

## Funwork #2

Due on February 2, 2009

1. Consider the problem of solving a jigsaw puzzle that consists of  $N$  pieces. Is this problem P, non-P, or NP? Justify your answer.
2. Exercise 5.9 from TEXT on page 75. Verify your calculations using MATLAB's Symbolic Toolbox.
3. Compute the linear,  $l(x_1, x_2)$ , and quadratic,  $q(x_1, x_2)$ , approximations of the function

$$f = f(x_1, x_2) = x_1^3 + x_1x_2 - x_1^2x_2^2,$$

at the point  $\mathbf{x}^{(0)} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ .

4.

- (a) Find
- $Df(\mathbf{x})$
- of

$$f(\mathbf{x}) = \mathbf{x}^T \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} \mathbf{x} - \mathbf{x}^T \begin{bmatrix} -2 \\ 3 \end{bmatrix} + \pi$$

- (b) Find the Hessian of

$$f(\mathbf{x}) = \frac{1}{2} \mathbf{x}^T \begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix} \mathbf{x} + \mathbf{x}^T \begin{bmatrix} 1 \\ -3 \end{bmatrix} + \log 3$$

5. For the function

$$f = f(x_1, x_2) = e^{x_1x_2^2},$$

- (a) find the gradient of
- $f$
- at
- $\mathbf{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$
- ;

- (b) find the rate of increase of
- $f$
- at the point
- $\mathbf{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$
- in the direction
- $\mathbf{d} = \begin{bmatrix} -3 & 4 \end{bmatrix}^T$
- ;

- (c) find the direction of maximum rate of increase at  $\mathbf{x} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ . What is the rate of increase in this direction?

6. Find the range of values of the parameter  $\xi$  for which the function

$$f(x_1, x_2, x_3) = 2x_1x_3 - x_1^2 - x_2^2 - 5x_3^2 - 2\xi x_1x_2 - 4x_2x_3,$$

is negative semi-definite.

7. For the function

$$f = f(x_1, x_2) = \frac{1}{3}x_1^3 + \frac{1}{2}x_1^2 + 2x_1x_2 + \frac{1}{2}x_2^2 - x_2 + 5,$$

- (a) find points that satisfy the first-order necessary conditions for the extremum;
- (b) which point is a strict local minimizer? Justify your answer.