

# Multidimensional Scaling

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## Outline

- **Motivation**
- **An Example on Haptic Texture Perception**
- **Summary**

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## Motivation

- In general, multidimensional stimuli lead to higher *information transfer*
- *Perceptual* dimensionality is related to, but not necessarily identical to, *physical* dimensionality
- Real-world stimuli are usually complex and multidimensional. How do we determine their associated *perceptual* dimensionality?
  - ◆ Example 1. Face recognition
  - ◆ Example 2. Color perception
  - ◆ Example 3. Haptic surface texture perception

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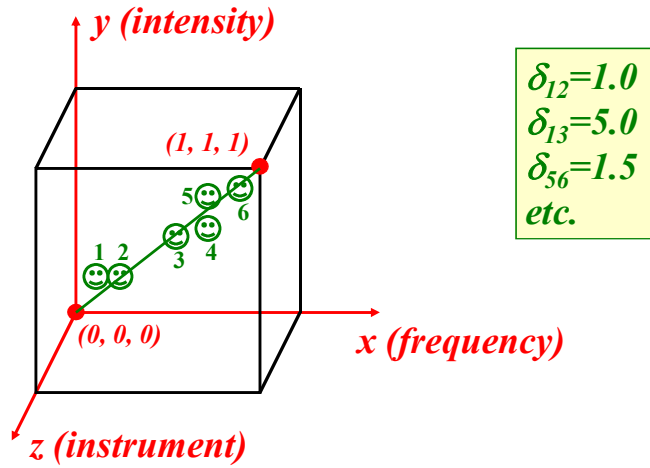
## Multidimensional Scaling (MDS) — An Overview

- MDS is a technique that lets us investigate the underlying dimensionality associated with a stimulus set.
- Given a set of  $n$  objects
  - ◆ Obtain “dissimilarity” measures  $\delta_{rs}$  for each pair of objects  $(r, s)$
  - ◆ Search for a low dimensional perceptual space, where each object is represented by a point
  - ◆ Ensure that the distances between the points in perceptual space,  $\{d_{rs}\}$ , match the original dissimilarities  $\{\delta_{rs}\}$ .

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## MDS — The Idea



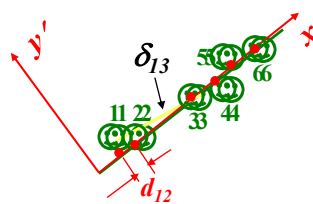
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## MDS — The Result(s)

Dissimilarity  
Judgments

$\delta_{12}=1.0$   
 $\delta_{13}=5.0$   
 $\delta_{56}=1.5$   
*etc.*



Recovered  
Perceptual  
Distances

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## An Example: Texture Perception

- Hollins et al., *P&P*, 1993.
- Stimuli: 17 texture samples
- Procedure: passive stimulation
- Dissimilarity Scores
  - ◆ Grouping (i.e., similarity scores)
  - ◆ Co-occurrence scores (0.0 or 1.0)
  - ◆ Dissimilarity = 1 – Co-occurrence
- MDS analysis (ALSCAL, SAS)

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## Co-occurrence Matrix Average over All Subjects

| Stimulus  | Felt | Straw | Wax<br>Paper | Cork | Tile | Card-<br>board |
|-----------|------|-------|--------------|------|------|----------------|
| Felt      | 1.00 |       |              |      |      |                |
| Straw     | .00  | 1.00  |              |      |      |                |
| Wax paper | .05  | .00   | 1.00         |      |      |                |
| Cork      | .10  | .05   | .30          | 1.00 |      |                |
| Tile      | .05  | .00   | .60          | .55  | 1.00 |                |
| Cardboard | .05  | .00   | .60          | .60  | .95  | 1.00           |

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(cont.)

