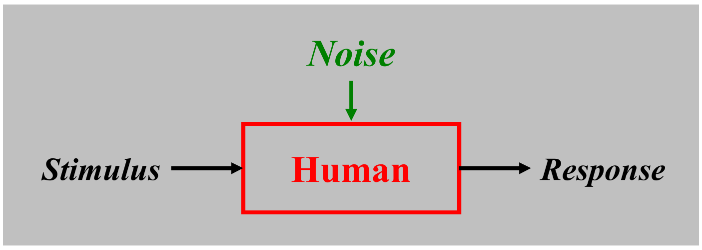
Introduction to Information Theory

Motivation

- **Peripheral to central limitations**
- **Miller's "magic number 7 ± 2"**
- **Humans as noisy communication channels**



Information & Uncertainty

- Information
  - Definition, Condition, Amount
- Uncertainty
  - An example: number game
    - 1 (0 question)
    - 1 2 (1 question)
    - 1 2 3 4 (2 questions)
    - 1 2 3 4 5 6 7 8 (3 questions)
  - An *intuitive* measure of uncertainty:
    $U = \log_2 k$ (k: # of *equally likely* outcomes)

Uncertainty (cont.)

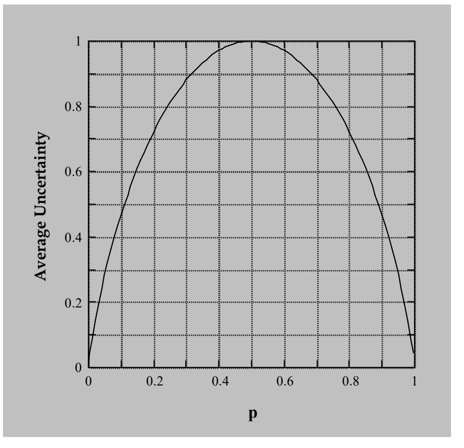
- $U = \log_2 k$ (k: # of *equally likely* outcomes)
- Uncertainty of a given outcome X_i
  - $U_i = \log_2[1/P(X_i)] = -\log_2 P(X_i)$
- If $P(X_i) = 1/k$ (X_i: an outcome, $i \in [1, k]$)
  - Then $U_i = \log_2 k$
- Average uncertainty:
  - $U = \Sigma\, P(X_i)\, U_i$
  - $U = -\Sigma\, P(X_i) \log_2[P(X_i)]$

Average Uncertainty

- Shannon's measure: $U = -\sum P(X_i) \log_2[P(X_i)]$
- Unit for uncertainty and information: *bits*

dichotomous distribution with *p* and *q*=1–*p*

average uncertainty: $-p\log_2 p - q\log_2 q$



The Absolute Identification (AI) Experiment

Procedures

- One-interval experiment
- Stimuli: S_i, $i \in [1, k]$ (k > 2)
- Responses: R_j, $j \in [1, k]$
- One-to-one mapping ($S_i \Leftrightarrow R_i$)
- On each trial, one of the stimuli S_i is presented with an *a priori* probability of $P(S_i)$
- Subject makes a response with R_j
- Trial-by-trial correct-answer feedback is optional

S-R Confusion Matrix (e.g., k = 5)

	R_1	R_2	R_3	R_4	R_5	
S_1	14	3	2	0	1	20
S_2	0	13	2	3	1	19
S_3	4	3	11	1	0	19
S_4	2	0	2	15	1	20
S_5	5	3	2	0	12	22
	25	22	19	19	15	100

# of times S_2 was Presented.

# of times the joint event (S_3, R_4) occurred.

# of times R_5 was called.

IS and IR

- IS (*Information in Stimulus*)
  - IS is the average uncertainty in stimulus

$$IS = -\sum_{i=1}^{k} P(S_i)\log_2 P(S_i)$$

  - If all stimuli are equally likely, then

$$IS = \log_2 k$$

- IR (*Information in Response*)
  - IR is the average uncertainty in response

$$IR = -\sum_{i=1}^{k} P(R_j)\log_2 P(R_j)$$

IT (Information Transfer)

- Also called "mutual information"
- IT = reduction in uncertainty
- For a particular (S_i, R_j) pair:
  - U(S_i) *before*: $-\log_2 P(S_i)$
    - Assuming that P(S_i) is constant throughout the exp.
  - U(S_i) *after*: $-\log_2 P(S_i \mid R_j)$
  - IT(S_i, R_j) = $-\log_2 P(S_i) - [-\log_2 P(S_i \mid R_j)]$

$$IT(S_i, R_j) = \log_2 \frac{P(S_i \mid R_j)}{P(S_i)}$$

- Average IT = $\Sigma\Sigma$ P(S_i, R_j) IT(S_i, R_j)
- IT: the degree of correlation between S's and R's

In-Class Demo

- Go to “Online Experiments”
- Go down to “Part IV. Information Theory”
- Select “1. Grayscale Identification”
- Click on “Start Experiment”
- Select the option with “feedback”
- Record the results in your notebook!

Discussion

- Compare an absolute-identification experiment with a 1-I 2AFC experiment
- *IS*
- *IR*
- *IT*
- 2^{IT}

Things to Remember

Uncertainty

- **Uncertainty of a given outcome X_i:**
  $U_i = -\log_2 P(X_i)$
- **Average uncertainty:**
  $U = E[U_i] = -\Sigma P(X_i) \log_2 P(X_i)$
- **Uncertainty for *k* equally likely outcomes:**
  $U = U_i = \log_2 k$

(Memorize!)

IS, IR, IT

- **IS and IR:**

$$IS = -\sum_{i=1}^{k} P(S_i)\log_2 P(S_i)$$

$$IR = -\sum_{j=1}^{k} P(R_j)\log_2 P(R_j)$$

- **IT = reduction in uncertainty**

$$IT = \sum_{j=1}^{k}\sum_{i=1}^{k} P(S_i, R_j)\log_2 \frac{P(S_i|R_j)}{P(S_i)}$$

(Memorize!)

Readings

- **Chap. 5: Macmillan, N.A. & Creelman, C.D. (2005).** ***Detection Theory: A User's Guide*.**
- **Chap. 10: Macmillan, N.A. & Creelman, C.D. (2005).** ***Detection Theory: A User's Guide*.**