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Fast and accurate frequency-sweep calculations using asymptotic waveform evaluation and the combined-field integral equation

Dan Jiao, Xian-Yang Zhu, and Jian-Ming Jin

Center for Computational Electromagnetics, Department of Electrical and Computer Engineering
University of Illinois, Urbana

Abstract. The method of asymptotic waveform evaluation (AWE) is applied to the combined-field integral equation (CFIE) to achieve fast and accurate frequency-sweep calculations of electromagnetic scattering and radiation by three-dimensional conducting and dielectric objects. The employment of the CFIE eliminates the interior resonance problem suffered by both the electric-field integral equation and the magnetic-field integral equation. It is shown that the use of AWE can speed up the calculation by more than an order of magnitude. It is also shown that when combined with the complex frequency hopping technique, the AWE method can produce an accurate solution within a prespecified frequency band. Numerical examples are presented to demonstrate the performance of the proposed method.

1. Introduction

Many electromagnetic applications require the calculation of the response of a device over a wide frequency band rather than at one or a few isolated frequencies. For example, for radar target recognition, one has to compute the radar cross section (RCS) of a target over a wide frequency band to generate range profiles and synthetic aperture radar (SAR) images. For the analysis of antennas, especially wideband antennas, one has to calculate the input impedance at many frequency points. Such calculations can be very time consuming when a traditional frequency domain numerical method is used because a set of algebraic equations must be solved repeatedly at many frequency points. The number of algebraic equations is proportional to the electrical size of the problem and can be large for most applications. Therefore there is an urgent need to find efficient solution techniques for determining the frequency response over a frequency band. Some efforts in this direction include the methods of model-based parameter estimation (MBPE) [*Burke et al.*, 1989; *Kottapalli et al.*, 1991] and impedance matrix interpolation [*Newman*, 1988], both of which have been applied to the method of moments.

A technique similar to the MBPE is the method of asymptotic waveform evaluation (AWE) [*Pillage and Rohrer*, 1990], which was originally developed for high-speed circuit analysis. In AWE, the transfer function of a circuit is expanded into a series, and the circuit model is then approximated with a lower-order transfer function by matching moments. There have been many papers published on AWE, and it is beyond the scope of this paper to give a complete review of the literature. The reader is referred to *Chiprout and Nakhla* [1994] for a detailed description of the method. A good tutorial article [*Smith et al.*, 1997] on AWE is also available for electromagnetics researchers. The AWE method has recently been applied to the finite element and finite difference analysis of electromagnetic problems [*Yuan and Cendes*, 1993; *Kolbehdari et al.*, 1996; *Li et al.*, 1996; *Gong and Volakis*, 1996; *Polstyanko et al.*, 1997; *Zhang and Jin*, 1998; *Bracken et al.*, 1998; *Zhang and Lee*, 1998]. In these applications, the implementation of AWE is straightforward since the resultant matrix equation has a simple frequency dependence. (In the limited

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applications of AWE to integral equations, the dependence of the scalar Green's function on frequency is often neglected in order to arrive at a simplified final numerical system [*Bracken et al.*, 1998].)

A very useful solution technique for electromagnetic scattering and radiation by conducting and dielectric objects is the method of moments (MOM), which solves a surface integral equation (SIE) for the electric current on the surface of an object. This method is advantageous because (1) it limits the unknown to the current on the surface of an object, and (2) it satisfies the radiation condition via the Green's function. However, the method results in a dense matrix that is computationally expensive to generate and invert. Since this matrix depends on frequency in a complex manner, one has to repeat the calculations at each frequency to obtain the solution over a band of frequencies. Recently, *Reddy et al.* [1998] applied AWE to the MOM solution of the electric-field integral equation (EFIE) for the fast computation of the RCS of a perfect electric conductor (PEC) object. They showed that the AWE method requires less CPU time to obtain a frequency response and it can speed up the RCS calculation by a factor of 3, compared with the direct calculation over a band of interest.

In this paper, the AWE method is applied to the combined-field integral equation (CFIE) to achieve fast and accurate frequency-sweep calculations of electromagnetic scattering and radiation by three-dimensional (3-D) PEC and dielectric objects. The employment of the CFIE eliminates the interior resonance problem suffered by both the EFIE and the magnetic-field integral equation (MFIE). It is shown that the use of AWE can speed up the calculation by more than an order of magnitude. It is also shown that when combined with the complex frequency hopping (CFH) technique, the AWE method can produce an accurate solution within a prespecified frequency band. Numerical examples are presented to demonstrate the performance of the proposed method.

