# ECE 202 - Linear Circuit Analysis II

Final Exam Solutions

December 19, 2008

## Solution 1

Breaking $F(s)$ into partial fractions,

$$\begin{aligned} F(s) &= \frac{4s^2-9}{s(s-9)} \\ &= 4+\frac{1}{s}+\frac{35}{s-9} \\ \Rightarrow f(t) &= 4\delta(t)+[1+35e^{9t}]u(t) \\ \Rightarrow A &= 9 \end{aligned}$$

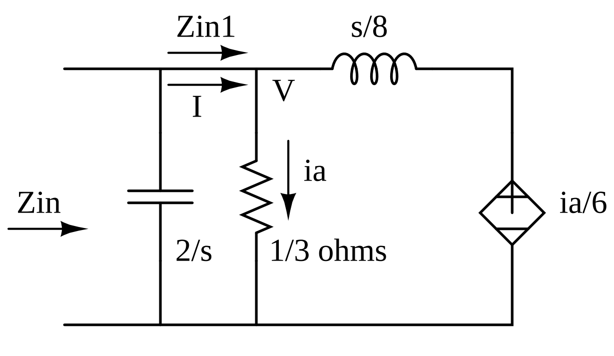
Hence (3) is the correct answer.

## Solution 2

$$\begin{aligned} f(t) &= 3[u(t)-u(t-1)]-2[u(t-1)-u(t-2)] \\ &= 3u(t)-5u(t-1)+2u(t-2) \\ \Rightarrow F(s) &= \frac{3}{s}-\frac{5e^{-s}}{s}+\frac{2e^{-2s}}{s} \end{aligned}$$

Hence (1) is the correct answer.

## Solution 3




Consider the impedance $Z_{in1}$ looking right at the node with voltage V. We can then write,

$$\begin{aligned} V &= \frac{i_a}{3} \\ -(I-i_a)\frac{s}{8}-\frac{i_a}{6}+\frac{i_a}{3} &= 0 \end{aligned}$$

$$
\begin{aligned}
I = \frac{i_a}{3}\left(3+\frac{4}{s}\right) &= V\left(3+\frac{4}{s}\right) \\
\Rightarrow Z_{in1} = \frac{V}{I} &= \frac{s}{3s+4} \\
\Rightarrow Z_{in} &= (2/s) \mid\mid Z_{in1} \\
&= \frac{2s}{s^2+6s+8}
\end{aligned}
$$

Hence (3) is the correct answer.

# Solution 4

Transfer function is given by the standard expression,

$$
\begin{aligned}
H(s) = \frac{V_{out}(s)}{V_{in}(s)} &= -\frac{Z_2}{Z_1} \\
Z_2 &= (2/s) \mid\mid 1 \\
&= \frac{2}{s+2} \\
Z_1 &= 1 + (1/s) \\
&= \frac{s+1}{s} \\
\Rightarrow H(s) &= -\frac{2s}{(s+1)(s+2)}
\end{aligned}
$$

Hence (4) is the correct answer.

# Solution 5

Using voltage division,

$$
\begin{aligned}
H(s) = \frac{V_o}{V_i} &= \frac{\frac{1}{s}}{\frac{1}{s}+s+2} \\
&= \frac{1}{s^2+2s+1} \\
&= \frac{1}{(s+1)^2} \\
\Rightarrow h(t) &= te^{-t}u(t)
\end{aligned}
$$

Hence (5) is the correct answer.

# Solution 6

$$
\begin{aligned}
v_i(t) &= 2[u(t) - u(t-1)] \\
V_i(s) &= 2(1-e^{-s})\frac{1}{s} \\
V_o(s) = H(s)V_i(s) &= \frac{2(1-e^{-s})}{s(s+1)^2} \\
&= 2(1-e^{-s})\left[\frac{1}{s} - \frac{1}{(s+1)^2} - \frac{1}{s+1}\right] \\
\Rightarrow v_o(t) &= [2 - 2te^{-t} - 2e^{-t}]u(t) - [2 - 2(t-1)e^{-(t-1)} - 2e^{-(t-1)}]u(t-1)
\end{aligned}
$$

Hence (4) is the correct answer.

