ECE 202 Summer 2009

Exam 3

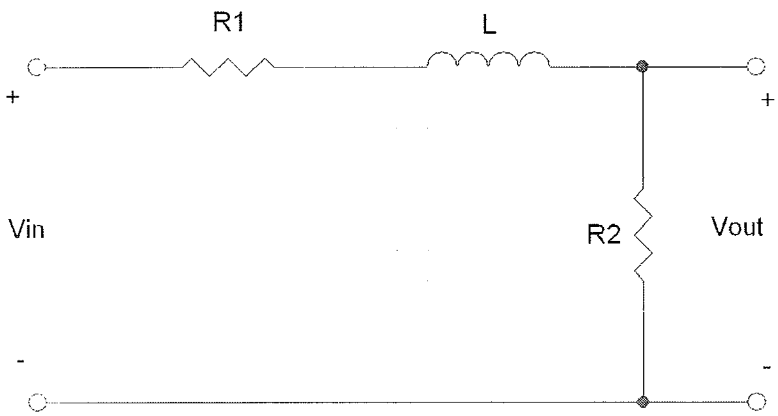
7/24/09

Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

Question 1: (25 Points)

For the circuit below, find:

1) The filter classification (High-pass, Low-pass, Band-pass, or Band-reject) Why?
2) Find H_{max}
3) Find the cutoff frequency/frequencies



1) Low-Pass filter. The inductor is a short at <u>low</u> frequencies, and an open @ high frequencies. Therefore, low frequencies will be passed to the output, and high frequencies will be supressed.

2) H_{max} occurs @ $s=0$, and the inductor is a short

∴ V divider $H_{max} = \frac{R_2}{R_1+R_2}$

3) $H(s) = \frac{R_2}{R_1+R_2+sL}$

@ $\omega_c \Rightarrow |H(j\omega_c)| = \frac{H_{max}}{\sqrt{2}} = \frac{|R_2|}{|R_1+R_2+j\omega_c L|}$

$$\frac{R_2}{\sqrt{2}(R_1+R_2)} = \frac{R_2}{|R_1+R_2+j\omega_c L|}$$

$$\frac{R_2}{\sqrt{2}\,(R_1+R_2)} = \frac{R_2}{|R_1+R_2+j\omega_c L|}$$

$$\sqrt{2}\,\frac{(R_1+R_2)}{R_2} = \frac{|R_1+R_2+j\omega_c L|}{R_2}$$

$$\sqrt{2}\,\frac{(R_1+R_2)}{R_2} = \frac{\sqrt{(R_1+R_2)^2+\omega_c^2 L^2}}{R_2}$$

$$2\,\frac{(R_1+R_2)^2}{R_2^2} = \frac{(R_1+R_2)^2}{R_2^2} + \frac{\omega_c^2 L^2}{R_2^2}$$

$$\frac{(R_1+R_2)^2}{\cancel{R_2^2}} = \frac{\omega_c^2 L^2}{\cancel{R_2^2}}$$

$$\frac{(R_1+R_2)^2}{L^2} = \omega_c^2$$

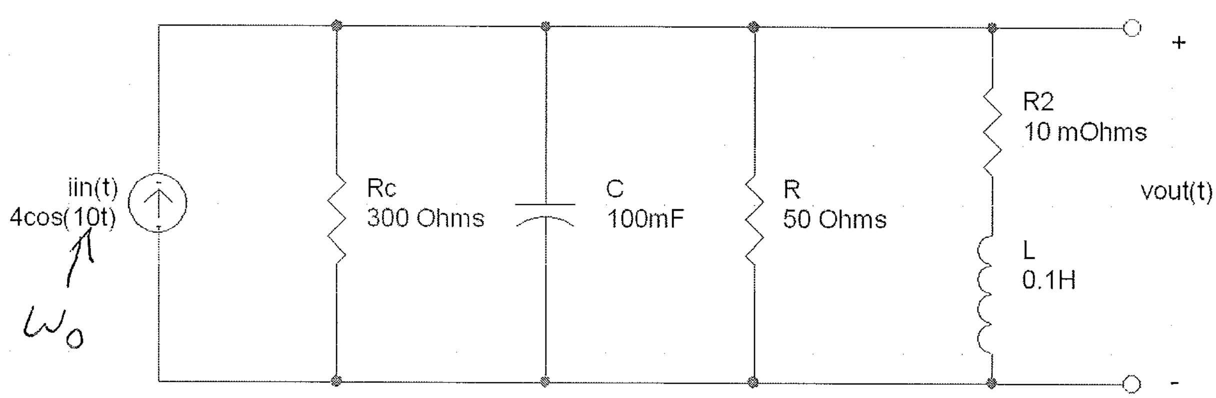
$$\boxed{\omega_c = \frac{R_1+R_2}{L}}$$

