ECE 202 - Linear Circuit Analysis II

Exam 2 Solutions

October 30, 2008

Solution 1

$$
\begin{aligned}
V_{out}(t) &= V_{in}(t) * h(t) \\
&= 30\sin[\pi(60t+1)] * 2\delta\left(t - \frac{1}{40}\right) \\
&= 60\sin\left[\pi\left\{60\left(t - \frac{1}{40}\right) + 1\right\}\right] \\
&= 60\sin\left[\pi\left(60t - \frac{1}{2}\right)\right] \\
&= 60\sin\left[60\pi t - \frac{\pi}{2}\right] = -60\cos(60\pi t)
\end{aligned}
$$

Hence (3) is the correct answer

Solution 2

A 40 dB/decade fall from 100 implies the term,

$$\frac{1}{1 + \left(\frac{s}{100}\right)^2}$$

Similarly, the flat portion from 1000 to 100000 implies the term,

$$1 + \left(\frac{s}{1000}\right)^2$$

A 20 dB/decade rise from 10^5 implies the term,

$$1 + \frac{s}{10^5}$$

Finally the flat portion from 10^7 implies the term,

$$\frac{1}{1 + \frac{s}{10^5}}$$

To account for the start at 80 dB, we must have a constant term equal to $10^{80/20} = 10^4$. The gain function can therefore be written as,

$$H(s) = 10^4 \frac{\left[1 + \left(\frac{s}{1000}\right)^2\right]\left(1 + \frac{s}{10^5}\right)}{\left[1 + \left(\frac{s}{100}\right)^2\right]\left(1 + \frac{s}{10^7}\right)}$$

Hence (8) is the correct answer

Solution 3

$$\begin{aligned}
V_{in}(t) &= 10u(t) \\
h(t) &= 3[u(t-3) - u(t-6)] + 8[u(t-6) - u(t-9)] \\
&= 3u(t-3) + 5u(t-6) - 8u(t-9) \\
I_{out}(t) &= V_{in}(t) * h(t) \\
&= V_{in}^{(1)}(t) * h^{(-1)}(t) \ (convolution\ algebra) \\
&= 10\delta(t) * [3r(t-3) + 5r(t-6) - 8r(t-9)] \\
&= 30r(t-3) + 50r(t-6) - 80r(t-9)
\end{aligned}$$

Hence (5) is the correct answer

Solution 4

From the frequency response plot,

$$\begin{aligned}
Q &= \frac{\omega_m}{B_\omega} \\
&= \frac{\omega_m}{\omega_2 - \omega_1} \\
&= \frac{65k}{80k - 50k} = 13/6
\end{aligned}$$

Hence (6) is the correct answer

Solution 5

$$\begin{aligned}
y(t) &= 3u(t) * h(t) \\
&= \int_{-\infty}^{\infty} h(\tau) 3u(t-\tau)\, d\tau \\
&= 3\int_{-\infty}^{t} h(\tau)\, d\tau \\
\Rightarrow y(6) &= 3\int_{-\infty}^{6} h(\tau)\, d\tau \\
&= 3\left[(4 \times 4) + \frac{1}{2} \times 2 \times (4+2)\right] = 66
\end{aligned}$$

Hence (7) is the correct answer

Solution 6

Source transformation on the left side using Norton and Thevenin theorems respectively results in the circuit shown in Figure 1. Again, transformation using Norton theorem results in a parallel RLC circuit shown in Figure 2 for which all results are well known.

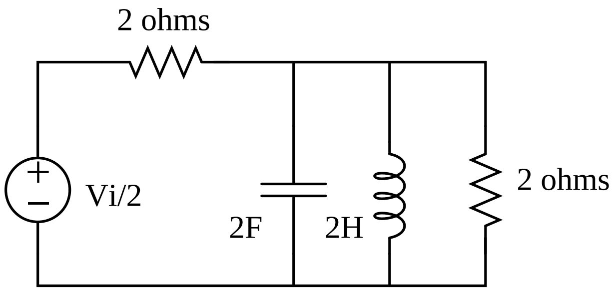

Figure 1: Source transformation

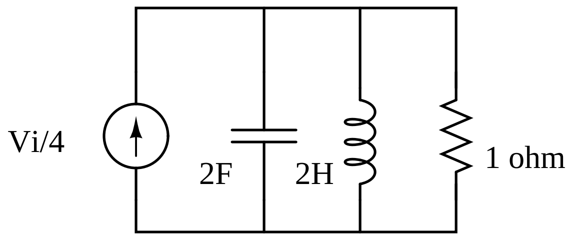

Figure 2: Parallel RLC circuit

Thus,

$$\begin{aligned} Q &= R\sqrt{\frac{C}{L}} = 1 \\ B_\omega &= \frac{1}{RC} = 0.5\ rad/s \end{aligned}$$

