ECE 202 Summer 2009

Final Exam

8/5/09

Problem	Score (Averages)	Maximum
0	5 /0	5
1	22.8/35	31
2	13.6 /30	26
3	16.1 /35	35
4	17.9 /30	30
5	14.6 /35	32
6	19.1 /35	35
Total	109 /200	156

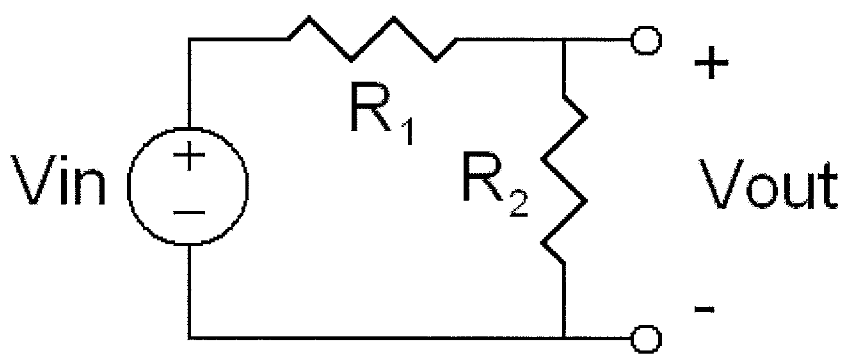
Question 1: (5 Extra Credit Points)

To get you started, here is a free problem. There will be NO partial credit.

For the circuit below, V_{out}=___5 V___

$R_1 = R_2 = 5\Omega$

$V_{in} = 10\ V$



Question 1: (35 Points)

For the circuit below, find:

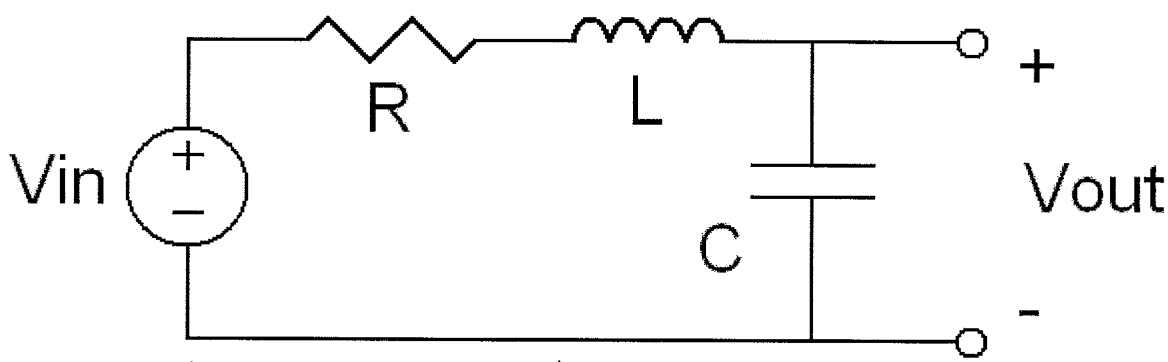
1) $v_{out}(0)$ by the initial value theorem
2) $v_{out}(\infty)$ by the final value theorem
3) $v_{out}(t)$ given that $v_{in}(t) = 2e^{-2t}$

R = 5Ω

L = 1H

C = 1/4 F

$$\mathcal{L}\{v_{in}(t)\} = \frac{2}{s+2}$$



$$H(s) = \frac{\frac{1}{sC}}{sL + R + \frac{1}{sC}} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{4}{s^2+5s+4} = \frac{4}{(s+1)(s+4)}$$

$$V_{out}(s) = \mathcal{L}\{v_{in}(t)\} \cdot \frac{4}{(s+1)(s+4)} = \frac{8}{(s+1)(s+2)(s+4)}$$

1) Initial value: $v(0^-) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left[\frac{8s}{(s+1)(s+2)(s+4)}\right] = \boxed{0}$

2) Final Value: $v(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0}\left[\frac{8s}{(s+1)(s+2)(s+4)}\right] = \boxed{0}$

3) $V_{out}(s) \Rightarrow \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+4} \Rightarrow 8 = A(s+2)(s+4) + B(s+1)(s+4) + C(s+1)(s+2)$

