ECE 202 Summer 2009

Exam 2

7/13/09

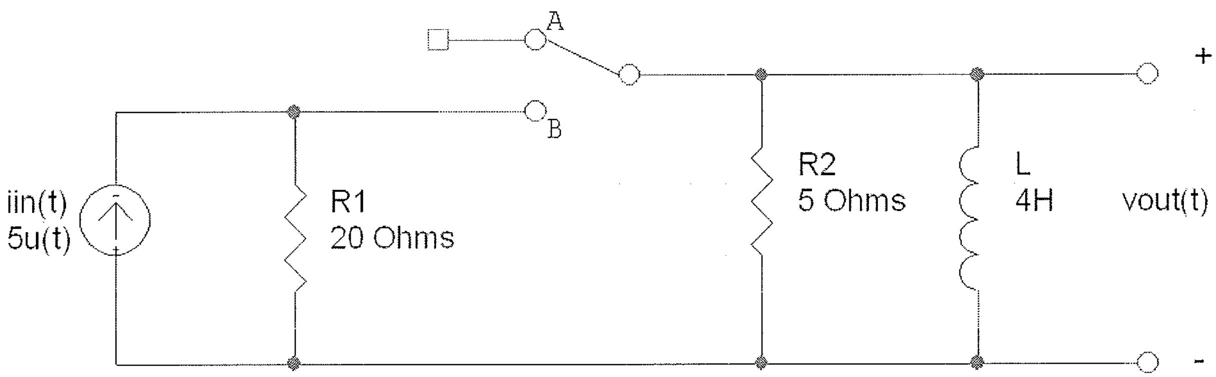
Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

Question 1: (25 Points)

In the circuit below,

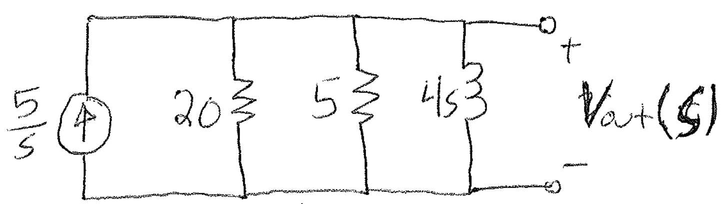
1) The switch has been in **position A** for a long time. (t<0)
2) Then, it switches to **position B** for 2 seconds. (0≤t<2)
3) Finally, it switches back to **position A**. (2≤t)

What is the voltage across the inductor at t=6?



@ $t<0$, $i_L = 0$; $i_L(0^-) = 0A$

Region 1: Position B



$20 \| 5 = 4$

Current division: $I_L = I_{in} \cdot \frac{\frac{1}{4s}}{\frac{1}{4} + \frac{1}{4s}} = \frac{5}{s(s+1)}$

Partial fractions $\frac{5}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$ @ $s=-1 \Rightarrow B=-5$
@ $s=0 \Rightarrow A=5$

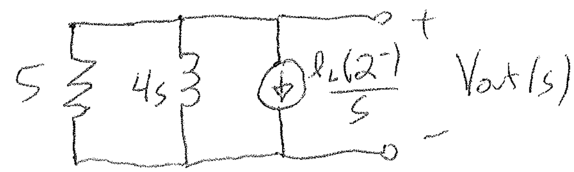
$0 \le t < 2$ $i_L(t) = 5u(t) - 5e^{-t}u(t)$