Question 2: (25 Points)

For the circuit below,

1) Find the BW
2) Find H_{max}

Hint: Using Q_{coil} formulas (located on back of exam), transform R2 and L as done in class, then combine the three resistances into R_{eq} to get H(s) for a parallel RLC circuit.



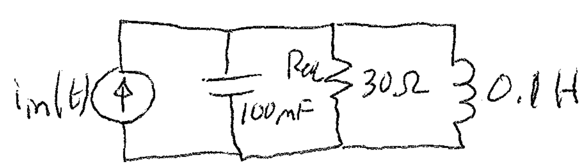
$$Q_{coil} = Q_L = \frac{\omega_0 L}{R_S} = \frac{10 \cdot 0.1}{10m} = 100$$

$Q > 6$, therefore,

$$\hat{L} = L = 0.1\ H \quad \& \quad \hat{R}_L = Q_L^2 R_2 = 100\Omega$$

$$R_{eq} = R_C \| \hat{R}_L \| R = \underline{30\Omega}$$

New circuit:



$$H(s) = \frac{s/C}{s^2 + \frac{1}{R_{eq}C}s + \frac{1}{LC}} \qquad H_m = \frac{|K|}{2\sigma_p} = \frac{\frac{1}{C}}{\frac{1}{R_{eq}C}} = R_{eq} = \boxed{30}$$

$$BW = 2\sigma_p = \frac{1}{R_{eq}C} = \frac{1}{30 \cdot .1} = \boxed{\frac{1}{3}\ \text{rad}}$$

Question 3: (25 Points)

For the two-pole Sallen-Key Low-Pass Normalized Butterworth-type filter below,**prove that this is a Normalized Butterworth low-pass filter** find new component values to **make ω_c = 1M rad/s and $C_1=C_2$ = 1nF.** Remember, A Normalized Butterworth filter has a cutoff frequency of 1 rad/s, and its two-pole form is $s^2 + \sqrt{2}s + 1$.

The transfer function of the circuit is:

$$H(s) = \frac{\frac{K}{R_1 R_2 C_1 C_2}}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-K}{R_2 C_2}\right)s + \frac{1}{R_1 R_2 C_1 C_2}}$$

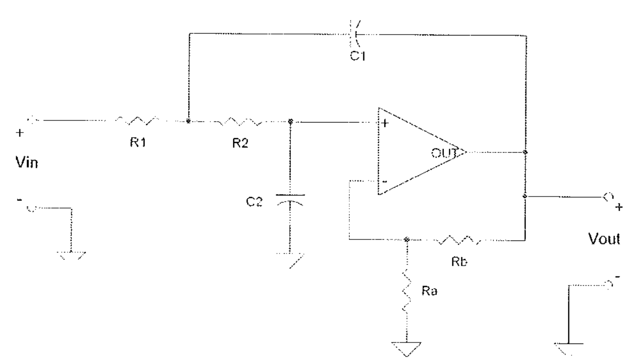
Where:

$$K = \frac{R_a + R_b}{R_a}$$

Given:

$$R_1 = 10\sqrt{2}\,;\ R_2 = 20\sqrt{2}\,;\ C_1 = \frac{1}{20}F\,;\ C_2 = \frac{1}{20}F\,;\ R_a = R_b$$

Hint: In other words, use the given formulas and the Normalized Butterworth equation to prove that it is a Butterworth function. Then, use frequency and magnitude scaling to get the correct new component values.



