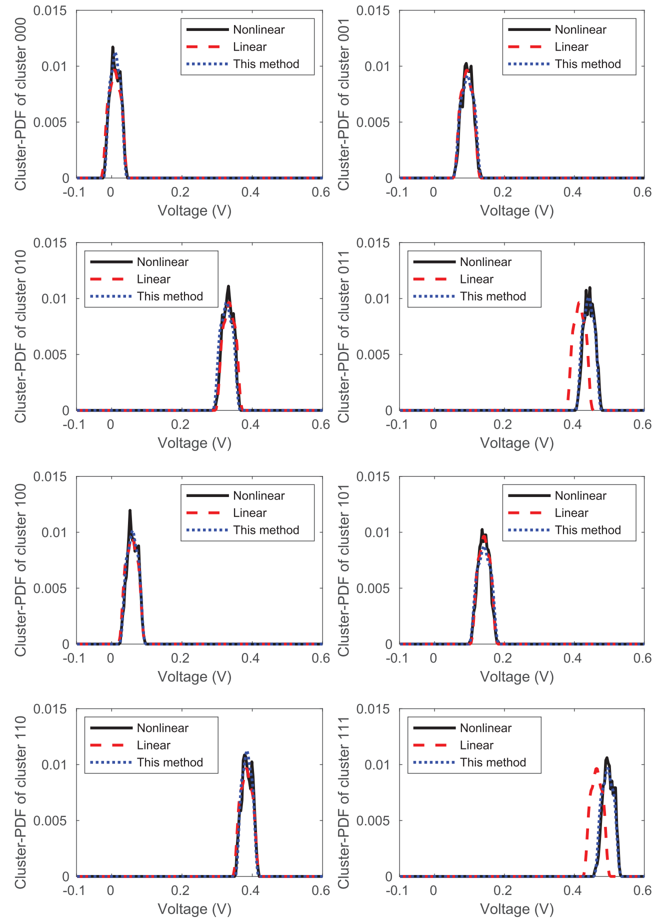


Fig. 4. PDFs of eight clusters for the example shown in Fig. 1(b).

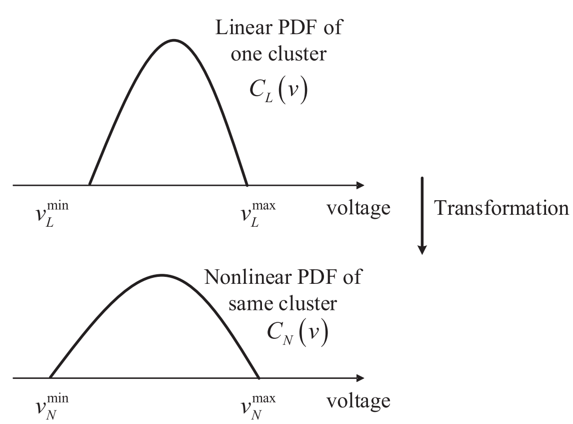


Fig. 5. Transformation between linear and nonlinear cluster PDF.

the nonlinear PDF and the linear one for the entire channel responses, as shown in Fig. 1(b), are quite different.

C. Obtaining the Nonlinear PDF

Since for each cluster, both nonlinear and linear PDFs follow the same Gaussian distribution, and the only difference is the mean μ and variance σ caused by different minimum and maximum values of y, the nonlinear PDF of each cluster can be obtained by a transformation of the linear PDF of the same cluster as shown in Fig. 5. The transformation can be realized at each time instant via a shift, a horizontal stretch, and a vertical stretch, satisfying the following two requirements.

1) The upper and lower bounds of the responses in each cluster are the actual maximum value ($v_N^{\max}$), and minimum value ($v_N^{\min}$) of the nonlinear responses of the channel instead of linear ones.
2) The total probability of the PDF of one cluster is $1/2^{m_s}$.