@ $s=-1$, $A = 8/3$

@ $s=-2$, $B = -4$

@ $s=-4$, $C = 4/3$

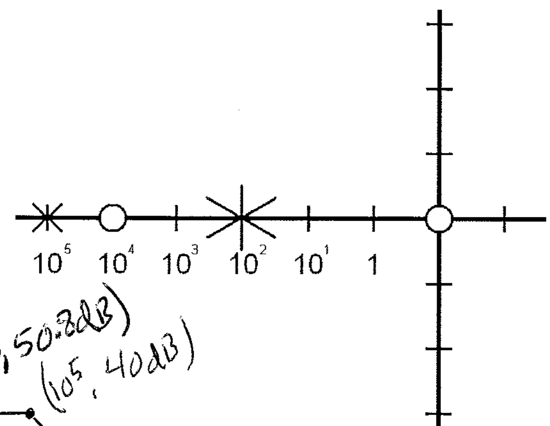
$$v_{out}(t) = \mathcal{L}^{-1}\left\{\frac{8/3}{s+1} + \frac{-4}{s+2} + \frac{4/3}{s+4}\right\} = \boxed{\left[\frac{8}{3}e^{-t} - 4e^{-2t} + \frac{4}{3}e^{-4t}\right]u(t)}$$

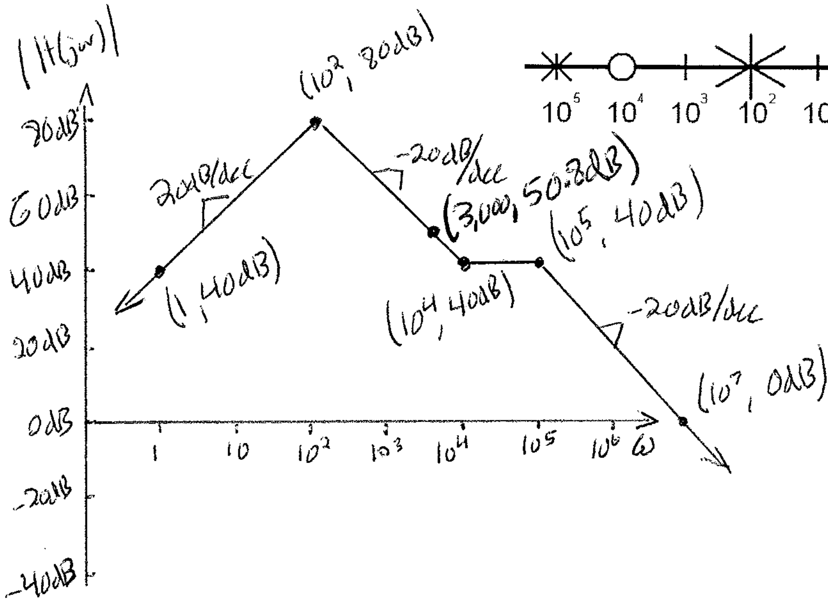
Question 2: (30 Points)

(a) For the pole-zero diagram below, draw out a bode diagram for the magnitude and frequency of H(s). On your diagram, label each point of interest (i.e. change of slope or zero crossing) with its magnitude and frequency. **Assume K = 100 + j0**.

(b) Find the exact magnitude (in dB) and phase of H(s) at ω = 3,000 rad/s. Label it on your plot. If your plot shows a different value, I will not take off extra. This will probably require a calculator.

Note: There is a double pole (two different poles) at ω = 100 rad/s.

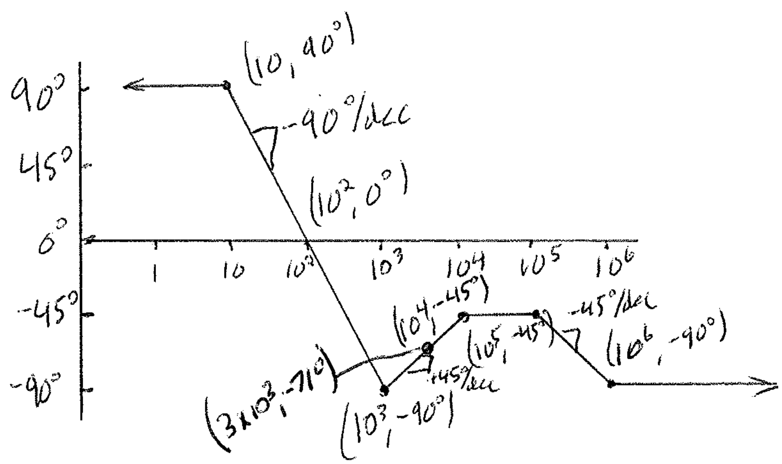




$$H(s) = 100 \frac{s\left(\frac{s}{10^4}+1\right)}{\left(\frac{s}{100}+1\right)^2\left(\frac{s}{10^5}+1\right)}$$

$$H(j3000) = 347 \angle -71^\circ$$

$$|H(j3000)| = 50.8 \text{ dB}$$



Question 3: (35 Points)