Proof:

$$s^2 + \sqrt{2}s + 1 = s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-K}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}$$

if $R_a = R_b$

$$K = \frac{R_a + R_b}{R_a} = \frac{2R_a}{R_a} = 2$$

$$\sqrt{2} = \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-K}{R_2C_2}\right) \qquad 1 = \frac{1}{R_1R_2C_1C_2}$$

$$\sqrt{2} = \left(\frac{1}{10\sqrt{2}\cdot\frac{1}{20}} + \frac{1}{20\sqrt{2}\cdot\frac{1}{20}} - \frac{1}{20\sqrt{2}\cdot\frac{1}{20}}\right) \qquad 1 = \frac{1}{10\sqrt{2}\cdot 20\sqrt{2}\cdot\frac{1}{20}\cdot\frac{1}{20}}$$

$$\sqrt{2} = \frac{1}{10\sqrt{2}\frac{1}{20}} \Rightarrow 2 = \frac{1}{10\cdot\frac{1}{20}} = 2 \checkmark \qquad 1 = \frac{1}{\frac{10\cdot 2}{20}} = 1 \checkmark$$

First, we frequency scale from 1 rad/s to 1Mrad/s

$$K_F = 1 \times 10^6$$

$$C_{1_{new}} = C_{2_{new}} = \frac{C_{1,2_{old}}}{1\times10^6} = \frac{1}{20}\times10^{-6}F = 50nF$$

Next, we use magnitude scaling

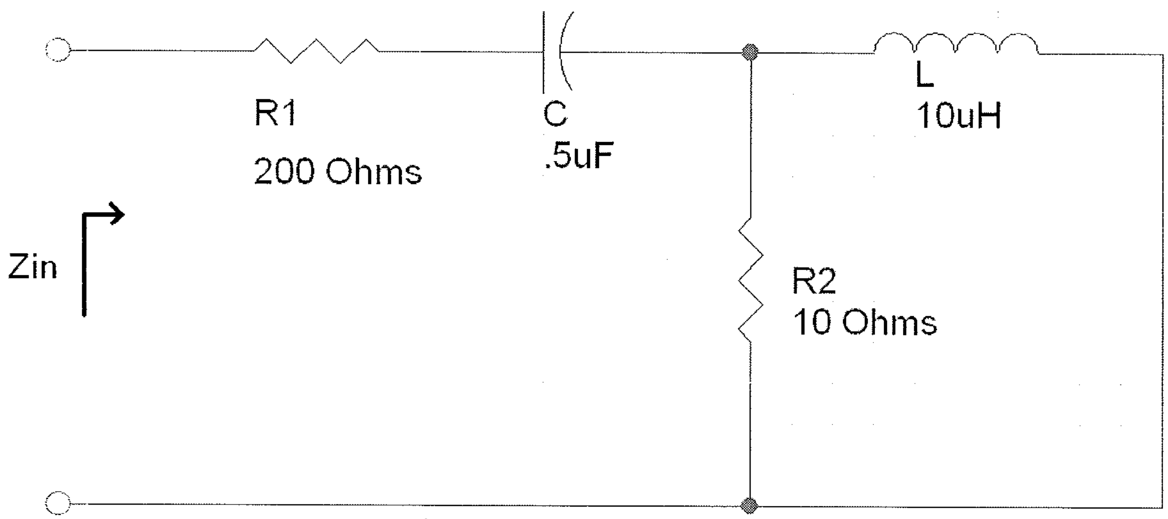
$$\frac{C_{1,2_{old}}}{K_m} = 1nF \Rightarrow K_m = 50, \quad \boxed{C_1 = C_2 = 1nF}$$

$$R_{1_{old}}\cdot K_m = R_{1_{new}} = 50\cdot 10\sqrt{2} = \boxed{500\sqrt{2}\,\Omega}$$

$$R_{2_{old}}\cdot K_m = R_{2_{new}} = 50\cdot 20\sqrt{2} = 1000\sqrt{2}\,\Omega = \boxed{\sqrt{2}\,k\Omega}$$

Question 4: (25 Points)

Find the resonant frequency of Z_{in}



$$Z_{in} = R_1 + \frac{1}{j\omega C} + \frac{1}{\frac{1}{R_2} + \frac{1}{j\omega L}} = R_1 - \frac{j}{\omega C} + \frac{jR_2\omega L}{j\omega L + R_2} \cdot \frac{R_2 - j\omega L}{R_2 - j\omega L}$$

$$Z_{in}(j\omega) = R_1 - \frac{j}{\omega C} + \frac{jR_2^2\omega L + R_2\omega^2 L^2}{R_2^2 + \omega^2 L^2}$$

$$= R_1 + \frac{R_2\omega^2 L^2}{R_2^2 + \omega^2 L^2} + j\left[\frac{R_2^2\omega L}{R_2^2 + \omega^2 L^2} - \frac{1}{\omega C}\right]$$

