

ECE 202 Summer 2009

Exam 1

6/29/09

Problem	Averages Score	Max
1	13.8 / 20	20
2	16.4 / 20	20
3	25.1 / 30	30
4	22.5 / 30	30
Total	77.7 / 100	94

Question 1: (20 Points)

$$\text{Find } \mathcal{L} \left\{ \sin\left(\frac{\pi}{2}t\right) \delta(t-3)u(t) - 2te^{-3t}u(t) \right\}$$

Def. of ~~Laplace~~ ← Split up
(linearity)

$$= \int_0^{\infty} \sin\left(\frac{\pi}{2}t\right) \delta(t-3) e^{-st} u(t) dt$$

Sifting property

$$\int_0^{\infty} \sin\left(\frac{3\pi}{2}\right) \delta(t-3) e^{-3s} dt$$

$$= \sin\left(\frac{3\pi}{2}\right) e^{-3s} \cdot \int_0^{\infty} \delta(t-3) dt$$

$$- 2te^{-3t}u(t)$$

Table # 6

$$\frac{-2}{(s+3)^2}$$

$$F(s) = -e^{-3s} - \frac{2}{(s+3)^2}$$

Question 2: (20 Points)

$$\text{Find: } \mathcal{L}^{-1} \left\{ \frac{-s^3 + 6s^2 + 4s - 8}{(s+1)(s+2)(s^2+4)} \right\}$$

Partial Fractions

$$\frac{-s^3 + 6s^2 + 4s - 8}{(s+1)(s+2)(s^2+4)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{Cs+D}{s^2+4}$$

$$-s^3 + 6s^2 + 4s - 8 = A(s+2)(s^2+4) + B(s+1)(s^2+4) + (Cs+D)(s+1)(s+2)$$

$$\textcircled{1} s = -1 \Rightarrow 1 + 6 - 4 - 8 = A(1)(5) \Rightarrow \underline{A = -1}$$

$$\textcircled{2} s = -2 \Rightarrow 8 + 24 - 8 - 8 = B(-1)(8) \Rightarrow \underline{B = \frac{16}{-8} = -2}$$

$$\textcircled{3} s = 0 \Rightarrow \begin{aligned} -8 &= A(2)(4) + B(1)(4) + D(1)(2) \\ 2D &= -8 + 8 + 8 \Rightarrow \underline{D = 4} \end{aligned}$$

$$\textcircled{4} s = 1 \Rightarrow \begin{aligned} -1 + 6 + 4 - 8 &= A(3)(5) + B(2)(5) + (C(1) + D)(2)(3) \\ 1 &= -15 - 20 + 6C + 24 \\ 12 &= 6C \Rightarrow \underline{C = 2} \end{aligned}$$

$$F(s) = \frac{-1}{s+1} - \frac{2}{s+2} + \frac{2s}{s^2+4} + \frac{4}{s^2+4}$$

Table #5
Table #5
Table #9
Table #8
 $\omega = 2$

$$\mathcal{L}^{-1}\{F(s)\} = f(t) = [-e^{-t} - 2e^{-2t} + 2\cos(2t) + 2\sin(2t)]u(t)$$

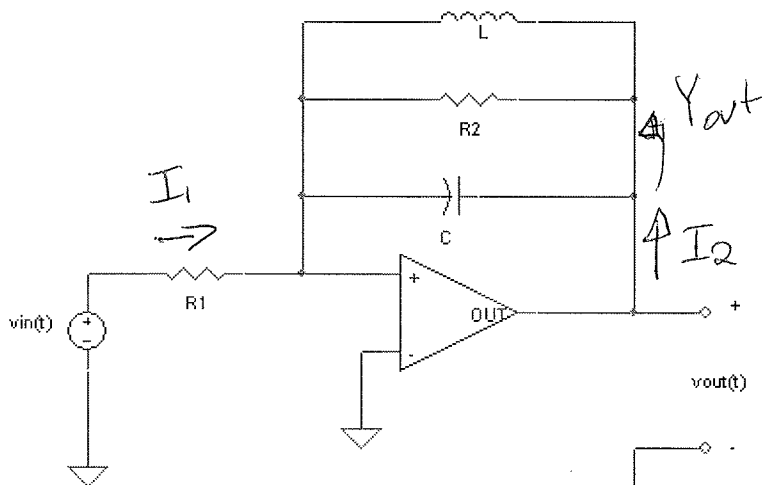
Question 3: (30 Points)