2. Formulation

Consider an arbitrarily shaped 3-D conducting object illuminated by an incident field $\mathbf{E}^i(\mathbf{r})$. The EFIE is given by

$$\hat{n} \times \iint_S \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}(\mathbf{r}')d\mathbf{r}' = \frac{1}{jk\eta}\,\hat{n} \times \mathbf{E}^i(\mathbf{r}) \qquad (1)$$

on S, where S denotes the conducting surface of the object, $\hat{n}$ is an outwardly directed normal, and $\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is the well-known free-space dyadic Green's function, given by

$$\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2}\right) g(\mathbf{r},\mathbf{r}')$$
$$g(\mathbf{r},\mathbf{r}') = \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \qquad (2)$$

with $\bar{\mathbf{I}}$ being the unit dyad. Also, $\mathbf{J}(\mathbf{r}')$ denotes the unknown surface current density at a point $\mathbf{r}'$ on the surface S, k is the free-space wavenumber, and η is the free-space wave impedance. For a closed conducting object, the MFIE is given by

$$\frac{1}{2}\mathbf{J}(\mathbf{r}) - \hat{n} \times \iint_S \nabla g(\mathbf{r},\mathbf{r}') \times \mathbf{J}(\mathbf{r}')d\mathbf{r}' = \hat{n} \times \mathbf{H}^i(\mathbf{r}) \quad (3)$$

on S, where $\mathbf{r}$ approaches S from the outside. When $\mathbf{r} = \mathbf{r}'$, the integral in (3) is interpreted in the principal value sense. The CFIE [*Mautz and Harrington*, 1978; *Huddleston et al.*, 1986] for a closed conducting object is simply a linear combination of the EFIE and the MFIE and is of the form

$$\alpha\,\text{EFIE} + (1-\alpha)\,\frac{1}{jk}\,\text{MFIE}, \qquad (4)$$

where α is the combination parameter ranging from 0 to 1.

To solve the CFIE numerically, the conducting surface S is subdivided into small triangular elements and the current on the surface is expanded using the Rao-Wilton-Glisson (RWG) basis function $\mathbf{f}_n(\mathbf{r})$ [*Rao et al.*, 1982]:

$$\mathbf{J}(\mathbf{r}) = \sum_{n=1}^{N} I_n \mathbf{f}_n(\mathbf{r}), \qquad (5)$$

where N is the number of unknowns. Applying Galerkin's method to (4) results in a matrix equation

$$\mathbf{Z}(k)I(k) = V(k) \qquad (6)$$

in which the impedance matrix Z and vector V have the elements given by

$$Z_{mn}(k) = \alpha \iint_{T_m}\iint_{T_n}\left[\mathbf{f}_m(\mathbf{r})\cdot\mathbf{f}_n(\mathbf{r}') - \frac{1}{k^2}\nabla\cdot\mathbf{f}_m(\mathbf{r})\nabla\cdot\mathbf{f}_n(\mathbf{r}')\right] \times g(\mathbf{r},\mathbf{r}')d\mathbf{r}'d\mathbf{r}$$

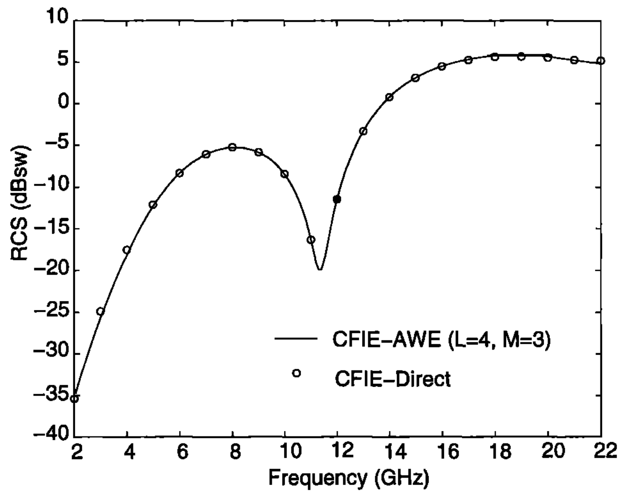

Figure 1. Radar cross section (RCS) frequency response of a cube with a side length of 1 cm from 2 to 22 GHz. The asymptotic waveform evaluation (AWE) result is obtained using one frequency expansion point at 12 GHz with $L = 4$, $M = 3$.

$$+(1-\alpha)\frac{1}{jk}\left\{\frac{1}{2}\iint_{T_m}\mathbf{f}_m(\mathbf{r})\cdot\mathbf{f}_n(\mathbf{r})d\mathbf{r} - \iint_{T_m}\mathbf{f}_m(\mathbf{r})\cdot\hat{n}\times\iint_{T_n}\nabla g(\mathbf{r},\mathbf{r}')\times\mathbf{f}_n(\mathbf{r}')d\mathbf{r}'d\mathbf{r}\right\} \tag{7}$$

$$V_m(k) = \frac{1}{jk\eta}\iint_{T_m}[\alpha\mathbf{E}^i(\mathbf{r}) + (1-\alpha)\eta\hat{n}\times\mathbf{H}^i(\mathbf{r})]\cdot\mathbf{f}_m(\mathbf{r})d\mathbf{r} \tag{8}$$

where T_m and T_n denote the support of $\mathbf{f}_m$ and $\mathbf{f}_n$, respectively.