Mathematically, the transformation can be written as

$$C_N(v,t) = \alpha C_L\left(\alpha v - \alpha v_N^{\min} + v_L^{\min}, t\right) \tag{16}$$

where

$$\alpha = \frac{v_L^{\max} - v_L^{\min}}{v_N^{\max} - v_N^{\min}} \tag{17}$$

and $C_N(v,t)$, $C_L(v,t)$ are, respectively, the nonlinear and linear PDF of the cluster, $v_L^{\max}$, $v_L^{\min}$, $v_N^{\max}$, and $v_N^{\min}$ are the maximum and minimum values of the linear responses, and nonlinear ones of the cluster, respectively. The linear PDF $C_L(v,t)$ can be obtained by the method described in Section II-A. For each cluster, since the m_s significant bits are fixed, the convolution only needs to be performed $(m - m_s)$ times as follows:

$$C_L(v,t) = f_{-m+1}(v,t) \circledast f_{-m+2}(v) \circledast \cdots \circledast f_{-m_s+1}(v,t). \tag{18}$$

The nonlinear PDF of the whole 2^m responses, $P_N(v,t)$, can then be obtained by the sum of the nonlinear PDF of each cluster, and hence

$$P_N(v,t) = \sum_{i=1}^{2^{m_s}} C_N^i(v,t). \tag{19}$$

In Fig. 4, we have shown the nonlinear PDF obtained by the proposed method of each cluster. The summation of them is plotted in Fig. 1 labeled as *This method*. As can be seen, it agrees very well with the reference *Nonlinear PDF* obtained from a brute-force simulation.

D. BER Computation and Summary of the Procedure

With the nonlinear PDF obtained, the BER can be readily evaluated by performing a numerical integration based on (3).

In summary, the proposed fast BER analysis has five steps for nonlinear signaling.

1) Determine significant bits and define clusters.
2) Find the bounds (maximum and minimum values) of the nonlinear responses in each cluster, using the method of [27].
3) Compute the nonlinear PDF of each cluster using (16).
4) Sum up the nonlinear PDF of each cluster to obtain the total nonlinear PDF.
5) Compute BER via the integration of the nonlinear PDF.

IV. Derivative Check (DC) for Error Assessment and Accuracy Control

The method in [27] has a relative error check, but it is the error of the rank-k model, not the error of physical parameters of interest, although the smaller the former, the less the latter. Here, we develop a DC method to evaluate and control error. This error control method reveals directly the error of the worst case eye without the need for knowing the whole responses. Hence, we can use it to find the bounds of each cluster of channel responses more accurately, which further improves the accuracy of PDF and hence, BER. It is worth mentioning that this DC method can be very effective because the method in [27] has found an accurate global optimum in terms of worst case eye, and hence, a derivative computation can help assess the error and further refine the accuracy at the global optimal point. If the DC is built upon another method that fails to find the global optimum, then it won't be effective owing to its local nature in optimization.

For each cluster, after using the method of [27] to find the maximum and minimum values of its responses (called cluster bounds) at each time instant, the bit patterns that produce the bounds are also known. We then change each bit in turn to obtain the derivative of the response with respect to each bit. The smaller the derivative is, the closer the current bound is to the true bound, and hence, the smaller the error.

Consider a time t_n, let $\bar{\mathbf{b}}^n$ be the bit patterns that produce the upper or lower bound of the cluster, the partial derivative of the nonlinear response y with respect to bit b_i at t_n can be evaluated as

$$\frac{\partial y(t_n, \bar{\mathbf{b}}^n)}{\partial b_i} = y(t_n, \bar{b}^n_{-m+1}, \ldots, b_i = 1, \ldots, \bar{b}^n_1) - y(t_n, \bar{b}^n_{-m+1}, \ldots, b_i = 0, \ldots, \bar{b}^n_1) \tag{20}$$

in which only the value of b_i is different, while the values of the rest bits in $\bar{\mathbf{b}}^n$ do not change. The sign of this derivative can tell us whether we need to change the value of b_i from $\bar{b}_i^n$ to $1 - \bar{b}_i^n$ to increase or decrease the response. Specifically.

1) If $\partial y(t_n, \bar{\mathbf{b}}^n)/\partial b_i > 0$.
   a) If $\bar{\mathbf{b}}^n$ corresponds to an upper bound of the cluster responses, i.e., a maximum value, then we change the value of b_i if its current value is 0, since such a change increases y, and hence, yields a larger upper bound.
   b) If $\bar{\mathbf{b}}^n$ corresponds to a lower bound of the cluster responses, i.e., a minimum value, then we change the value of b_i if its current value is 1, since such a change decreases y, and hence, yields a smaller lower bound.
2) If $\partial y(t_n, \bar{\mathbf{b}}^n)/\partial b_i < 0$.
   a) If $\bar{\mathbf{b}}^n$ corresponds to an upper bound of the cluster responses, then we change the value of b_i if its current value is 1.
   b) If $\bar{\mathbf{b}}^n$ corresponds to a lower bound of the cluster responses, then we change the value of b_i if its current value is 0.