Find: 1) The transfer function $H(s) = \frac{V_{out}(s)}{V_{in}(s)}$

2) The **step** response

Given: $C=1\text{ F}$ $L=1/3\text{ H}$ $R_1=1/16$ $R_2=1/4$

No Initial Conditions



$$I_1 = -I_2$$

$$I_1 = \frac{V_{in}(s)}{R_1}$$

$$I_2 = -V_{out} \left[sC + \frac{1}{R_2} + \frac{1}{sL} \right]$$

$$I_2 = -V_{out} \left[sC + \frac{1}{R_2} + \frac{1}{sL} \right]$$

$$\frac{V_{in}(s)}{R_1} = -V_{out}(s) \left[sC + \frac{1}{R_2} + \frac{1}{sL} \right]$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-\frac{1}{R_1}}{sC + \frac{1}{R_2} + \frac{1}{sL}} = \frac{-s}{R_1 C} \frac{1}{s^2 + \frac{1}{R_2 C} s + \frac{1}{LC}}$$

$$\text{Or, } H(s) = \frac{-16s}{s^2 + 4s + 3} = \boxed{\frac{-16s}{(s+1)(s+3)}}$$

2) Step Response

$$S.R. = \mathcal{L}^{-1} \left\{ \frac{H(s)}{s} \right\}$$

$$\frac{H(s)}{s} = \frac{1}{s} \cdot \frac{-16s}{(s+3)(s+1)} = \frac{A}{s+1} + \frac{B}{s+3}$$

$$-16 = A(s+3) + B(s+1)$$

$$\text{@ } s = -1 \quad -16 = 2A \Rightarrow \underline{A = -8}$$

$$\text{@ } s = -3 \quad -16 = -2B \Rightarrow \underline{B = 8}$$

$$S.R. = \mathcal{L}^{-1} \left\{ \frac{8}{s+3} - \frac{8}{s+1} \right\}$$

$$S.R. = \left[8e^{-3t} - 8e^{-t} \right] u(t)$$

Question 4: (30 Points)

Find: $V_{out}(s)$ due to step input

Given: $v_c(0^-) = 4V$

$i_L(0^-) = 1A$

$C = 1/4 F$

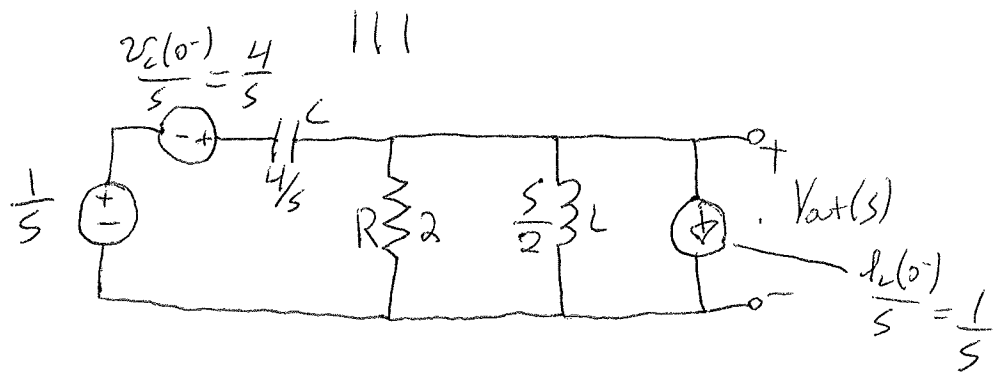
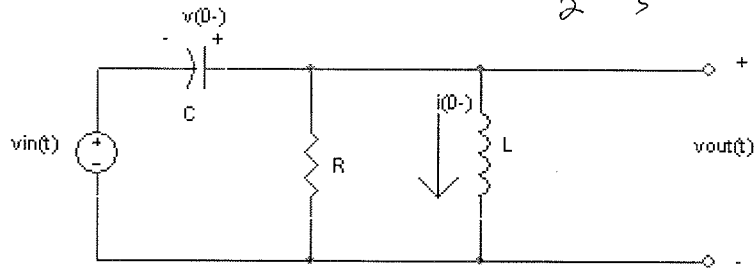
$L = 1/2 H$