To obtain the solution of (6) over a wide frequency band, we approximate $I(k)$ with a reduced-order rational Padé function:

$$I(k) = \frac{\sum_{i=0}^{L} a_i(k-k_0)^i}{1+\sum_{j=1}^{M} b_j(k-k_0)^j}, \tag{9}$$

in which k_0 is the expansion point. To determine the $L + M + 1$ unknown coefficients in (9), one can sample $I(k)$ at $L + M + 1$ frequency points to construct a set of linear equations to solve for a_i and b_j. However, this requires the solution of the matrix equation at the $L + M + 1$ frequency points, which is rather inefficient.

The AWE method determines a_i and b_j by first expanding $I(k)$ into a Taylor series:

$$I(k) = \sum_{n=0}^{L+M} m_n(k-k_0)^n. \tag{10}$$

Substituting this into (6), expanding the impedance matrix $\mathbf{Z}(k)$ and the excitation vector $V(k)$ into a Taylor series, and finally matching the coefficients of the equal powers of $k - k_0$ on both sides yield the recursive relation for the moment vectors:

$$m_0 = \mathbf{Z}^{-1}(k_0)V(k_0) \tag{11}$$

$$m_n = \mathbf{Z}^{-1}(k_0)\left[\frac{V^{(n)}(k_0)}{n!} - \sum_{i=1}^{n}\frac{\mathbf{Z}^{(i)}(k_0)m_{n-i}}{i!}\right] \quad n \geq 1, \tag{12}$$

where $\mathbf{Z}^{(i)}$ denotes the ith derivative of $\mathbf{Z}(k)$ and likewise $V^{(n)}$ denotes the nth derivative of $V(k)$. These derivatives are determined analytically in this work (see the appendix).

Once the moment vectors are obtained, the unknown coefficients in the rational function are calculated by substituting (10) into (9), multiplying (9) by the denominator of the Padé expansion, and matching the coefficients of the equal powers of $k - k_0$. This leads to the matrix equation

$$\begin{bmatrix} m_L & m_{L-1} & m_{L-2} & \cdots & m_{L-M+1} \\ m_{L+1} & m_L & m_{L-1} & \cdots & m_{L-M+2} \\ m_{L+2} & m_{L+1} & m_L & \cdots & m_{L-M+3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{L+M-1} & m_{L+M-2} & m_{L+M-3} & \cdots & m_L \end{bmatrix}$$

Table 1. CPU Time Comparison for Radar Cross Section Calculations for the Perfect Electric Conductor Cube

	AWE Method (One Expansion Point)	Direct Method (21 Frequency Points)	Speedup
This paper	112.9 s	2,319.6 s	20.6
Reddy et al. [1998]	2,875 s	7,434 s	2.6

AWE, asymptotic waveform evaluation.

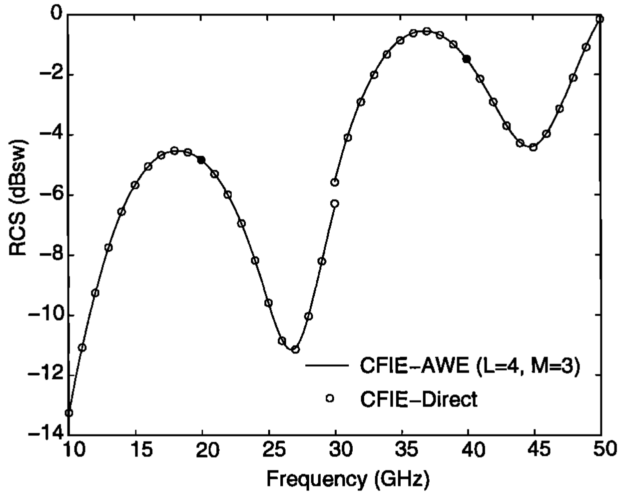

Figure 2. RCS frequency response of a sphere with a radius of 0.318 cm from 10 to 50 GHz. The AWE result is obtained using two frequency expansion points at 20 and 40 GHz with $L = 4$, $M = 3$.

$$\times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_M \end{bmatrix} = - \begin{bmatrix} m_{L+1} \\ m_{L+2} \\ m_{L+3} \\ \vdots \\ m_{L+M} \end{bmatrix}, \tag{13}$$

which can be solved for b_j. With b_j, the unknown coefficients a_i can then be calculated as

$$a_i = \sum_{j=0}^{i} b_j m_{i-j} \qquad 0 \le i \le L. \tag{14}$$

Clearly, with AWE the impedance matrix $\mathbf{Z}(k)$ is inverted only once, which is the main reason for the efficiency of the method.