By using the above approach, we can check the $m - m_s$ insignificant bits and determine a new bit pattern $\bar{\bar{\mathbf{b}}}^n$ that can produce a more accurate bound. The error of the worst case eye obtained from rank-revealing can be assessed by evaluating the average difference between the new bound and the previous bound at r time instants as

$$\text{err} = \frac{1}{r}\sum_{n=1}^{r}\left|y\left(t_n, \bar{\bar{\mathbf{b}}}^n_{\max}\right) - y\left(t_n, \bar{\mathbf{b}}^n_{\max}\right)\right| + \frac{1}{r}\sum_{n=1}^{r}\left|y\left(t_n, \bar{\bar{\mathbf{b}}}^n_{\min}\right) - y\left(t_n, \bar{\mathbf{b}}^n_{\min}\right)\right| \tag{21}$$

where the subscripts max and min represent bit patterns associated with upper and lower bounds, respectively. If this error is small but it is larger than a prescribed accuracy, we can perform the DC one or a few more times to improve the accuracy. If this error is relatively large, it means the selected k responses are not enough and we need to go back to the rank-revealing part to add more nonlinear simulations to get closer to the global optimal point before applying the DC.

V. Accuracy and Computational Cost

The proposed method utilizes the fact that the PDF of each cluster of responses is governed by a Gaussian distribution to find the BER of a nonlinear channel. Its accuracy can be well controlled by the number of significant bits used to partition clusters and the accuracy of cluster bounds. The latter is ensured by the RR method and the DC, and controlled by the desired accuracy. As for the number of significant bits m_s, if more significant bits are chosen, the error of BER is smaller. However, the computational cost will also increase. The smallest number of significant bits is 1, i.e., using the current bit. We find that, in general, using no greater than 3 significant bits is sufficient in obtaining a good accuracy in BER.

The computational cost of the proposed method can be analyzed step by step. The cost of Steps 1, 3, and 4 shown in Section III-D is trivial. Step 2 requires the use of the RR for finding the nonlinear responses of each cluster and DC for error control, from which the cluster bounds are determined. The complexity of the RR algorithm is $O(k^2r)$ in time and $O(kr)$ in memory, where k is the rank. The number of nonlinear simulations is k, which is much smaller than 2^m, and independent of 2^m. For the DC of the response at one time instant, we need to perform additional $2(m - m_s)$ nonlinear simulations to obtain the derivative with respect to each of the $m - m_s$ insignificant bits, for both the upper and lower bounds of one cluster. But the total number of additional nonlinear simulations invoked for the DC is small and usually less than k. Overall, the computational cost of the proposed method is dominated by the $O(k)$ nonlinear simulations since the time for RR and DC is negligible, and hence, the method is efficient.

In the circuits simulated in this work, we examined both small m_s and larger ones such as $m_s = 9$. If even larger m_s is encountered, using 2^{m_s} clusters directly may not be efficient. In this case, one can divide the m_s significant bits into g groups, and apply the proposed method recursively. For example, let the g groups be sig_1, sig_2, …, and sig_g, respectively, from the least significant to the most significant. To obtain the PDF, first, the proposed method can be used to generate the PDF of all nonlinear responses due to insignificant bits (denoted by insig) and significant bits in sig_1 group, denoted by $P(\text{insig}, \text{sig}_1)$. After getting $P(\text{insig}, \text{sig}_1)$, we treat sig_2 as significant bits, and the combined sig_1 and insig bits as insignificant bits, and apply the proposed method again to obtain the PDF due to insig, sig_1, and sig_2, and thereby $P(\text{insig}, \text{sig}_1, \text{sig}_2)$. To be specific, this is to perform Steps 1 to 4 shown in Section III-D, where the Step 4 is modified to shift and stretch the $P(\text{insig}, \text{sig}_1)$ based on the bounds found in Step 1 for the cluster defined by sig_2 bits. Basically, $P(\text{insig}, \text{sig}_1)$ is nothing but the PDF for the cluster 0 whose sig_2 bits are all 0. Since sig_2 now makes the significant bits, for other clusters whose sig_2 bits assume another set of values, their PDF gets shifted to the new bounds found in Step 1, and stretched accordingly such as the area under the PDF remains the same. We continue the aforementioned process until we find $P(\text{insig}, \text{sig}_1, \text{sig}_2, \ldots, \text{sig}_g)$, which is the total nonlinear PDF. Since each group has a small number of significant bits, now m_s/g, the computational cost is small.