Find:

$$\underbrace{[r(t) + 2r(t-2) - 4r(t-3)]}_{f(t)} * \underbrace{[\cos(t)[u(t) - u(t-4\pi)]]}_{h(t)}$$

$$f(t) * h(t) = \frac{\partial^2 f(t)}{\partial t^2} * \iint h(t)\,dt\,dt$$

$$\frac{\partial f(t)}{\partial t} = u(t) + 2u(t-2) - 4u(t-3)$$

$$\frac{\partial^2 f(t)}{\partial t} = \delta(t) + 2\delta(t-2) - 4\delta(t-3)$$

$$\int h\,dt = \int_{-\infty}^{t} \cos(t)\,u(t)\,dt - \int_{-\infty}^{t} \cos(t)\,u(t-4\pi)\,dt$$

$$= u(t)\int_0^t \cos(t)\,dt - u(t-4\pi)\int_{4\pi}^{t} \cos(t)\,dt$$

$$= u(t)\left[\sin(t)\right]_0^t - u(t-4\pi)\left[\sin(t)\right]_{4\pi}^{t}$$

$$= u(t)\sin(t) - u(t-4\pi)\sin(t)$$

$$\iint h\,dt\,dt = \int_{-\infty}^{t} \sin(t)\,u(t)\,dt - \int_{-\infty}^{t} \sin(t)\,u(t-4\pi)\,dt$$

$$= u(t)\int_0^t \sin(t)\,dt - u(t-4\pi)\int_{4\pi}^{t} \sin(t)\,dt$$

$$= u(t)\left[-\cos(t)\right]_0^t - u(t-4\pi)\left[-\cos(t)\right]_{4\pi}^{t}$$

$$k(t) \equiv \iint h\,dt\,dt = u(t)[1-\cos(t)] - u(t-4\pi)[1-\cos(t)]$$

$$f * h = k(t) + 2k(t-2) - 4k(t-3)$$

$$= u(t)[1-\cos(t)] - u(t-4\pi)[1-\cos(t)]$$
$$+ 2u(t-2)[1-\cos(t-2)] - 2u(t-2-4\pi)[1-\cos(t-2)]$$
$$- 4u(t-3)[1-\cos(t-3)] + 4u(t-3-4\pi)[1-\cos(t-3)]$$

Question 4: (30 Points)

Find H(s) and put it in its simplified form. Simplified form means that the highest power of 's' in the denominator has a coefficient of 1, and there are no sub-fractions with 's' in the denominator. This does **not** mean you have to do partial fractions.

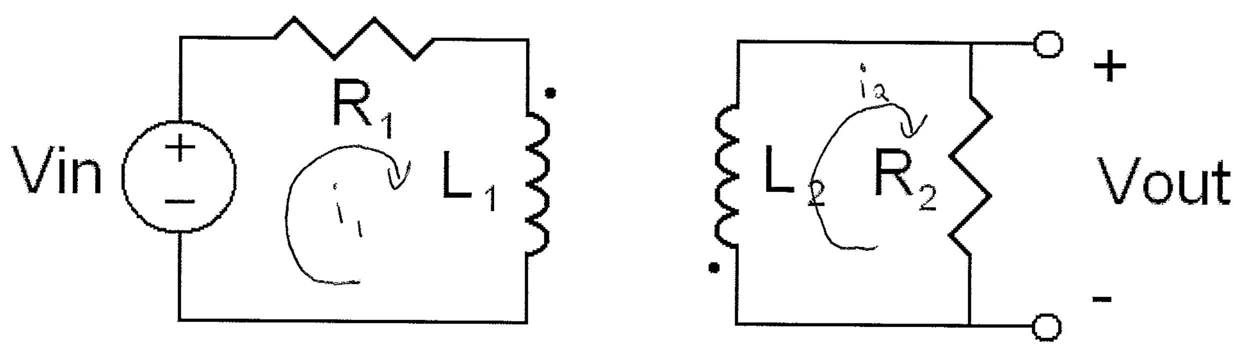
R_1=1Ω

R_2=2Ω

L_1=3H

L_2=6H

M=4H



$V_{in} = i_1 R_1 + i_1 sL_1 + i_2 sM \qquad -V_{out} = i_2 sL_2 + i_1 sM \qquad V_{out} = i_2 R_2$