With AWE, one obtains a solution that is accurate at frequencies near the point of expansion. The accuracy of the solution decreases when the frequency moves away from the point of expansion. In many practical applications, one is often required to find the solution over a specified frequency band. In such cases, one point of expansion may not be able to yield an accurate solution over the entire band, and multiple points of expansion may become necessary. These points can be selected automatically using the CFH technique [*Kolbehdari et al.*, 1996], which can be realized with a simple binary search algorithm [*Zhang and Jin*, 1998], as described below.

Given a frequency band $[f_1, f_2]$ and the error tolerance ϵ for the desired quantity, which is the RCS σ in this paper, the following steps can be used to select the points of expansion to generate a solution of a desired accuracy within the given frequency band. (1) Let $f_{\rm min} = f_1$ and $f_{\rm max} = f_2$. (2) Apply AWE at $f_{\rm min}$ and $f_{\rm max}$ and obtain $\sigma_1(f)$ and $\sigma_2(f)$. (3) Choose $f_{\rm mid} = (f_{\rm max} + f_{\rm min})/2$ and calculate $\sigma_1(f_{\rm mid})$ and $\sigma_2(f_{\rm mid})$. (4) If $|\sigma_1(f_{\rm mid}) - \sigma_2(f_{\rm mid})| < \epsilon$, stop. (5) Otherwise, apply AWE at $f_{\rm mid}$ and repeat the steps above for the subregions $[f_{\rm min}, f_{\rm mid}]$ and $[f_{\rm mid}, f_{\rm max}]$.

Note that in the binary search algorithm described above, one can also use more points to check the accuracy of the solution. For example, one can choose two points at $f_{\rm comp1} = f_{\rm min} + 0.4(f_{\rm max} - f_{\rm min})$ and $f_{\rm comp2} = f_{\rm min} + 0.6(f_{\rm max} - f_{\rm min})$ and check the accuracy at these two points. This can reduce the probability of false termination of the binary search. The implementation described here is one of the two approaches of the CFH technique. The other approach constructs one rational transfer function over the entire frequency band by matching the moments calculated at different frequency points. As pointed out by *Kolbehdari et al.* [1996], the approach employed in this work is generally more efficient for generating frequency responses.

3. Numerical Examples

To demonstrate the efficiency and accuracy of the proposed method, a number of numerical examples

Table 2. CPU Time Comparison for Radar Cross Section Calculations for the Perfect Electric Conductor Sphere

	AWE Method (Two Expansion Points)	Direct Method (41 Frequency Points)	Speedup
This paper	234.4 s	2,982.9 s	12.7
Reddy et al. [1998]	5,648 s	16,032 s	2.8

Table 3. CPU Timings for Radar Cross Section Calculations on a Digital Personal Workstation

Problem	AWE Method		Direct Method		Speedup
	Expansion Points	CPU Time	Frequency Points	CPU Time	
Figure 3	1	10.6 s	61	341.1 s	32.3
Figure 4	7	1,989.3 s	84	23,220 s	11.7

are considered. Before presenting some of these examples, we note that the interior resonance problem suffered by the EFIE and the MFIE is well known, and therefore there is no need to demonstrate the problem here. In fact, our simulation showed indeed that when the expansion point of AWE is close to the frequency of interior resonance, both the EFIE and the MFIE can exhibit significant errors, whereas the CFIE solution is always more accurate.

The first two examples illustrate the performance of the AWE implemented in this work. These two examples are the same as those employed by *Reddy et al.* [1998], who have provided detailed information on the CPU time for their implementation. To render the comparison meaningful, we carried out the computation using the same type of computer (SGI-Indigo 2 with 250-MHz IP22 processor and 64-Mb memory). The first scatterer is a 1 cm × 1 cm × 1 cm PEC cube with broadside incidence. The number of unknowns is 576. With a frequency step of 1 GHz, it takes the direct method 2319.6 s to obtain the solution from 2 to 22 GHz. With one expansion point and a seventh-order Taylor expansion ($L = 4, M = 3$), the AWE produces an accurate solution with 0.1-GHz increments over the entire band in 112.9 s, which is a speedup of 20.6. The result is shown in Figure 1, and a comparison of the CPU time is given in Table 1, which shows clearly that our method is significantly faster than the one by *Reddy et al.* [1998] even though our implementation of the CFIE requires calculation of more integrals than the implementation of only the EFIE.