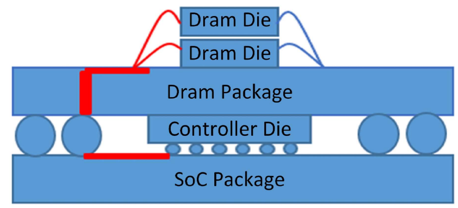


Fig. 6. Nonlinear LPDD5 Channel with a 13-bit channel memory.

VI. Simulation Results

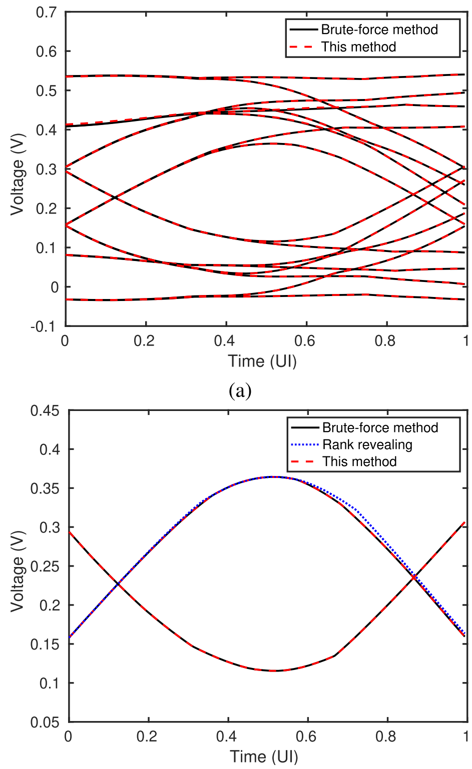
In this section, we perform the signaling analysis of a number of real-world nonlinear channels to validate the performance of the proposed method to obtain accurate BER. The results are compared with either nonlinear circuit simulator HSPICE which is an industry-standard, or reference results provided by Intel that are validated by measurements.

A. Nonlinear LPDDR5 Channel With 13 Bits

The first example is a nonlinear LPDDR5 channel with 13-bits of memory as shown in Fig. 6, which is the example used for explaining concepts in Section III. The number of exhaustive nonlinear simulations of this example is 8191, which can be used to validate the accuracy of the proposed method within a feasible run time. The exhaustive nonlinear simulation results are generated by using HSPICE. The nonlinearity of this channel is due to the nonlinear drivers used for power efficiency. The data rate is 6400 MT/s and the sampling rate is 1 ps. The significance value of each bit has been shown in Fig. 2, based on which the last three bits are chosen as the significant bits. Therefore, the entire nonlinear response is categorized into eight clusters for BER analysis.

The proposed method only performs 397 nonlinear simulations, among which 232 are simulated in the step of RR to obtain cluster responses, and the additional 165 simulations are used for the DC. The CPU time of the RR and the DC are 4.2 and 3.6 s, respectively. The resultant cluster bounds are shown to agree very well with those generated from a brute-force exhaustive nonlinear simulation, as can be seen from Fig. 7(b). The EH and EW obtained from the proposed method are 248.8 mV and 115.8 ps, which are identical to those of the brute-force method as shown in Table I. The EH and EW obtained by RR without using the DC are 248.8 mV, and 116.5 ps, respectively, and hence, the DC method improves the accuracy. In addition, the DC predicts that the error shown in (21) of the worst case eye obtained by the RR is 2.44 mV, which is the same as the true error (obtained by using the brute-force method). After one step of the DC, the error shown in (21) becomes zero. This shows that the DC is an effective method for error assessment and accuracy improvement. The worst case eyes obtained by the three methods are compared in Fig. 7(c). Good agreement among the three and improved accuracy using DC can be observed.

We then proceed to compute BER, where 1001 V intervals are used. The time cost is 1.8 s only. The BER computed from the proposed method is plotted in Fig. 8(b), which is shown to agree very well with that obtained by the brute-force



(b)

Fig. 7. Signaling analysis of a nonlinear LPDD5 channel. (a) Cluster bounds obtained by the proposed method in comparison with the brute-force method (commercial HSPICE). (b) Worst case eye obtained by the brute-force method (commercial HSPICE), the RR, the proposed method, and the linear method (PDA).