$i_L(2^-) = 5(1 - e^{-2})$

Region 2: $2 \le t$ $t' = t-2$

Circuit starts @ $t'=0$

Position A



Want: $V_{out}(t)$ @ $t=6$

$$V_{out} = -\frac{i_L(2^-)}{s} \cdot \frac{1}{\frac{1}{4s}+\frac{1}{5}}$$

$$= -i_L(2^-) \cdot \frac{1}{s} \cdot \frac{5s}{s+\frac{5}{4}}$$

$$= -5\, i_L(2^-) \cdot \frac{1}{s+\frac{5}{4}}$$

$$v_{out}(t) = -5\, i_L(2^-)\, e^{-\frac{5}{4}t'}\, u(t')$$

$$v_{out}(6) = -5(1-e^{-2}) \cdot 5 \cdot e^{-\frac{5}{4}(6-2)}\, u(6-2)$$

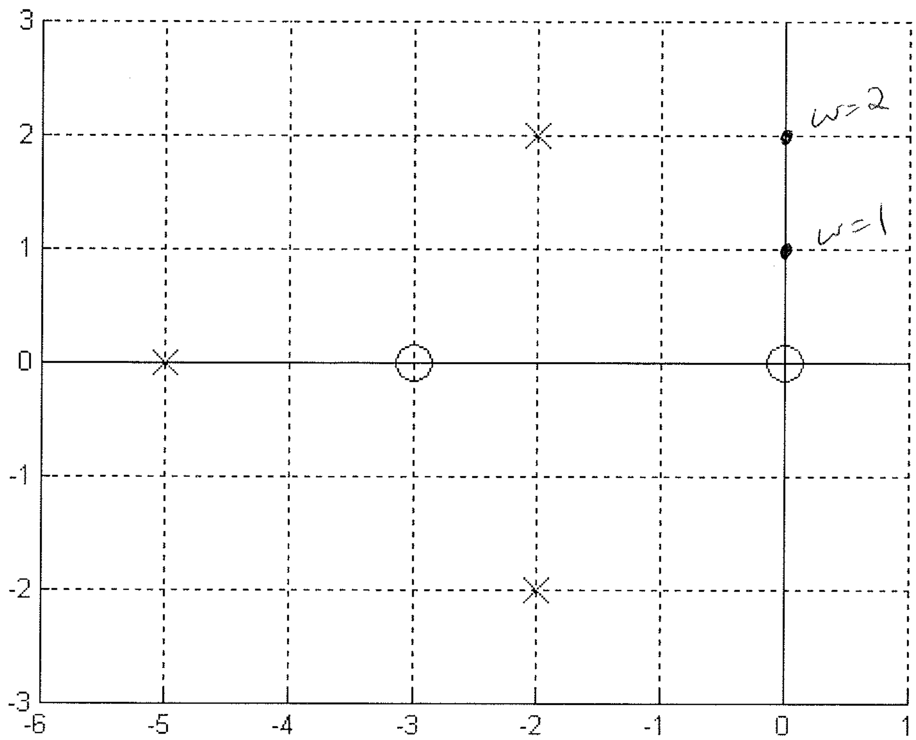
$$\boxed{v_{out}(6) = 25(e^{-2}-1)\, e^{-5}}$$

Question 2: (25 Points)

Given the graph below with |H(j1) |= 10 and a DC phase of 0°.

1) Find H(s)
2) Find Magnitude and Phase of H(s) at ω=2

$\angle K = 0°$



$$H(s) = K\,\frac{s(s+3)}{(s+5)(s^2+4s+8)}$$

Need to find |K|

$$H(j1) = 10 = \frac{|K|\cdot 1\cdot\sqrt{10}}{\underbrace{\sqrt{26}}_{\sqrt{2}\sqrt{13}}\cdot\sqrt{5}\cdot\sqrt{13}} \Rightarrow |K| = 130$$

$$\boxed{H(s) = 130\,\frac{s(s+3)}{(s+5)(s^2+4s+8)}}$$

$$|H(j2)| = \frac{130\cdot\cancel{2}\cdot\sqrt{13}}{\cancel{2}\cdot\sqrt{20}\cdot\sqrt{29}} = \boxed{65\sqrt{\frac{13}{145}}}$$

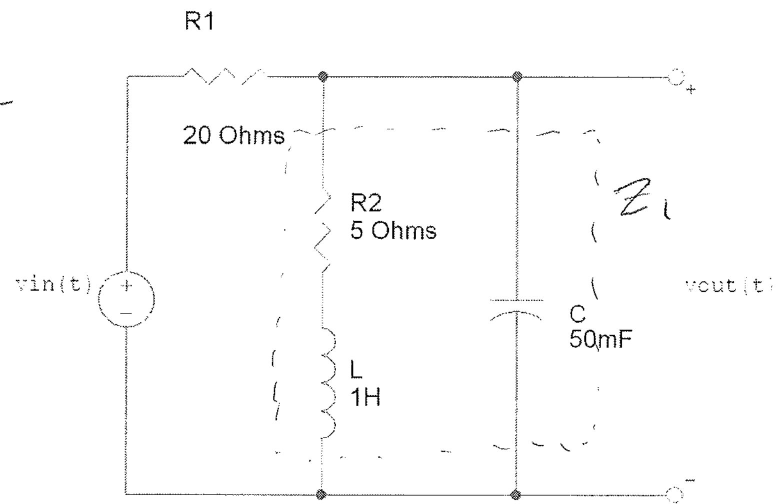
$$\angle H(j2) = 0° + 90° + \operatorname{atan}\left(\tfrac{2}{3}\right) - \operatorname{atan}\left(\tfrac{4}{2}\right) - 0° - \operatorname{atan}\left(\tfrac{2}{5}\right)$$