## Solution 7

The equivalent impedance of the LC combination at $\omega$=0.5 is zero. Thus voltage at the inverting terminal of the op-amp is $v_o$. By virtual ground principle of ideal op-amp,

$$v_o(t) \quad = \quad v_s(t) = 2\sin(0.5t + 30^\circ)$$

Hence (5) is the correct answer.

## Solution 8

When the switch is closed, $v_5(t) = v_3(t) = v_2(t)$. Thus applying principle of conservation of charge,

$$\begin{aligned} v_5(t) &= \frac{5 \times 8}{5+3+2} \\ &= 4 \end{aligned}$$

Hence (4) is the correct answer.

## Solution 9

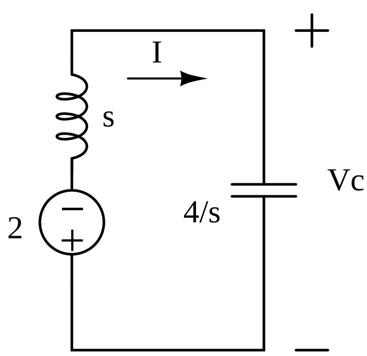
Here common mistake is to forget converting the circuit from time domain to s-domain and keeping the voltage source as 2 instead of (2/s). Analyzing the equivalent circuit in s-domain, and applying voltage division,

$$\begin{aligned} V_{out} &= \frac{2}{s}\left(\frac{\frac{4}{s}}{\frac{4}{s} + \frac{s}{2} + 3}\right) \\ &= \frac{16}{s(s+2)(s+4)} \end{aligned}$$

Hence (1) is the correct answer.

## Solution 10

Just before t=0 the circuit is in steady state and the inductor acts as a short, while the capacitor is open. Thus initial current through the inductor=4/2=2 A. The circuit thus looks as follows for $t > 0$ :-



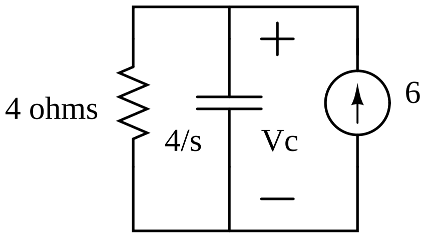
Applying KVL around the loop,

$$
\begin{aligned}
I &= \frac{sV_c}{4} \\
V_c + Is + 2 &= 0 \\
V_c\left(1 + \frac{s^2}{4}\right) + 2 &= 0 \\
V_c &= -\frac{8}{s^2+4} \\
&= -4\left(\frac{2}{s^2+4}\right) \\
\Rightarrow v_c(t) &= -4\sin(2t)u(t)
\end{aligned}
$$

Hence (5) is the correct answer.

# Solution 11

Just before t=0, the capacitor is charged to an initial voltage of 24V. The equivalent s-domain circuit for *zero input response* is as shown below. Clearly, $V_c$ is given by,




$$
\begin{aligned}
V_c &= 6(4 \mid\mid 4/s) \\
&= \frac{24}{s+1} \\
\Rightarrow v_c(t) &= 24e^{-t}u(t)
\end{aligned}
$$

Hence (6) is the correct answer.

# Solution 12

The equivalent admittance between the two terminals is given by,

$$
\begin{aligned}
Y_{eq}(s) &= Cs + \left(\frac{s^2+6}{s} + 1\right)^{-1} \\
&= Cs + \frac{s}{s^2+s+6} \\
&= s\left(C + \frac{1}{s^2+s+6}\right) \\
Y_{eq}(j3) &= j3\left(C + \frac{1}{-3+3j}\right) \\
&= j3\left(C + \frac{-1-j}{6}\right)
\end{aligned}
$$

Clearly, for resonance, $Y_{eq}(j3)$ should be real, which is possible if C=1/6.
Hence (2) is the correct answer.