@ ω_r, $Z_{in} = Re + j \cdot 0$

$$\therefore \frac{1}{\omega_r C} = \frac{R_2^2\omega_r L}{R_2^2 + \omega_r^2 L^2}$$

$$\omega_r^2 R_2^2 LC = R_2^2 + \omega_r^2 L^2$$

$$\omega_r^2\left[R_2^2 LC - L^2\right] = R_2^2$$

$$\omega_r = \sqrt{\frac{R_2^2}{R_2^2 LC - L^2}} = \sqrt{\frac{100}{100 \cdot 5e^{-12} - 1e^{-10}}} = \sqrt{\frac{100}{5e^{-10} - 1e^{-10}}} = \sqrt{\frac{100}{4e^{-10}}}$$

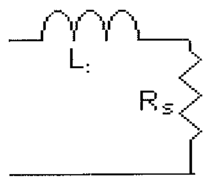
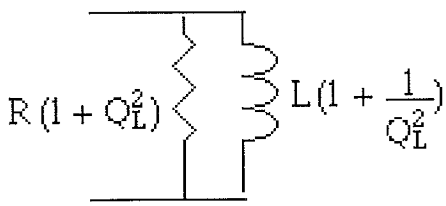
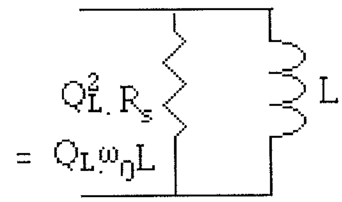
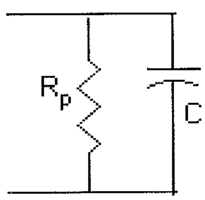
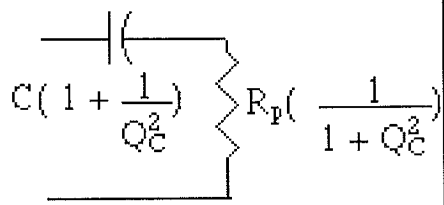
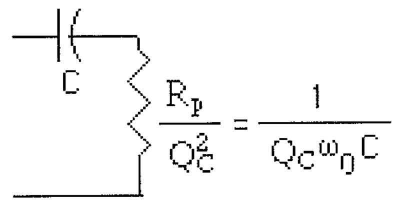
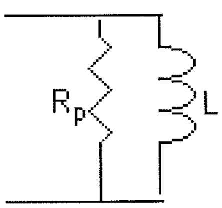
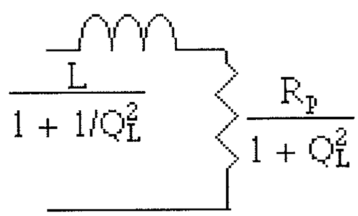
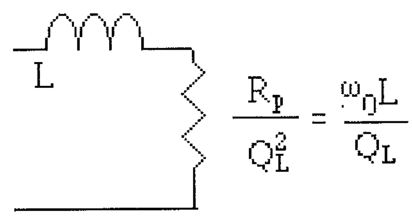
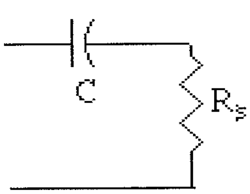
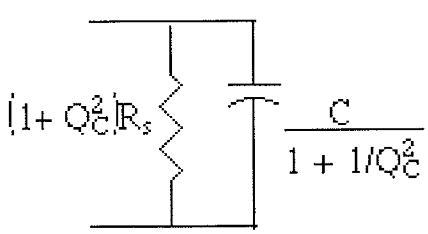
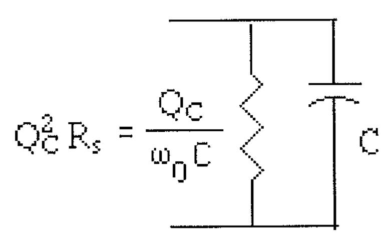
$$= \frac{10}{\sqrt{4e^{-10}}} = \frac{10}{2e^{-5}} = 5e^5 = \boxed{500{,}000 \text{ rad/s}}$$

