ECE 202 - Linear Circuit Analysis II

Exam 1 Solutions

September 25, 2008

Solution 1

$$F(s) = ln(\frac{s+2}{s+1})$$

Time shift property gives,

$$L(f(t-2)) = e^{-2s}F(s)$$

Again, frequency shift property gives,

$$\begin{aligned} L(e^t f(t-2)) &= e^{-2(s-1)}F(s-1) \\ &= \frac{e^{-2s}}{e^{-2}} ln(\frac{s+1}{s}) \end{aligned}$$

Hence (1) is the correct answer

Solution 2

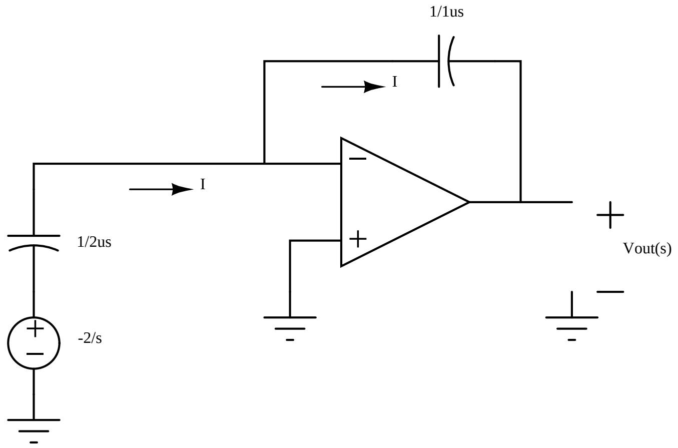
$$\begin{aligned} f(t) &= \frac{1}{2}(t+1)\{u(t-1)-u(t-3)\} \\ &= \frac{1}{2}[(t-1)u(t-1)-(t-3)u(t-3)+2u(t-1)-4u(t-3)] \end{aligned}$$

Application of time shift property gives,

$$\begin{aligned} F(s) &= \frac{1}{2}[\frac{e^{-s}}{s^2} - \frac{e^{-3s}}{s^2} + 2\{\frac{e^{-s}}{s} - 2\frac{e^{-3s}}{s}\}] \\ &= \frac{1}{s}(e^{-s}-2e^{-3s}) + \frac{0.5}{s^2}(e^{-s}-e^{-3s}) \end{aligned}$$

Hence (1) is the correct answer

Solution 3

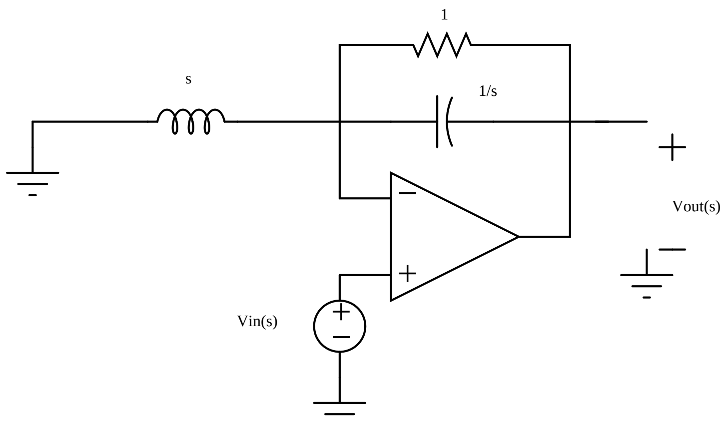


The equivalent circuit when switch is moved to position B is drawn above
Writing KVL equations assuming ideal op amp,

$$\begin{aligned} V_{out} + \frac{I}{1\mu s} &= 0 \\ -\frac{2}{s} - \frac{I}{2\mu s} &= 0 \\ -\frac{2}{s} = \frac{I}{2\mu s} &= -\frac{V_{out}}{2} \\ or\ V_{out} &= \frac{4}{s} \\ Hence\ v_{out}(t) &= 4u(t) \\ &= 4V, t = 0^{+} \end{aligned}$$

Hence (2) is the correct answer

Solution 4



The voltage at the inverting terminal of the op amp is V_{in} due to virtual ground
Applying KCL at the inverting terminal we get,

$$(V_{out} - V_{in})(s + 1) = \frac{V_{in}}{s}$$

$$\begin{aligned} V_{out} &= V_{in}(\frac{1}{s^2+s}+1) \\ or\ H(s) = \frac{V_{out}}{V_{in}} &= \frac{s^2+s+1}{s^2+s} \end{aligned}$$