The second scatterer is a PEC sphere with a radius of 0.318 cm. With a frequency step of 1 GHz, it takes the direct method 2982.9 s to obtain the solution from 10 to 50 GHz. With two expansion points and a seventh-order Taylor expansion ($L = 4, M = 3$), the AWE produces an accurate solution with 0.1-GHz increments over the entire band in 234.4 s, which is a speedup of 12.7. The smaller speedup in this case is due to the fact that the AWE solution actually extends beyond the band of comparison, which is obvious in Figure 2. The number of unknowns is 432 at 20 GHz and 768 at 40 GHz. A comparison of the CPU time is given in Table 2.

Next, the relation between the bandwidth of AWE and its order is demonstrated using the sphere of radius 0.318 cm. In the above examples, the order of AWE is seven. When this order is increased to 10, the AWE produces an accurate solution from 10 to 70 GHz using only one expansion point at 40 GHz. The CPU time is only 10.6 s on a Digital personal workstation (500-MHz Alpha 21164 processor), compared with 341.1 s using the direct method on the same computer. The speedup is given in Table 3, and the result is shown in Figure 3.

The next example is to demonstrate the performance of the CFH technique. The scatterer is the 1-m-long NASA almond [*Woo et al.*, 1993]. It takes the

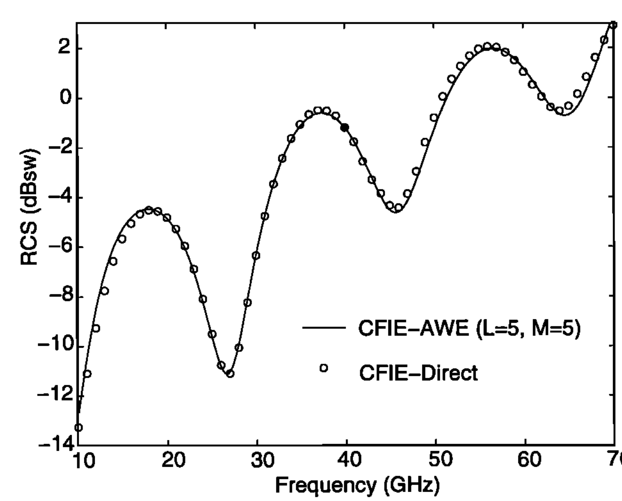

Figure 3. RCS frequency response of a sphere with a radius of 0.318 cm from 10 to 70 GHz. The AWE result is obtained using one frequency expansion point at 40 GHz with $L = 5$, $M = 5$.

direct method 23,220 s on a Digital personal workstation to calculate the RCS at 84 frequency points from 0 to 1.7 GHz. The number of unknowns varies from 1560 to 2148 during the frequency sweep. With the AWE method, it takes only 1989.3 s to calculate the RCS over the entire band using seven expansion points. The error tolerance ϵ in RCS was chosen to be very small (0.2 dB) in order to obtain a very smooth curve (Figure 4). This choice results in the relatively small speedup of 11.7. With a slightly larger error tolerance, the number of expansion points can be reduced, leading to a larger speedup. The results presented in Figure 4 show the RCS for two different incidence angles and two different polarizations.