Laplace Transform Pairs

Item Number	$f(t)$	$\mathcal{L}\{f(t)\}$=F(s)
1	$K\delta(t)$	K
2	$Ku(t) or\ K$	$\frac{K}{s}$
3	$r(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$\frac{1}{s+a}$
6	$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)\,u(t)$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)\,u(t)$	$\frac{s}{s^2+\omega^2}$
10	$e^{-at}\sin(\omega t)\,u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
11	$e^{-at}\cos(\omega t)\,u(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
12	$t\sin(\omega t)\,u(t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
13	$t\cos(\omega t)\,u(t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14	$\sin(\omega t+\varphi)\,u(t)$	$\frac{s\sin\varphi+\omega\cos\varphi}{s^2+\omega^2}$
15	$\cos(\omega t+\varphi)\,u(t)$	$\frac{s\cos\varphi-\omega\sin\varphi}{s^2+\omega^2}$
16	$e^{-at}[\sin(\omega t)-\omega t\cos(\omega t)]u(t)$	$\frac{2\omega^3}{[(s+a)^2+\omega^2]^2}$
17	$te^{-at}\sin(\omega t)u(t)$	$2\omega\frac{s+a}{[(s+a)^2+\omega^2]^2}$
18	$e^{-at}\left[C_1\cos(\omega t)+\left(\frac{C_2-C_1a}{\omega}\right)\sin(\omega t)\right]u(t)$	$\frac{C_1s+C_2}{(s+a)^2+\omega^2}$
19	$2\sqrt{A^2+B^2}e^{-at}\cos\left[\omega t-\tan^{-1}\left(\frac{B}{A}\right)\right]u(t)$	$\frac{A+jB}{s+a+j\omega}+\frac{A-jB}{s+a-j\omega}$
20	$2\sqrt{A^2+B^2}te^{-at}\cos\left[\omega t-\tan^{-1}\left(\frac{B}{A}\right)\right]u(t)$	$\frac{A+jB}{(s+a+j\omega)^2}+\frac{A-jB}{(s+a-j\omega)^2}$

Laplace Properties

Property	$f(t)$	$\mathcal{L}\{f(t)\}$=F(s)
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F(s) + a_2 F_2(s)$
Time Shift	$f(t-T)u(t-T)$	$e^{-sT}F(s)$
Multiplication by t	$tf(t)u(t)$	$-\frac{d}{ds}F(s)$
Multiplication by t^n	$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$
Frequency Shift	$e^{-at}f(t)$	$F(s+a)$
Time Differentiation	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
Second-order Differentiation	$\frac{d^2 f(t)}{dt^2}$	$s^2 F(s) - sf\,0 — f^{(1)}(0^-)$
*n*th-order Differentiation	$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f^{(1)}(0^-) - \cdots - f^{n-t}(0^-)$
Time Integration	(a) $\int_{-\infty}^{t} f(q)dq$ (b) $\int_{0^-}^{t} f(q)dq$	$\frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(q)dq}{s}$ $\frac{F(s)}{s}$
Time or Frequency Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$

Original Circuit	Exact Equivalent Circuit at ω_0	Approximate Equivalent circuit, for high Q, ($Q_L > 6$ and $Q_C > 6$) and ω within $(1 \pm 0.05)\omega_0$
 L, R_s; $Q_L(\omega_0) = \frac{\omega_0 L}{R_s}$	 $R(1+Q_L^2)$, $L(1+\frac{1}{Q_L^2})$	 $Q_L^2 . R_s = Q_L \omega_0 L$, L
 R_p, C; $Q_c(\omega_0) = \omega_0 R_p C$	 $C(1+\frac{1}{Q_C^2})$, $R_p(\frac{1}{1+Q_C^2})$	 C, $\frac{R_p}{Q_C^2} = \frac{1}{Q_C \omega_0 C}$
 R_p, L; $Q_L(\omega_0) = \frac{R_p}{\omega_0 L}$	 $\frac{L}{1+1/Q_L^2}$, $\frac{R_p}{1+Q_L^2}$	 L, $\frac{R_p}{Q_L^2} = \frac{\omega_0 L}{Q_L}$
 C, R_s; $Q_C(\omega_0) = \frac{1}{\omega_0 R_s C}$	 $[1+Q_C^2]R_s$, $\frac{C}{1+1/Q_C^2}$	 $Q_C^2 R_s = \frac{Q_C}{\omega_0 C}$, C