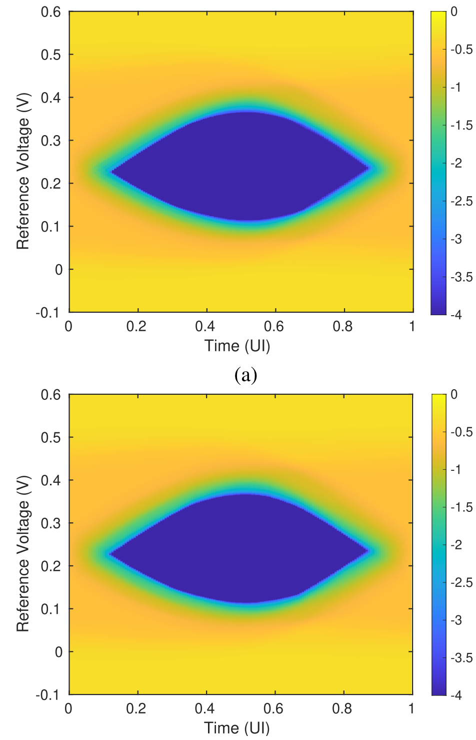
TABLE I
COMPARISON OF EYE METRICS OBTAINED BY LINEAR METHOD (PDA), RR, RR WITH DC, AND BRUTE-FORCE METHOD (HSPICE SIMULATION)

Results	RR	RR with DC	Brute-force (HSPICE Simulation)
EW (ps)	116.5	115.8	115.8
EH (mV)	248.8	248.8	248.8

TABLE II
RELATIVE ERROR OF BER OBTAINED BY THIS METHOD

Cases of this method	3 significant bits	9 significant bits
Time instant = 50	1.68%	0.27%
Time instant = 80	1.71%	0.23%
Time instant = 120	1.06%	0.27%
Voltage = 0.23 V	4.73%	1.71%

method shown in Fig. 8(a). For further comparison, we plot a vertical cut at the 80th time instant in Fig. 9, for two choices of significant bits. They are shown to agree very well with the reference brute-force solution. The relative errors of more BER bathtubs figures (horizontal and vertical cuts of the BER) are listed in Table II, for two choices of significant bits. As can be seen, the error of the proposed method is small, and also controllable. As theoretically expected, when the number of significant bits increases, the error decreases.



(b)

Fig. 8. BER diagram (in logarithmic scale) obtained by this method in comparison with the BER from an exhaustive nonlinear simulation (HSPICE). (a) BER obtained from a brute-force nonlinear simulation (HSPICE). (b) BER obtained by this method.

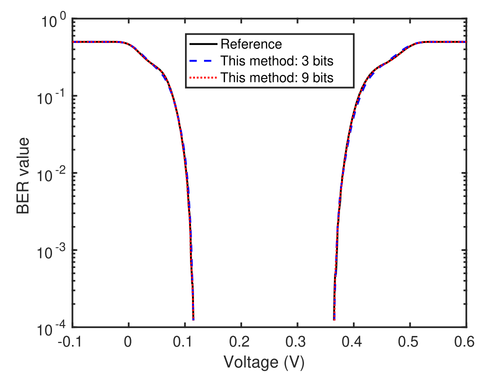
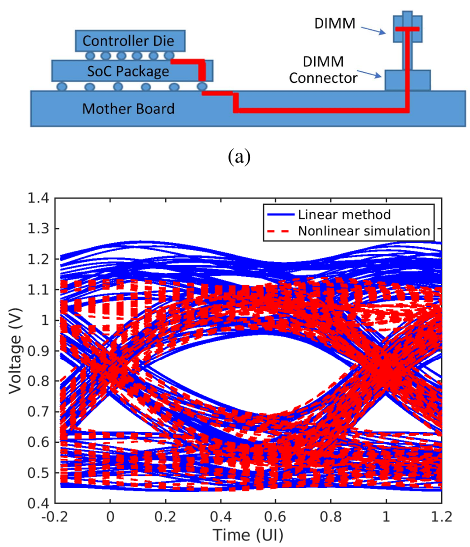


Fig. 9. Comparison of bathtub obtained by brute-force method (HSPICE) and this method. Vertical-cut bathtub at time instant 80.

B. Nonlinear DDR5 Channel With 25 Bits

The second example is a nonlinear 1DPC [one (dual in-line memory module (DIMM) per channel] DDR5 channel for a test chip comprising package, board, connector, DIMM, etc., as shown in Fig. 10(a). Its channel memory is 25 bits. The nonlinearity is due to the nonlinear driver in this example. In Fig. 10(b), we plot the eye-diagrams generated by the nonlinear simulation using commercial HSPICE, in comparison with that from the linear-based peak distortion analysis (PDA) method. As can be seen, even though only a partial set of bit patterns are simulated in HSPICE (complete ones are not feasible), the two already are very different, indicating the strong nonlinearity of the circuit. An LTI-based analysis hence would yield a big error if used to simulate this example.