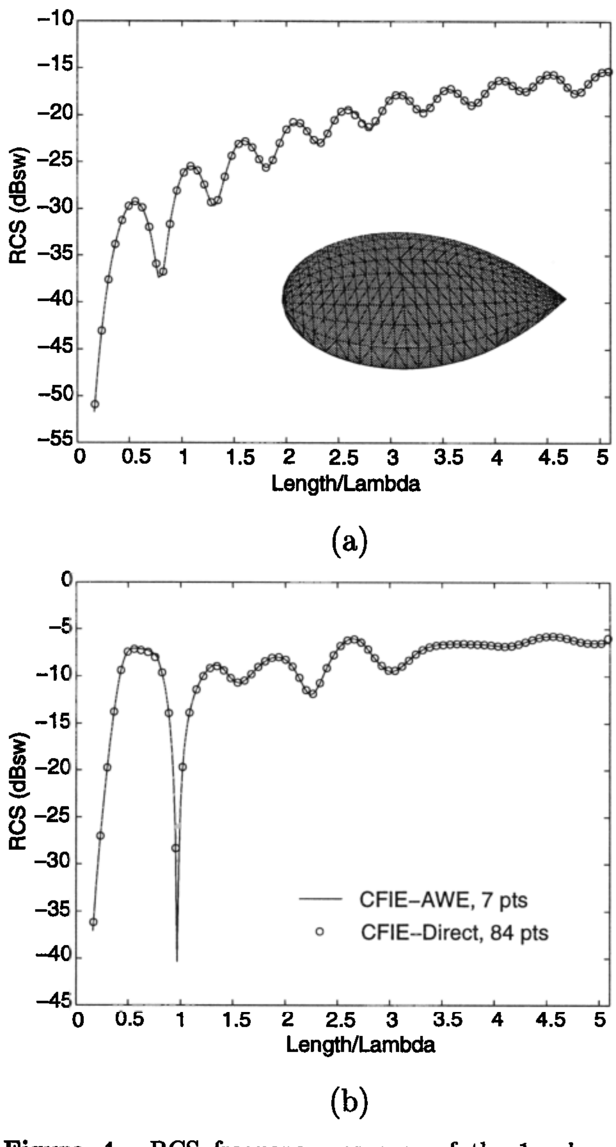

Figure 4. RCS frequency response of the 1-m-long NASA almond from 0 to 1.7 GHz. (a) VV polarization with $\theta^{\text{inc}} = 90°$ and $\phi^{\text{inc}} = 0°$. (b) HH polarization with $\theta^{\text{inc}} = 45°$ and $\phi^{\text{inc}} = 45°$.

The AWE method can also be implemented to compute the input impedance of antennas and the mutual coupling between antennas. The basic formulation is similar to what is described in section 2. However, when a wire antenna is attached to a conducting body, one has to design a model to simulate the current flow at the junction. To demonstrate the usefulness of the AWE method, we implemented it into the MOM code described by *Chao* [1998] and considered two antenna configurations. These configurations were chosen because they have been analyzed previously by *Georgakopoulos* [1998] and *Georgakopoulos et al.* [1998] and the measured data are available for comparison. The first configuration consists of two inverted-L antennas on a finite ground plane (Figure 5) and the second consists of two loop antennas on a finite ground plane (Figure 6). In both cases, the agreement between the calculated and measured results is very good.

In addition to PEC objects treated above, the AWE method is also applicable to dielectric objects. A formulation that is widely used for scattering by a dielectric object is the so-called PMCHW [*Mautz and Harrington*, 1979], named after *Poggio and Miller* [1973], *Chang and Harrington* [1977], and *Wu and Tsai* [1977], who originally developed the formulation. In this formulation, the EFIE for the field inside the dielectric object is combined with the EFIE for the field outside the object to form a combined equation. Similarly, the MFIE for the field inside the object is combined with the MFIE for the field outside the object to form another combined equation. These two equations are then solved by the MOM in a manner similar to the one described in section 2. This formulation is found to be free of interior resonances and yields accurate and stable solutions. Figure 7a shows the backscatter RCS of a dielectric sphere having radius 0.5 cm and relative permittivity $\epsilon_r = 2.56$ from 0 to 30 GHz. With a frequency step of 0.5 GHz, it takes the direct method 21,095 s to obtain the solution on a Digital personal workstation. In contrast, the AWE method produces the solution with 0.01-GHz increments in 1891 s on the same computer. Figure 7b shows the similar results for a 1 cm × 1 cm × 1 cm dielectric cube with broadside incidence.