Hence (4) is the correct answer

Solution 7

Here $\omega = 0.5$, thus the parallel RLC combination resonates at this frequency. The equivalent circuit is reduced to the one shown in Figure 3(remember that there is a parasitic resistance R_p of the order of kilo ohms associated with the capacitor, that is how we get RLC in parallel),
The voltages at the inverting and non-inverting terminals of an ideal op-amp are equal.
Thus,

$$\begin{aligned} V_+ &= V_- \\ V_s &= V_o \frac{R_p}{R_p + 1} \\ &\approx V_o \\ or\ V_o &= V_s = 2\sin(0.5t + 30^\circ) \end{aligned}$$

Hence (5) is the correct answer

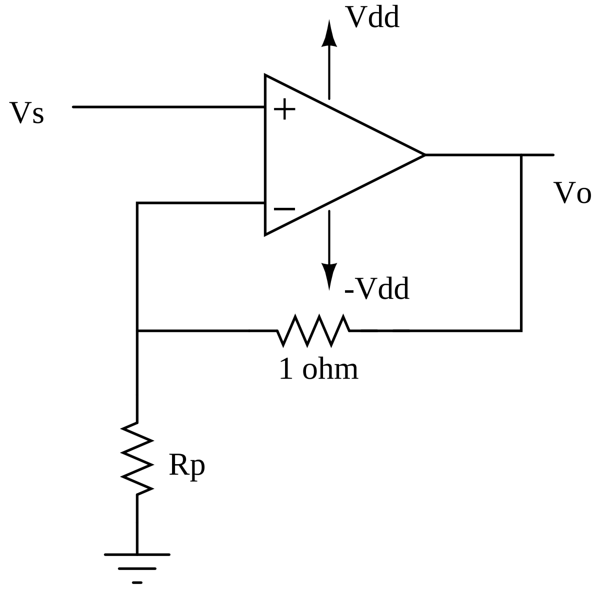

Figure 3: Simplified circuit

Solution 8

$$
\begin{aligned}
H(s) &= -\frac{2s+3}{s^2+2s+7} \\
H(j2) &= -\frac{4j+3}{-4+4j+7} \\
&= -1 = 1\angle 180^\circ \\
\mathbf{V}_s &= 3\angle 45^\circ \\
\Rightarrow |\mathbf{V}_{out}| &= |\mathbf{V}_s||H(j2)| = 3 \\
\angle \mathbf{V}_{out} &= \angle \mathbf{V}_s + \angle H(j2) = 225^\circ
\end{aligned}
$$

Hence (2) is the correct answer

Solution 9

$$
\begin{aligned}
v_{out}(t) &= 2h(t) - h(t-1) + 3h(t-2) \\
\Rightarrow v_{out}(2.5) &= 2h(2.5) - h(1.5) + 3h(0.5) \\
&= 0 - (2-1.5) + 3 = 2.5
\end{aligned}
$$

Hence (4) is the correct answer

Solution 10

$$
\begin{aligned}
\mathbf{I} &= \frac{20\angle 0^\circ}{2} = 10\angle 0^\circ \\
V_L(j\omega_r) &= j\omega_r L\mathbf{I} \\
\Rightarrow |V_L(j\omega_r)| &= \omega_r L|\mathbf{I}| \\
&= \frac{1}{\sqrt{LC}} L|\mathbf{I}| \\
&= \sqrt{\frac{L}{C}}|\mathbf{I}| = 2 \times 10 = 20
\end{aligned}
$$

Hence (3) is the correct answer

Solution 11

$$
\begin{aligned}
H(s) &= K\frac{s}{s^2 + 2\sigma_p s + \omega_p^2} \\
&= K\frac{s}{s^2 + \left(\frac{\omega_p}{Q_p}\right)s + \omega_p^2} \\
&= 10\frac{s}{(s+3-4j)(s+3+4j)} \\
&= 10\frac{s}{(s+3)^2 + 16} \\
&= 10\frac{s}{s^2 + 6s + 25} \\
\Rightarrow \omega_p &= 5 \\
Q_p &= 5/6
\end{aligned}
$$

Hence (1) is the correct answer