## Solution 13

Applying KVL to the s-domain equivalent circuit,

$$\begin{aligned} V_s - I_L(R+Ls) - \frac{(\beta+1)I_L}{Cs} &= 0 \\ \Rightarrow \frac{V_s}{I_L} &= R + Ls + \frac{(\beta+1)}{Cs} \\ \Rightarrow Y_{eq}(s) &= \left[R + Ls + \frac{(\beta+1)}{Cs}\right]^{-1} \end{aligned}$$

Clearly, the circuit is resonant when,

$$\begin{aligned} \omega_r^2 LC &= (\beta+1) \\ or\ \omega_r &= \sqrt{\frac{\beta+1}{LC}} \end{aligned}$$

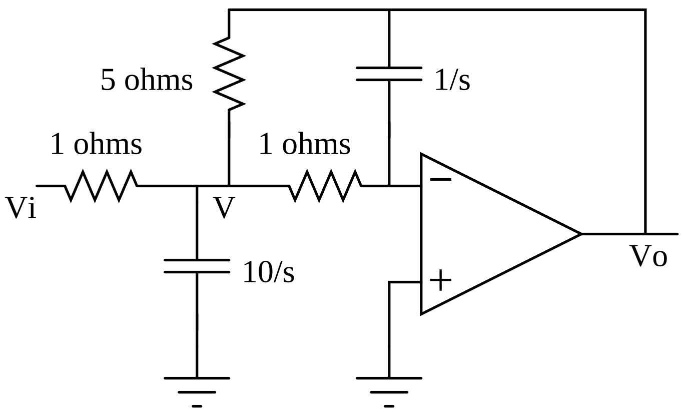
Hence (6) is the correct answer.

## Solution 14

$$\begin{aligned} K_m &= \frac{50}{0.05} = 1000 \\ \omega_{old} &= \frac{1}{\sqrt{0.5\times 8}} = 0.5 \\ K_f &= \frac{\omega_{new}}{\omega_{old}} = 100/0.5 = 200 \\ \Rightarrow C_{new} &= \frac{C_{old}}{K_m K_f} \\ &= \frac{8}{1000\times 200} = 40\ \mu F \end{aligned}$$

Hence (2) is the correct answer.

## Solution 15




The following KCL node equations can be written,

$$(V_i - V) + (0 - V) + (-\frac{sV}{10}) + \left(\frac{V_o - V}{5}\right) = 0$$

$$sV_o = -V$$

Eliminating V from the above equations, we get,

$$\frac{V_o}{V_i} = \frac{-10}{s^2 + 22s + 2}$$

This is clearly the transfer function of a low pass filter with a DC gain(s=0) of -10/2=-5. Hence (2) is the correct answer.

## Solution 16

Let input voltage be V and input current be I from the top. Here care has to be taken to write the proper sign for drop due to mutual inductance. We can write, using KVL and dot convention,

$$\begin{aligned} V - 5I - 3sI + (2sI) - \frac{I}{6s} - 5sI + (2sI) &= 0 \\ \Rightarrow Z_{in}(s) = \frac{V}{I} &= 5 + 4s + (6/s) \end{aligned}$$

Hence (5) is the correct answer.

## Solution 17

Using convolution algebra,

$$\begin{aligned} f(t) * h(t) &= f^1(t) * h^{-1}(t) \\ &= 3\delta(t) * h^{-1}(t) \\ &= 3h^{-1}(t) \\ &= 3\int_{-\infty}^{t} t^2[u(t) - u(t-2)]\, dt \\ &= \begin{cases} 0,\ t < 0 \\ t^3,\ 0 \le t \le 2 \\ 8,\ t \ge 2 \end{cases} \end{aligned}$$

Hence (1) is the correct answer.