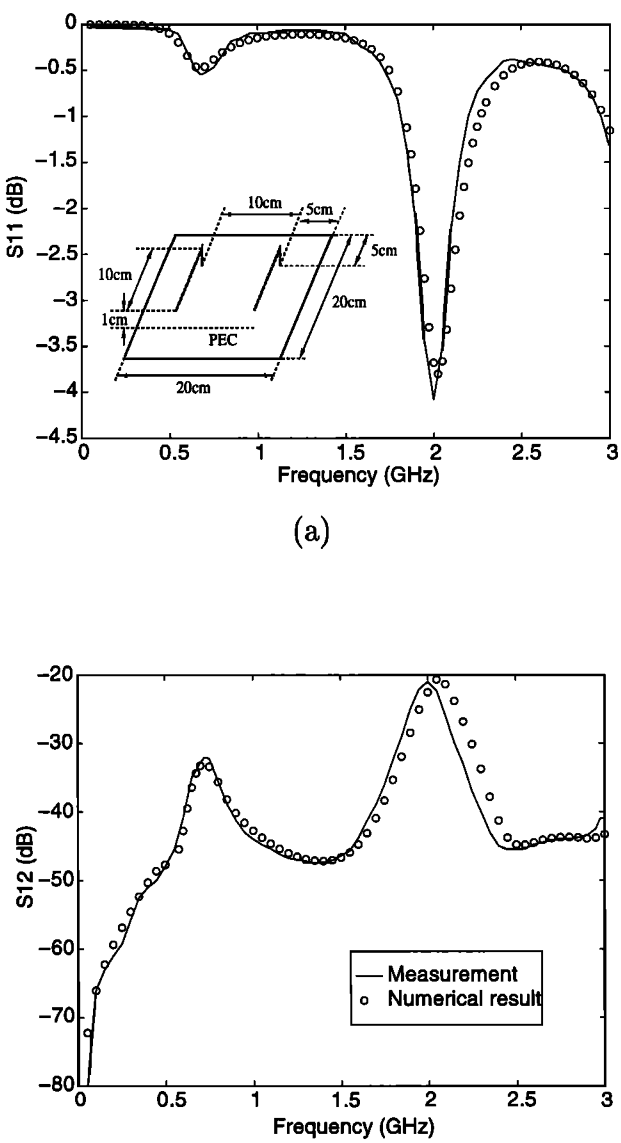

Figure 5. The S parameters of two inverted-L antennas on a finite ground plane. (a) S_{11}. (b) S_{12}.

4. Concluding Remarks

In this paper, the AWE method in conjunction with the CFH technique was applied to the solution of the CFIE to achieve fast and accurate frequency-sweep calculations of electromagnetic scattering and radiation by 3-D PEC and dielectric objects. The employment of the CFIE eliminates the interior resonance problem suffered by both the EFIE and the MFIE. It was shown that the use of AWE can speed up the calculation by more than an order of magnitude. It was also shown that when combined with the CFH technique, the AWE method could produce an accurate solution within a prespecified frequency band. Numerical examples were presented to demonstrate the performance of the proposed method.

The major drawback of the AWE method is the increased memory requirements. To implement AWE,

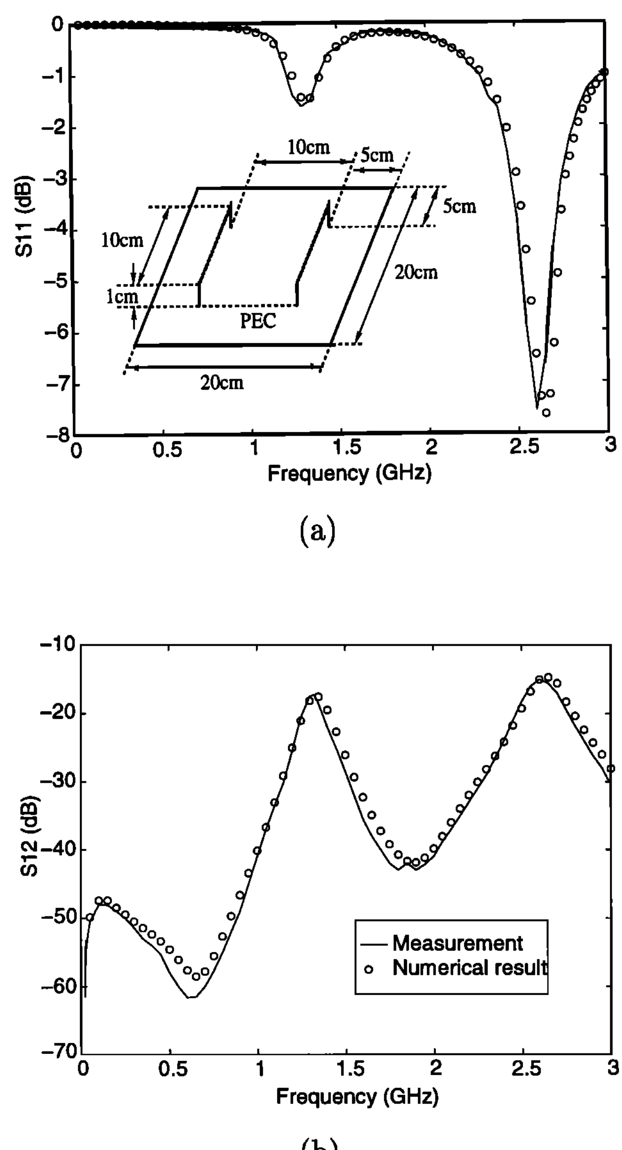

Figure 6. The S parameters of two loop antennas on a finite ground plane. (a) S_{11}. (b) S_{12}.