(a)

(b)

Fig. 10. Nonlinear DD5 channel with a 25-bit channel memory. (a) Circuit of nonlinear DD5 channel. (b) Comparison of eye-diagram obtained by nonlinear simulation (Commercial HSPICE) and linear method (PDA).

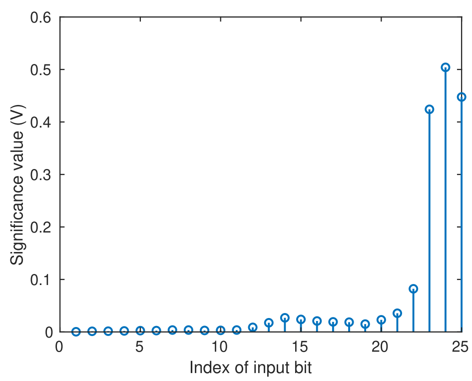


Fig. 11. Significance values of 25 input bits.

The data rate is 5600 MT/s and the sampling rate is 1 ps. The significance value of each bit is shown in Fig. 11, based on which the last three bits are chosen as significant bits. As a result, eight clusters are used in the proposed BER analysis.

The proposed method uses 603 nonlinear simulations in total, among which 386 nonlinear simulations are used for RR and the additional 217 are used for the DC. The run time of the proposed algorithm is 15 s. We use the BO [25], [27] method to validate the accuracy of cluster bounds obtained by the proposed method since an exhaustive nonlinear simulation is not feasible in this example. Since BO is costly, only the worst case 1 and 0 are optimized. To facilitate BO's optimization, we also provide BO with the worst case results obtained by RR [27] as the initial guess, because otherwise, the run time of BO can be too long to tolerate. Even with such a good initial guess, the BO method takes 3018 steps, and hence, simulates additional 3018 nonlinear cases to finish optimization, generating a worst case 1 and 0 shown in Fig. 12. Compared with BO, the proposed method is clearly more accurate since it produces a *worse* worst case result: the EW obtained by this method is 138.79 ps, which is 1.31 ps smaller

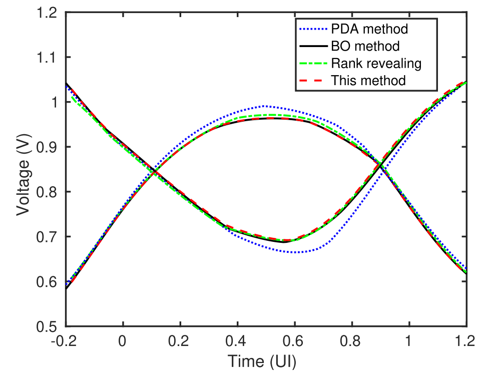
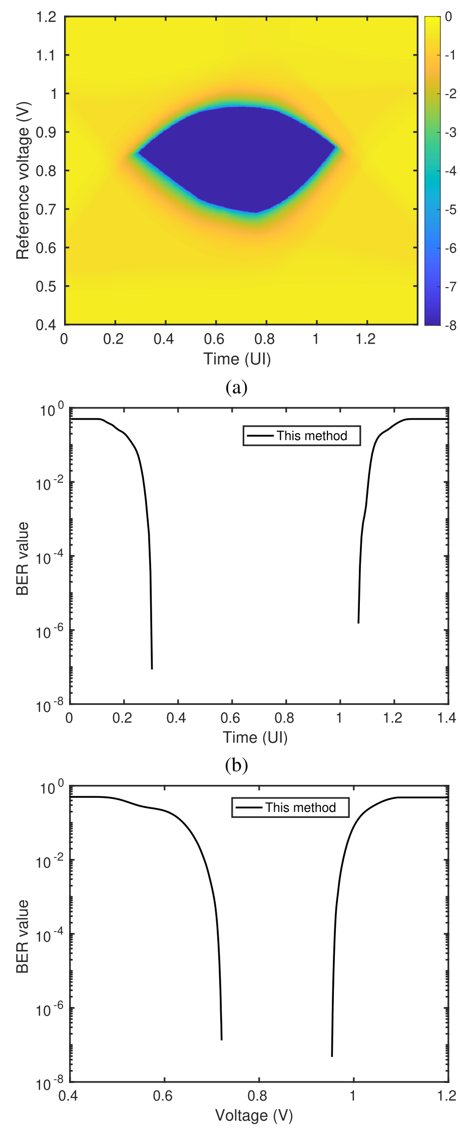


Fig. 12. Comparison of worst case eye obtained by this method, BO method and linear-based PDA method.