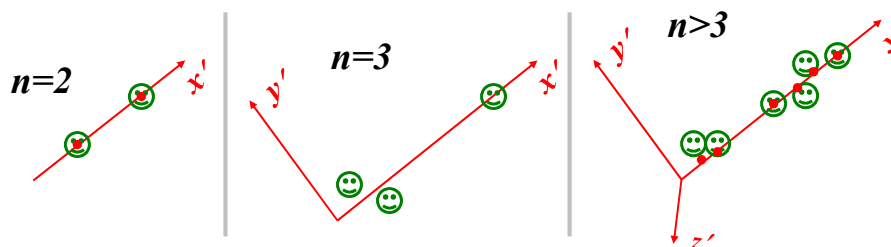
- Dissimilarity = 1 – Co-occurrence
- MDS analysis (ALSCAL, SAS)

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## How many dimensions?

- Given  $n$  objects, MDS analysis recovers  $(n-1)$  underlying dimensions

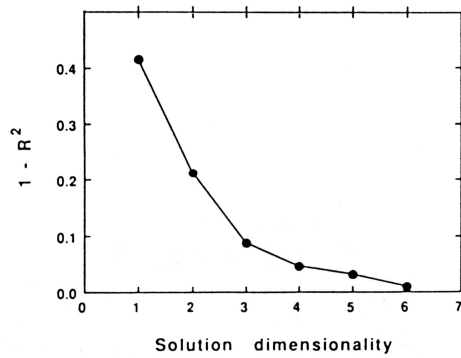


- Dimensionality is determined by examining S-Stress, Stress, and  $1-R^2$ , as a function of dimensions

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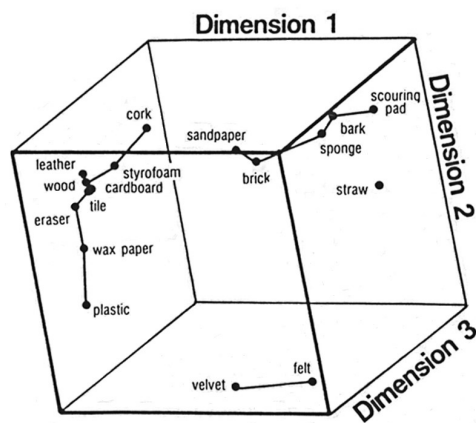
## 1-R<sup>2</sup> Plot from Hollins *et al.*



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## A 3-D Solution



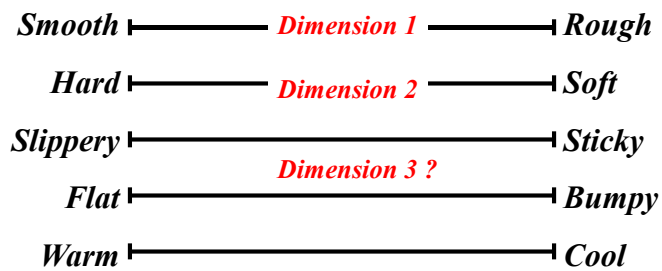
Cube representing the 3-dimensional multidimensional scaling solution.  
(From Hollins, Faldowski, Rao & Young, 1993)

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## Interpreting the MDS Solution

### ■ Adjective Rating



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## Summary of MDS

### ■ Experimental procedures

- ◆ Key is to obtain dissimilarity scores
- ◆ Grouping, similarity, dissimilarity
- ◆ Ordering (non-metric)
- ◆ etc.

### ■ Data analysis

- ◆ Use statistical packages such as SAS
- ◆ For  $n$  objects,  $(n-1)$  dimensional solution

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- Select solution dimensionality
  - ◆ S-Stress, Stress, and  $1 - R^2$
- Interpretation of MDS solutions
  - ◆ Adjective rating
- Known problems and limitations
  - ◆ Invariant to translation, rotation, reflection
  - ◆ May “discover” non-existent perceptual spaces
- Verification of MDS solution
  - ◆ Adjective rating
  - ◆ Matching experiments

**Color perception:  $X + rR = gG + bB$**

## Readings

- T. F. Cox and M. A. A. Cox,  
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- M. Hollins, R. Faldowski, S. Rao, and F.  
Young, “Perceptual dimensions of tactile  
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Psychophysics*, vol. 54, pp. 697-705, 1993.