$R = 2 \Omega$

$v_{in}(t) = u(t)$

$$R \parallel L = \frac{1}{\frac{1}{R} + \frac{1}{sL}}$$

$$= \frac{1}{\frac{1}{2} + \frac{2}{s}} = \frac{2s}{s+4}$$



Superposition -

1) Combined V_{in} & $\frac{v_c(0^-)}{s}$

$$V_{out}(s) = \left(\frac{1}{s} + \frac{4}{s} \right) \cdot \frac{\frac{2s}{s+4}}{\frac{4}{s} + \frac{2s}{s+4}}$$

2) $i_L(0^-)$

$$+ \frac{-1}{s} \cdot \frac{1}{\frac{s}{4} + \frac{1}{2} + \frac{2}{s}}$$

$$V_{out}(s) = \frac{5}{s} \cdot \frac{2s^2}{4s+16+2s^2} + \frac{-1}{s} \cdot \frac{4s}{s^2+2s+8}$$

$$V_{out}(s) = \frac{5s}{s^2+2s+8} - \frac{4}{s^2+2s+8}$$

$$V_{out}(s) = \frac{5s-4}{s^2+2s+8}$$

Laplace Transform Pairs

Item Number	$f(t)$	$\mathcal{L}\{f(t)\}=F(s)$
1	$K\delta(t)$	K
2	$Ku(t)$ or K	$\frac{K}{s}$
3	$r(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$\frac{1}{s+a}$
6	$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)u(t)$	$\frac{\omega}{s^2 + \omega^2}$
9	$\cos(\omega t)u(t)$	$\frac{s}{s^2 + \omega^2}$
10	$e^{-at} \sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2 + \omega^2}$
11	$e^{-at} \cos(\omega t)u(t)$	$\frac{s+a}{(s+a)^2 + \omega^2}$
12	$t \sin(\omega t)u(t)$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
13	$t \cos(\omega t)u(t)$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
14	$\sin(\omega t + \varphi)u(t)$	$\frac{s \sin \varphi + \omega \cos \varphi}{s^2 + \omega^2}$
15	$\cos(\omega t + \varphi)u(t)$	$\frac{s \cos \varphi - \omega \sin \varphi}{s^2 + \omega^2}$
16	$e^{-at} [\sin(\omega t) - \omega t \cos(\omega t)]u(t)$	$\frac{2\omega^3}{[(s+a)^2 + \omega^2]^2}$
17	$te^{-at} \sin(\omega t)u(t)$	$2\omega \frac{s+a}{[(s+a)^2 + \omega^2]^2}$
18	$e^{-at} \left[C_1 \cos(\omega t) + \left(\frac{C_2 - C_1 a}{\omega} \right) \sin(\omega t) \right] u(t)$	$\frac{C_1 s + C_2}{(s+a)^2 + \omega^2}$
19	$2\sqrt{A^2 + B^2} e^{-at} \cos \left[\omega t - \tan^{-1} \left(\frac{B}{A} \right) \right] u(t)$	$\frac{A + jB}{s + a + j\omega} + \frac{A - jB}{s + a - j\omega}$
20	$2\sqrt{A^2 + B^2} t e^{-at} \cos \left[\omega t - \tan^{-1} \left(\frac{B}{A} \right) \right] u(t)$	$\frac{A + jB}{(s + a + j\omega)^2} + \frac{A - jB}{(s + a - j\omega)^2}$