(a)

(b)

(c)

Fig. 13. BER analysis results obtained by this method. (a) BER in logarithmic scale. (b) Horizontal-cut bathtub at 0.85 V. (c) Vertical-cut bathtub at time instant 100.

than BO; and the EH obtained by this method is 270.94 mV, which is 4.31 mV smaller than BO. In addition, we simulate this example with a linear-based PDA method [6]. As can be seen from Fig. 12 and Table III, an LTI-based approach fails

TABLE III
COMPARISON OF WORST CASE EYE ANALYSIS OBTAINED BY BO METHOD, THIS METHOD AND PDA METHOD

Results	BO	This method	PDA
Nonlinear simulation no.	3018	603	1024
EW (ps)	140.10	138.79	144.48
EH (mV)	275.25	270.94	318.50

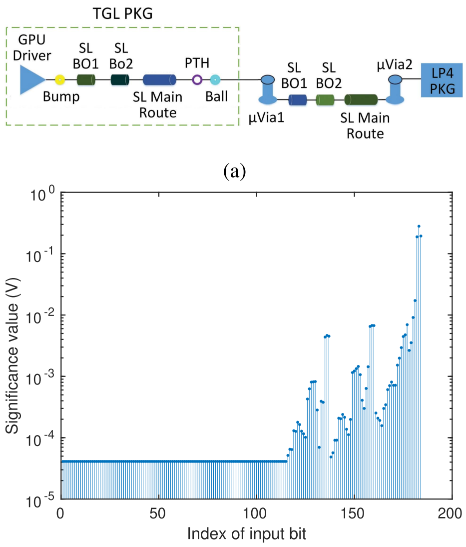


Fig. 14. LPDDR5 channel with 184 bits. (a) Channel. (b) Significance values of all bits.

to capture nonlinear effects in this example, and the worst case eye predicted by the LTI approach is significantly off from the accurate results. The EH and EW predicted by the rank-revealing are 278.43 mV, and 141.09 ps, respectively, for this example. The DC predicts an error of 9.9 mV for the worst case eye-diagram obtained by the RR, and the accuracy is further improved after the DC that can be seen from Fig. 12.

We then proceed to compute BER. In total, 4001 V intervals are used for BER analysis, costing 1 min 27 s. The resultant BER is plotted in Fig. 13(a). A horizontal cut at 0.85 V and a vertical cut at time instant 100 are plotted in Fig. 13(b) and (c), respectively. The lowest BER obtained by the proposed method is shown to agree very well with its theoretical value, which is $1/2^{25} = 2.98 \times 10^{-8}$. A brute-force nonlinear simulation of BER for this example is not feasible because of the large number of bits.

C. LPDDR5 Channel With 184 Bits

The third example is a linear LPDDR5 Channel for DQ [double input–output (IO)] write direction, which is illustrated in Fig. 14(a). We include one victim and seven aggressors to study both ISI and crosstalk effects. The total number of bits considered in this simulation is 184. Although this channel is a linear channel, we treat it as a nonlinear one to test the capability of the proposed algorithm. This example serves as a good testing case since the bit number is large while the reference result is known (provided by Intel Corporation, which has a measured channel model using which a PDA and statistical analysis based in-house tool is applied to obtain the BER. The PDA and statistical analysis of Intel's in-house tool are experimentally validated as shown in [6]). The data rate is 6400 MT/s and the sampling time is 1 ps. In Fig. 14(b), we plot the significance values of all channel bits. As can be seen, among the 184 bits, the significance values of the last 3 bits in the victim channel are extremely larger than others. Hence, again, we choose the last 3 bits as the significant bits and thereby use eight clusters in the BER analysis.

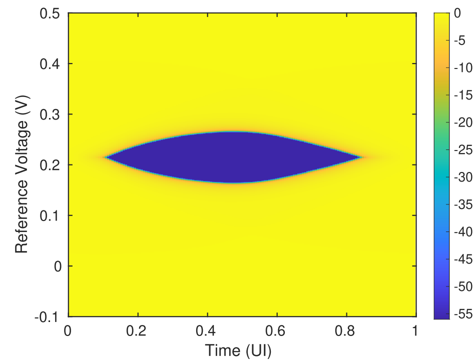


Fig. 15. BER diagram of a channel with 184 bits.