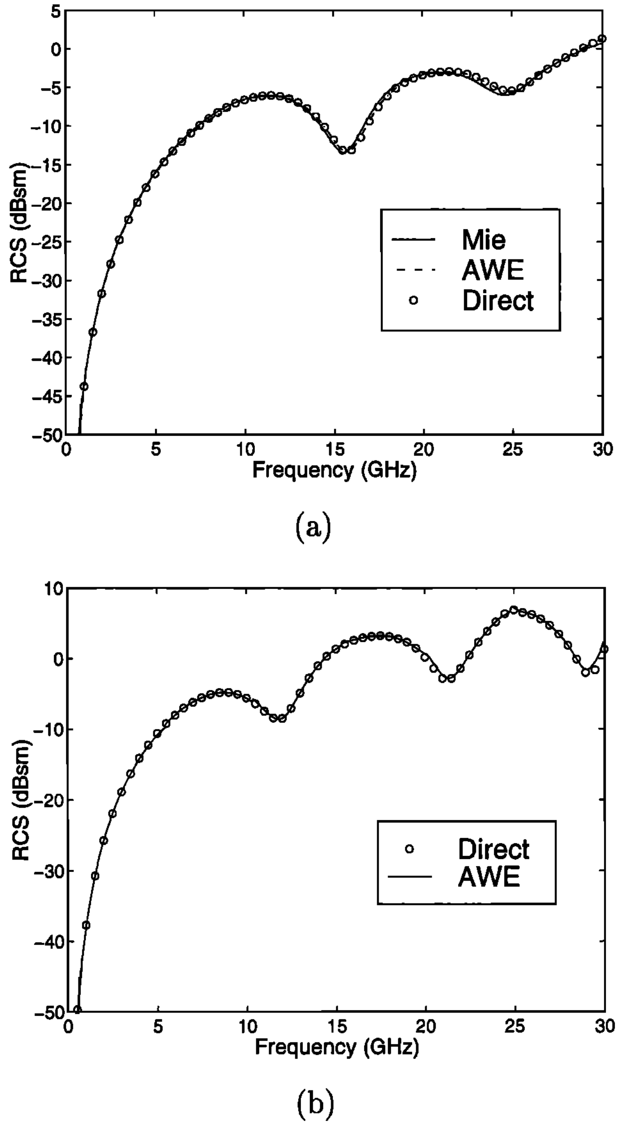

Figure 7. (a) RCS frequency response of a dielectric sphere with a radius of 0.5 cm and a relative permittivity of $\epsilon_r = 2.56$. (b) RCS frequency response of a dielectric cube with a side length of 1 cm and a relative permittivity of $\epsilon_r = 2.56$.

one has to store not only the impedance matrix but also the derivatives of the impedance matrix. This imposes a burden on computer resources for electrically large problems. It was suggested by *Reddy et al.* [1998] that this problem could be overcome by storing the derivative matrices out of core since they are required only for the matrix-vector multiplication. We have tested this approach and found that it could lead to such an increase in the solution time that a speedup would be insignificant. A better solution is to evaluate the matrix-vector multiplication directly, without generating the derivative matrices, using either the adaptive integral method (AIM) [*Bleszynski et al.*, 1996; *Wang et al.*, 1998] or the fast multipole method (FMM) [*Coifman et al.*, 1993]. This is possible because the derivatives with respect to frequency do not alter the translational invariance of the matrices. With the AIM, the memory requirements and computational complexity for the matrix-vector multiplication can be reduced from $O(N^2)$ to $O(N^{1.5})$ and $O(N^{1.5}\log N)$, respectively. With the FMM, these can be reduced to $O(N^{1.5})$. If the more complicated multilevel FMM [*Song et al.*, 1997] is employed, both the memory requirements and computational complexity can be reduced further to $O(N\log N)$.

Appendix: Derivative Formulas

The integrand in the EFIE can be written in the form

$$f(k) = \left(\frac{1}{k} - ak\right)\frac{e^{-jkr}}{r},$$

where a is a constant. Its nth derivative is given by

$$f^{(n)}(k) = \sum_{i=1}^{n-1} j^{i-1}\frac{(-1)^n n! r^{i-2}}{(i-1)!k^{n-i+2}}e^{-jkr} + n(-j)^{n+1}r^{n-2}e^{-jkr}\left(a + \frac{1}{k^2}\right) + (-j)^n r^{n-1}e^{-jkr}\left(\frac{1}{k} - ak\right).$$

A carefully designed recursive formula can allow the calculation of any derivatives. The integrand in the MFIE can be written in the form

$$g(k) = \left(1 + jkr\right)\frac{e^{-jkr}}{r^3},$$

whose nth derivative is given by

$$g^{(n)}(k) = je^{-jkr}(-jr)^{n-3}\left(1 + jkr - n\right).$$

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D. Jiao, J. M. Jin, and X. Y. Zhu, Center for Computational Electromagnetics, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801-2991. (e-mail: j-jin1@uiuc.edu)

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