Hence (6) is the correct answer

Solution 5

$$\begin{aligned} V_{in} &= I_{in}Z(s) \\ or\ H(s) = \frac{I_{in}}{V_{in}} &= \frac{1}{Z} \\ &= \frac{s+a}{s-(a-1)} \end{aligned}$$

For stability, poles should be in the left half plane
So $a-1<0$ or $a<1$
Hence (3) is the correct answer

Solution 6

Since the op amp is ideal, no current enters it, i.e. it's input current is zero
Hence the input impedance is simply the equivalent impedance of the two branches in parallel
Thus

$$\begin{aligned} Z_{in} &= (s+\frac{1}{s})||(s+\frac{1}{s}) \\ &= \frac{s^2+1}{2s} \end{aligned}$$

Hence (5) is the correct answer

Solution 7

From the pole-zero diagram, we can write,

$$H(s) = A\frac{(s+2)(s-2)}{(s+4j)(s-4j)}$$

Given that H(0)=-1, we get A=4
Hence,

$$\begin{aligned} H(s) &= \frac{4(s^2-4)}{s^2+16} \\ &= 4(1-\frac{20}{s^2+16}) \\ &= 4-20.\frac{4}{s^2+4^2} \end{aligned}$$

Thus impulse response is given by,

$$h(t) = L^{-1}(H(s)) = 4\delta(t) - 20\sin(4t)$$

Hence (4) is the correct answer

Solution 8

The impedances of the two branches are given by,

$$Z_1 = 1 + s \text{ and } Z_2 = s + \frac{2}{s}$$

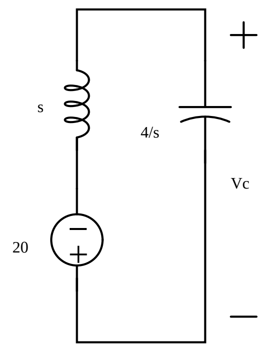
These two are in parallel, so the current I_{in} will divide in the inverse ratio of their impedances
Hence we get,

$$\begin{aligned} I_{out} &= I_{in}\frac{Z_2}{Z_1 + Z_2} \\ &= 1.\frac{1}{\frac{Z_1}{Z_2} + 1} \\ &= \frac{1}{\frac{s(s+1)}{s^2+2} + 1} \\ &= \frac{s^2 + 2}{2s^2 + s + 2} \end{aligned}$$

Hence (4) is the correct answer

Solution 9

At t=0- the current through the inductor reaches steady state and hence the current through the inductor at t=0- can be found out by considering the inductor to be shorted($v_L = L\frac{di_L}{dt} = 0$) and the capacitor to be open circuit(since the voltage across the capacitor also reaches steady state and $i_C = C\frac{dv_C}{dt} = 0$).
Simple circuit analysis thus gives $i_L(0-) = \frac{16}{0.8} = 20A$
The circuit after opening the switch S which incorporates initial conditions looks as shown below



Applying KCL at the common node of the capacitor and inductor, we get,

$$\begin{aligned} \frac{V_c + 20}{s} + \frac{V_c}{\frac{4}{s}} &= 0 \\ V_c(\frac{1}{s} + \frac{s}{4}) &= \frac{-20}{s} \\ V_c &= \frac{-80}{s^2 + 4} \\ or\ V_c &= -40.\frac{2}{s^2 + 2^2} \\ or\ v_c(t) &= -40\sin(2t), t > 0 \end{aligned}$$

Hence (1) is the correct answer

Solution 10

We can write KVL equations(in s domain) for the three loops as follows,
Loop 1:

$$V_{in} - I_1 - (I_1 - I_2)(s+1) = 0$$

Loop 2:

$$-(I_2 - I_1)(s+1) - \frac{I_2}{s} - (I_2 - I_3)2 = 0$$

Loop 3:

$$-(I_3 - I_2)2 - 6I_3 - 4sI_3 = 0$$

Rearranging the above equations and grouping similar terms gives the following equations,

$$\begin{aligned} I_1(2+s) - I_2(1+s) &= V_{in} \\ -I_1(1+s) + I_2(3+s+\frac{1}{s}) - 2I_3 &= 0 \\ -2I_2 + (8+4s)I_3 &= 0 \end{aligned}$$

These equations can be written in matrix form as,

$$\begin{pmatrix} 2+s & -1-s & 0 \\ -1-s & 3+s+\frac{1}{s} & -2 \\ 0 & -2 & 8+4s \end{pmatrix} \begin{pmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{pmatrix} = \begin{pmatrix} V_{in}(s) \\ 0 \\ 0 \end{pmatrix}$$

Hence (3) is the correct answer