ECE 580 Spring 2009

Linear Algebra Review

Due on January 26, 2009

1. Investigate the rank of the following matrix for different values of the parameter γ,

$$\boldsymbol{A} = \begin{bmatrix} 1 & \gamma & -1 & 2 \\ 2 & -1 & \gamma & 5 \\ 1 & 10 & -6 & 1 \end{bmatrix}.$$

2. Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & -1 & 3 & 2 \\ 2 & -1 & 3 & 0 & 1 \\ 3 & 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 1 & 1 \end{bmatrix}$$

Find the rank of the above matrix by first transforming it by means of the row elementary operations into an upper triangular form.

3. Consider the following system of equations,

$$\left.\begin{array}{rcl} x_1 + x_2 + 2x_3 + x_4 &=& 1 \\ x_1 - 2x_2 - x_4 &=& -2 \end{array}\right\}$$

Use Theorem 2.1 to check if the system has a solution. Then, use the method of the proof of Theorem 2.2 to find a general solution to the system.

4. Find the nullspace of

$$\boldsymbol{A} = \begin{bmatrix} 4 & -2 & 0 \\ 2 & 1 & -1 \\ 2 & -3 & 1 \end{bmatrix}$$

5. Find the transformation matrix $\boldsymbol{T}$ from $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ to $\{\boldsymbol{e}'_1, \boldsymbol{e}'_2, \boldsymbol{e}'_3\}$, where

(a) $\boldsymbol{e}'_1 = \boldsymbol{e}_1 + 3\boldsymbol{e}_2 - 4\boldsymbol{e}_3$, $\boldsymbol{e}'_2 = 2\boldsymbol{e}_1 - \boldsymbol{e}_2 + 5\boldsymbol{e}_3$, $\boldsymbol{e}'_3 = 4\boldsymbol{e}_1 + 5\boldsymbol{e}_2 + 3\boldsymbol{e}_3$.

(b) $\boldsymbol{e}_1 = \boldsymbol{e}'_1 + \boldsymbol{e}'_2 + 3\boldsymbol{e}'_3$, $\boldsymbol{e}_2 = 2\boldsymbol{e}'_1 - \boldsymbol{e}'_2 + 4\boldsymbol{e}'_3$, $\boldsymbol{e}_3 = 3\boldsymbol{e}'_1 + 5\boldsymbol{e}'_3$.

6. Given two bases, $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_4\}$ and $\{\boldsymbol{e}'_1, \boldsymbol{e}'_2, \boldsymbol{e}'_3, \boldsymbol{e}'_4\}$ of $\mathbb{R}^4$, where $\boldsymbol{e}'_1 = \boldsymbol{e}_1$, $\boldsymbol{e}'_2 = \boldsymbol{e}_1 + \boldsymbol{e}_2$, $\boldsymbol{e}'_3 = \boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3$, $\boldsymbol{e}'_4 = \boldsymbol{e}_1 + \boldsymbol{e}_2 + \boldsymbol{e}_3 + \boldsymbol{e}_4$, and the matrix representation of a linear transformation in $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \boldsymbol{e}_4\}$ of the form

$$\boldsymbol{A} = \begin{bmatrix} 2 & 0 & 1 & 0 \\ -3 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & 0 & 3 \end{bmatrix}.$$

Find the matrix representation of the linear transformation in the basis $\{\boldsymbol{e}'_1, \boldsymbol{e}'_2, \boldsymbol{e}'_3, \boldsymbol{e}'_4\}$.

7. Given two bases, $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ and $\{\boldsymbol{e}'_1, \boldsymbol{e}'_2, \boldsymbol{e}'_3\}$ of $\mathbb{R}^3$, where $\boldsymbol{e}_1 = 2\boldsymbol{e}'_1 + \boldsymbol{e}'_2 - \boldsymbol{e}'_3$, $\boldsymbol{e}_2 = 2\boldsymbol{e}'_1 - \boldsymbol{e}'_2 + 2\boldsymbol{e}'_3$, $\boldsymbol{e}_3 = 3\boldsymbol{e}'_1 + \boldsymbol{e}'_3$, and the matrix representation of a linear transformation in $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ of the form

$$\boldsymbol{A} = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find the matrix representation of the linear transformation in the basis $\{\boldsymbol{e}'_1, \boldsymbol{e}'_2, \boldsymbol{e}'_3\}$.

8. Find the basis in which the matrix

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 5 & 2 & 1 \\ -1 & 1 & 0 & 3 \end{bmatrix}$$

is diagonal

9. Determine if the following quadratic forms are positive definite, negative definite, positive semidefinite, negative semidefinite, or indefinite:

(a) $f(x_1, x_2, x_3) = x_2^2$;

(b) $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - x_1x_3$;

(c) $f(x_1, x_2, x_3) = x_1^2 + x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3$.

10. Find a transformation that brings the following quadratic form into the diagonal form described in the Text on pages 30–31,

$$f(x_1, x_2, x_3) = 4x_1^2 + x_2^2 + 9x_3^2 - 4x_1x_2 - 6x_2x_3 + 12x_1x_3.$$