$$\approx \boxed{38.5°}$$

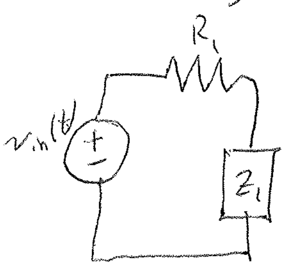
Question 3: (25 Points)

For the circuit below, find $v_{out}(t)$, given that

$$v_{in}(t) = 120\cos(5t + 30°)$$



$$Z_1 = \frac{1}{\frac{s}{20} + \frac{1}{s+5}} = \frac{20(s+5)}{s^2+5s+20}$$



V divider

$$V_{out}(s) = V_{in}(s) \cdot \frac{\frac{\cancel{20}(s+5)}{s^2+5s+20}}{\cancel{20} + \frac{\cancel{20}(s+5)}{s^2+5s+20}}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = H(s) = \frac{s+5}{s^2+5s+20+s+5} = \frac{s+5}{s^2+6s+25}$$

$$H(j5) = \frac{5+j5}{\cancel{-25}+j30+\cancel{25}} = \frac{5\sqrt{2}\angle 45°}{30\angle 90°}$$

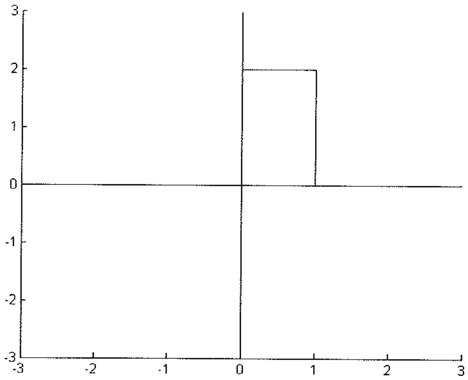
$$H(j5) = \frac{\sqrt{2}}{6}\angle -45°$$

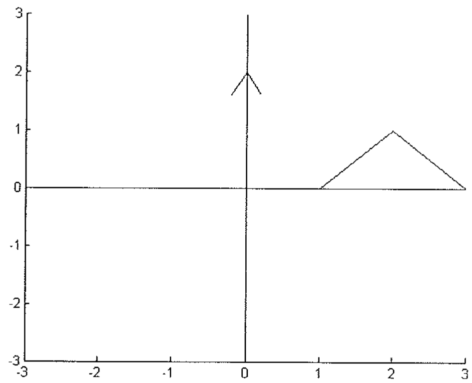
$$v_{out}(t) = 120 \cdot \frac{\sqrt{2}}{6} \cdot \cos(5t + 30° - 45°)$$

$$\boxed{v_{out}(t) = 20\sqrt{2}\cos(5t - 15°)}$$

Question 4: (25 Points)

Find: $f(t) * g(t)$

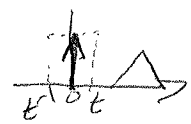




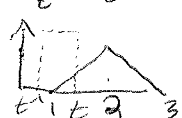
$f(t) = 2u(t) - 2u(t-1)$ $\qquad g(t) = 2\delta(t) + r(t-1) - 2r(t-2) + r(t-3)$

Region 0 $\quad t \le 0 \quad \boxed{0}$

Region 1 $\quad 0 \le t \le 1 \quad \boxed{4}$

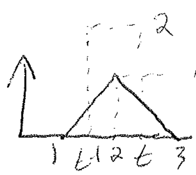


Region 2 $\quad 1 \le t \le 2$



$\int_1^t 2(t-1)\,dt = t^2 - 2t \Big|_1^t = t^2 - 2t + 1$

Region 3 $\quad 2 \le t \le 3$



Two parts:

$\int_{t-1}^{2} 2(t-1)\,dt + \int_2^t 2(3-t)\,dt$

$(t^2 - 2t)\Big|_{t-1}^{2} + \left[-t^2 + 6t\right]_2^t$

$4 - 4 - \left[(t-1)^2 - 2(t-1)\right] - t^2 + 6t + 4 - 12$

$-\left[t^2 - 2t + 1 - 2t + 2\right] - t^2 + 6t - 8$

$\boxed{-2t^2 + 10t - 11}$

Region 4 $\quad 3 \le t \le 4$

$$\int_{t-1}^{3} 2(3-t)\,dt = -t^2 + 6t\Big|_{t-1}^{3}$$

$$-9 + 18 - \left[-(t-1)^2 + 6(t-1)\right]$$

$$9 - \left[-(t^2 - 2t + 1) + 6t - 6\right]$$

$$9 - \left[-t^2 + 2t - 1 + 6t - 6\right]$$

$$\boxed{t^2 - 8t + 16}$$

Region 5 $\quad t \ge 4 \quad \boxed{0}$

Laplace Transform Pairs

Item Number	$f(t)$	$\mathcal{L}\{f(t)\}$=F(s)
1	$K\delta(t)$	K
2	$Ku(t) or K$	$\frac{K}{s}$
3	$r(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$\frac{1}{s+a}$
6	$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)\,u(t)$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)\,u(t)$	$\frac{s}{s^2+\omega^2}$
10	$e^{-at}\sin(\omega t)\,u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
11	$e^{-at}\cos(\omega t)\,u(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
12	$t\sin(\omega t)\,u(t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
13	$t\cos(\omega t)\,u(t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14	$\sin(\omega t+\varphi)\,u(t)$	$\frac{s\sin\varphi+\omega\cos\varphi}{s^2+\omega^2}$
15	$\cos(\omega t+\varphi)\,u(t)$	$\frac{s\cos\varphi-\omega\sin\varphi}{s^2+\omega^2}$
16	$e^{-at}[\sin(\omega t)-\omega t\cos(\omega t)]u(t)$	$\frac{2\omega^3}{[(s+a)^2+\omega^2]^2}$
17	$te^{-at}\sin(\omega t)u(t)$	$2\omega\frac{s+a}{[(s+a)^2+\omega^2]^2}$
18	$e^{-at}\left[C_1\cos(\omega t)+\left(\frac{C_2-C_1 a}{\omega}\right)\sin(\omega t)\right]u(t)$	$\frac{C_1 s+C_2}{(s+a)^2+\omega^2}$
19	$2\sqrt{A^2+B^2}e^{-at}\cos\left[\omega t-\tan^{-1}\left(\frac{B}{A}\right)\right]u(t)$	$\frac{A+jB}{s+a+j\omega}+\frac{A-jB}{s+a-j\omega}$
20	$2\sqrt{A^2+B^2}te^{-at}\cos\left[\omega t-\tan^{-1}\left(\frac{B}{A}\right)\right]u(t)$	$\frac{A+jB}{(s+a+j\omega)^2}+\frac{A-jB}{(s+a-j\omega)^2}$

Laplace Properties

Property	$f(t)$	$\mathcal{L}\{f(t)\}$=F(s)
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F(s) + a_2 F_2(s)$
Time Shift	$f(t-T)u(t-T)$	$e^{-sT}F(s)$
Multiplication by t	$tf(t)u(t)$	$-\frac{d}{ds}F(s)$
Multiplication by t^n	$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$
Frequency Shift	$e^{-at}f(t)$	$F(s+a)$
Time Differentiation	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
Second-order Differentiation	$\frac{d^2 f(t)}{dt^2}$	$s^2F(s) - sf\ 0 — f^{(1)}(0^-)$
*n*th-order Differentiation	$\frac{d^n f(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f^{(1)}(0^-) - \cdots - f^{n-t}(0^-)$
Time Integration	(a)$\int_{-\infty}^{t} f(q)dq$ (b)$\int_{0^-}^{t} f(q)dq$	$\frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(q)dq}{s}$ $\frac{F(s)}{s}$
Time or Frequency Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$