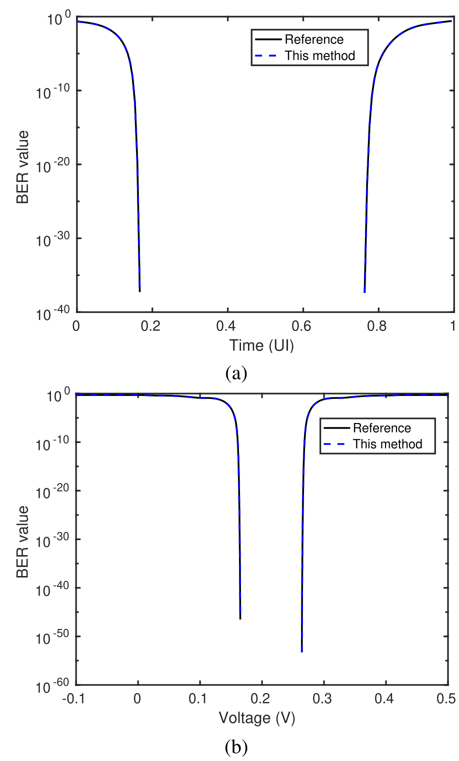


Fig. 16. Comparison of BER obtained by this method in comparison with industry reference results validated by measurements. (a) Horizontal-cut bathtub at 0.23 V. (b) Vertical-cut bathtub at time instant 74.

A brute-force simulation would require $2^{184} - 1$ simulations for BER analysis. In contrast, the proposed algorithm only simulates 1764 responses, where the RR uses 688 responses and the DC uses 1076 responses. The CPU run time of the algorithm is 3.2 s for RR and 1.4 s for the DC. The EH and EW are found to be 99.00 mV, and 112.79 ps, respectively, which are identical to the reference results provided by Intel. The CPU time of computing BER is 70 s. The resultant BER is plotted in Fig. 15, which is also identical to the reference BER. A horizontal cut of the BER at 0.23 V and a vertical cut at time instant 74 are plotted in Fig. 16 in comparison with reference results of the BER. Excellent agreement can be observed. The BER predicted by this method is as low as 4.2×10^{-56}, which agrees well with the theoretical prediction and reference result.

VII. Conclusion

The BER computation in large-scale nonlinear signaling analysis is a challenging problem that has not been well addressed. In this work, we develop a fast and accurate method for BER analysis of nonlinear circuits and systems. It is as efficient as the statistical method in [6] but suitable for both nonlinear and linear circuits. In this method, we only need to perform nonlinear simulations of $O(k)$ bit patterns, where k is a small number independent of the number of exhaustive nonlinear simulations. We then use the $O(k)$ simulation results to obtain the PDF of the nonlinear responses. We find that the input bits can be categorized into significant bits and insignificant ones based on their importance to the received signal, and the contribution of insignificant bits to a nonlinear channel response can be well characterized by a Gaussian distribution since it is the summation of a number of mutually independent random numbers. The maximum and minimum values of the distribution are very different from those predicted by a linear method. But they can be accurately found using the rank-revealing method that is generic for nonlinear signaling analysis. Using this finding, we separate the whole nonlinear responses into clusters based on significant bits and obtain the PDF of each cluster, the sum of which makes the whole PDF of the nonlinear responses. Depending on applications, the entire channel response may also be partitioned using different ways into clusters having a Gaussian PDF. For example, the $O(k)$ nonlinear responses found in this work can be used as $O(k)$ clusters. A DC method is also developed to assess the true error of the nonlinear signaling analysis results without the need for knowing the entire nonlinear responses. The method further improves the accuracy of the worst case eye and BER. The application of the proposed work to real-world large-scale nonlinear signaling analysis has demonstrated the accuracy, efficiency, and capacity of this work. A BER as low as 10^{-56} is accurately predicted by the proposed method. In addition to analyzing the BER of digital integrated circuits, the proposed work can be applied to address jitter and equalization problems and signal analysis in other applications.

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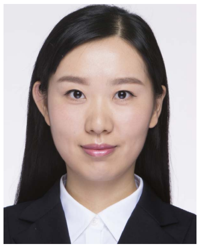

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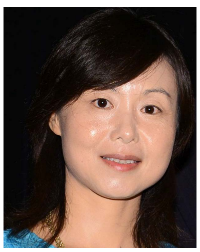

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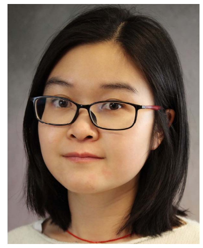

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