## Solution 18

Again, using convolution algebra,

$$\begin{aligned} f(t) * h(t) &= f^{-1}(t) * h^1(t) \\ &= f^{-1}(t) * 2\delta(t) \\ &= 2f^{-1}(t) \\ &= 2\int_{-\infty}^{t} f(\tau)\, d\tau \\ &= \begin{cases} 0,\ t < 0 \\ 6t,\ 0 \le t < 2 \\ 24 - 6t,\ 2 \le t < 4 \\ 0,\ t \ge 4 \end{cases} \end{aligned}$$

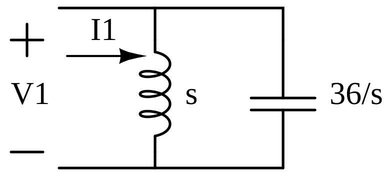
Hence (4) is the correct answer.

## Solution 19

We know,

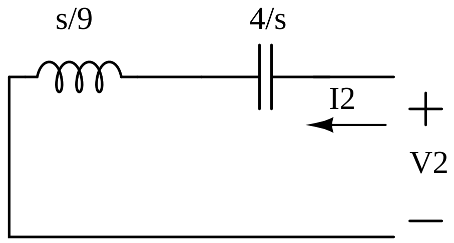
$$h_{11} = \frac{V_1}{I_1}|_{V_2=0}$$
$$h_{22} = \frac{I_2}{V_2}|_{I_1=0}$$

For finding $h_{11}$, we short circuit the output side and reflect the secondary impedance on the primary side. The reflected impedance becomes $a^2 Z_s = (9 \times 4/s) = 36/s$. Thus we get the following circuit,




$$\begin{aligned} h_{11} &= \frac{V_1}{I_1} \\ &= \frac{s(36/s)}{s + 36/s} \\ &= \frac{36s}{s^2 + 36} \end{aligned}$$

Similarly, for finding $h_{22}$, we open circuit the input side and reflect the primary impedance on the secondary side. The reflected impedance becomes $(1/a^2)Z_p = (1/9 \times s) = s/9$. Thus we get the following circuit,




$$\begin{aligned} h_{22} &= \frac{I_2}{V_2} \\ &= \left(\frac{s}{9} + \frac{4}{s}\right)^{-1} \\ &= \frac{9s}{s^2 + 36} \end{aligned}$$

Hence (2) is the correct answer.

## Solution 20

Note that when we short the input side, the voltage appearing on the primary side of the ideal transformer becomes $1\ \Omega \times I_1 = I_1$. Also, from the ideal transformer relation, $I_1/V_2 = n$. Thus,

$$
\begin{aligned}
t_{12} &= \frac{V_2}{I_1}|_{V_1=0} \\
&= \frac{1}{n}
\end{aligned}
$$

Hence (3) is the correct answer.

## Solution 21

$$
\begin{aligned}
[T] &= [T_A][T_B] \\
&= \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1+s & s \\ 1 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 3+s & s+2 \\ 3+s & s+2 \end{pmatrix}
\end{aligned}
$$

Hence (6) is the correct answer.

## Solution 22

If we keep reflecting the load impedance $Z_L$ from the right to the left most side, we will get the reflected load impedance $Z_r$ as,

$$
\begin{aligned}
Z_r &= [(N_1/N_2)^2(N_3/N_4)^2(N_5/N_6)^2]Z_L \\
&= [(2/3)^2(2/2)^2(6/2)^2]Z_L \\
&= 4Z_L
\end{aligned}
$$

Now, using *Maximum Power Transfer* theorem,

$$
\begin{aligned}
4Z_L &= 12 \\
\Rightarrow Z_L &= 3\ \Omega
\end{aligned}
$$

Hence (3) is the correct answer.

## Wish you all the best for your final grade ☺