ECE 580 Spring 2009

Funwork Set #3

Due on February 13

1. Use the Taylor series expansion to approximate $f(x) = \cos x$ about $x_0 = 0$. Plot f, the zeroth-order, the second-order, and the fourth-order approximations on the same plot, where $-6 \le x \le 6$. The range for the vertical axis should be $[-1.5, 1.5]$.

2. Use the Taylor series expansion to approximate $f(x) = \cos x$ about $x_0 = \pi/2$. Plot f, the zeroth-order, the first-order, and the third-order approximations on the same plot, where $-6 \le x \le 6$. The range for the vertical axis should be $[-1.5, 1.5]$.

3. Find the second-order Taylor series expansion of

$$f(x_1, x_2) = (x_2 - x_1)^4 + 8x_1x_2 - x_1 + x_2 + 3.$$

about $\boldsymbol{x}^{(1)} = \begin{bmatrix} -0.42 & 0.42 \end{bmatrix}^T$ and about $\boldsymbol{x}^{(2)} = \begin{bmatrix} 0.55 & -0.55 \end{bmatrix}^T$. Are the above points extremizers of f? What conditions do they satisfy? Are there any other points that satisfy FONC? Use MATLAB to plot contours of f and obtain a 3D plot of f. Also plot the second-order Taylor series approximations and compare the obtained plots with the plots of the function f itself.

4. For the function $f(x_1, x_2) = (2 + x_1)^2 + 5(1 - x_1 - x_2^2)^2$, find the equation of the tangent line to the contour line at $\boldsymbol{x} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. Obtain a 3D plot of the function and a plot of the function contours along with a plot of the tangent line.

5. Find minimizers and maximizers of the function

$$f(x) = x^4 - \frac{2}{3}x^3 - 2x^2 + 2x + 4.$$

Obtain a plot of the above function on the interval $[-2, 2]$.

6. Recall that we say that $g(k)$ is of the order-of-magnitude of $f(k)$ and write $g(k) = O(f(k))$ if for sufficiently large k and some positive constant C, we have

$$g(k) \le C f(k).$$

(a) what is the order-of-magnitude of

$$3k^3 + 2k^2 + 5?$$

(b) the above "big-O" notation can also be used when analyzing the accuracy of approximation formulas such as the Taylor series. In this case we write $g(h) = O(f(h))$ to mean

$$g(h) \leq Cf(h) \quad \text{as} \quad h \to 0$$

(Note that the above is equivalent to the previous definition if we take $h = 1/k$.) Now, we know that for $|h| < 1$,

$$\frac{1}{1-h} = 1 + h + h^2 + h^3 + \cdots$$

From this we can conclude that

$$\frac{1}{1-h} = 1 + h + O\left(h^2\right)$$

Find a constant C for the above order-of-magnitude estimation.

Hint: Consider the case when $|h| < 1/2$.

7. Many iterative optimization methods use a variable step size. The step size is determined by using a line search which involves locating the minimizer of a function of many variables in a specified direction $\boldsymbol{d}$. This involves the location of an interval in which the minimizer lies and then the interval is reduced. Once the uncertainty interval is determined, one can use the Golden Section search or the Fibonacci method to reduce the uncertainty interval.

We now describe a method that can be used to determine an interval containing the minimizer. We begin by evaluating the given function, say f, at an initial point $\boldsymbol{x}^{(0)}$. The next step is to evaluate the function at a second point which is a distance ε from $\boldsymbol{x}^{(0)}$, where ε is the chosen parameter, that is, we evaluate f at $\boldsymbol{x}^{(0)} + \varepsilon\boldsymbol{d}$. We then continue to evaluate f at new points, successively doubling the distance between the points. The process stops when the function increases between two consecutive evaluations. In the one-dimensional example in Figure 1, the function f increases between x_2 and x_3 and therefore the minimizer is bracketed in the interval $[x_1, x_3]$.

Consider now the function

$$f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \boldsymbol{x}$$

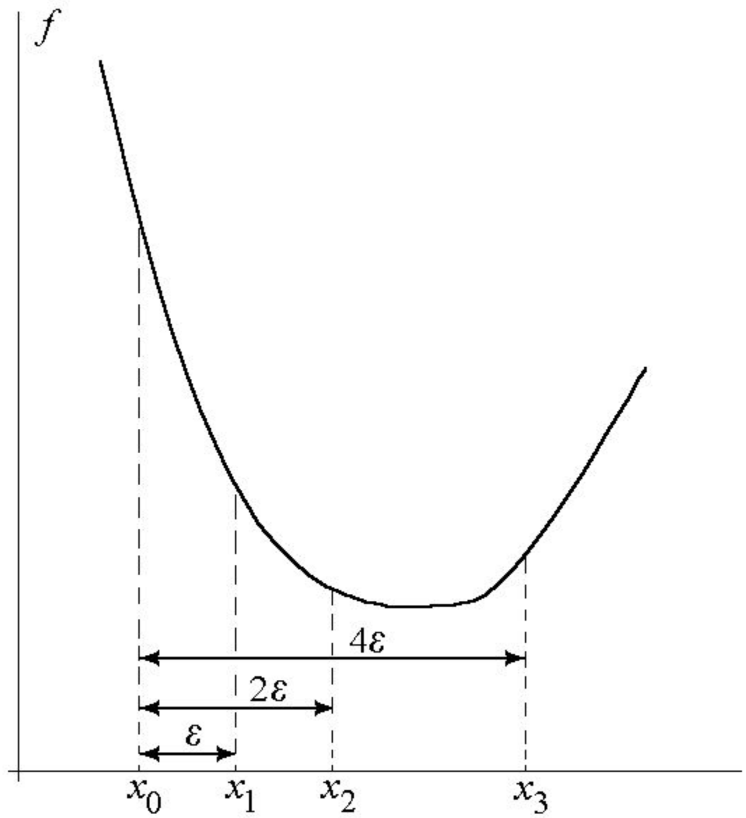

Figure 1: An illustration of the process of bracketing a minimizer.

with the initial guess $\boldsymbol{x}^{(0)} = \begin{bmatrix} 0.8 & -0.25 \end{bmatrix}^T$. Assume that the direction of travel is the negative gradient of f. Take $\varepsilon = 0.075$. Bracket the minimizer, that is, use the above described method to find an initial uncertainty region.

8. Apply the Golden Section search to the previous problem to reduce the uncertainty region width to 0.01. Organize the results of your computations in table format similar to that of Exercise **7.2**.

9. Repeat Problem **8** using the Fibonacci search.

10. Repeat Problem **8** using the Newton method.