$V_{in} = i_1(sL_1 + R_1) + i_2(sM) \qquad i_1 = -\frac{(V_{out} + i_2 sL_2)}{sM} \qquad i_2 = \frac{V_{out}}{R_2}$

$$V_{in} = \frac{-V_{out} - i_2 sL_2}{sM}(sL_1 + R_1) + i_2 sM$$

$$V_{in} = \frac{-V_{out} - \frac{V_{out} sL_2}{R_2}}{sM}(sL_1 + R_1) + \frac{V_{out} sM}{R_2}$$

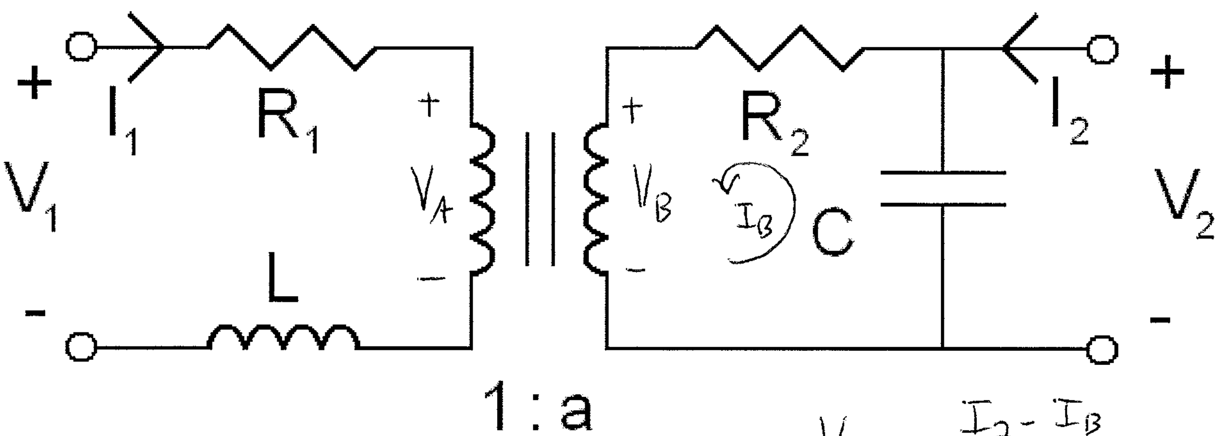
$$\frac{V_{out}}{V_{in}} = H(s) = \frac{1}{\frac{\left(-1 - \frac{sL_2}{R_2}\right)}{sM}(sL_1 + R_1) + \frac{sM}{R_2}} = \frac{sMR_2}{(-R_2 - sL_2)(sL_1 + R_1) + s^2M^2}$$

$$H(s) = \frac{-sMR_2}{s^2[L_1L_2 - M^2] + s[L_1R_2 + L_2R_1] + R_1R_2} = \frac{\frac{-sMR_2}{L_1L_2 - M^2}}{s^2 + s\left[\frac{L_1R_2 + L_2R_1}{L_1L_2 - M^2}\right] + \frac{R_1R_2}{L_1L_2 - M^2}}$$

$$= \frac{\frac{-8s}{2}}{s^2 + s\left[\frac{6+6}{2}\right] + \frac{2}{2}} = \boxed{\frac{-4s}{s^2 + 6s + 1}}$$

Question 5: (35 Points)

Find the four z-parameters of the two-port circuit below.



$$V_1 = I_1 R_1 + V_A + sLI_1$$

$$aV_A = V_B$$

$$\frac{-I_1}{a} = I_B$$

$$V_2 = \frac{I_2 - I_B}{sC}$$

$$V_2 = \frac{I_2 + \frac{I_1}{a}}{sC} = \frac{I_2}{sC} + \frac{I_1}{asC}$$

Need to find V_A

I_B loop eqn $\Rightarrow$ $0 = -V_B - I_B R_2 + V_2$

$$V_B + I_B R_2 = V_2$$

$$aV_A + \frac{-I_1}{a} R_2 = \frac{I_2}{sC} + \frac{I_1}{asC}$$

$$V_A = \frac{I_1}{a^2} R_2 + \frac{I_1}{a^2 sC} + \frac{I_2}{asC}$$

$$V_1 = I_1 R_1 + \frac{I_1}{a^2} R_2 + \frac{I_1}{a^2 sC} + \frac{I_2}{asC} + sL I_1$$

$$V_1 = I_1 \left[R_1 + sL + \frac{R_2}{a^2} + \frac{1}{a^2 sC} \right] + I_2 \frac{1}{asC}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} R_1 + sL + \frac{R_2}{a^2} + \frac{1}{a^2 sC} & \frac{1}{asC} \\ \frac{1}{asC} & \frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Question 6: (35 Points)

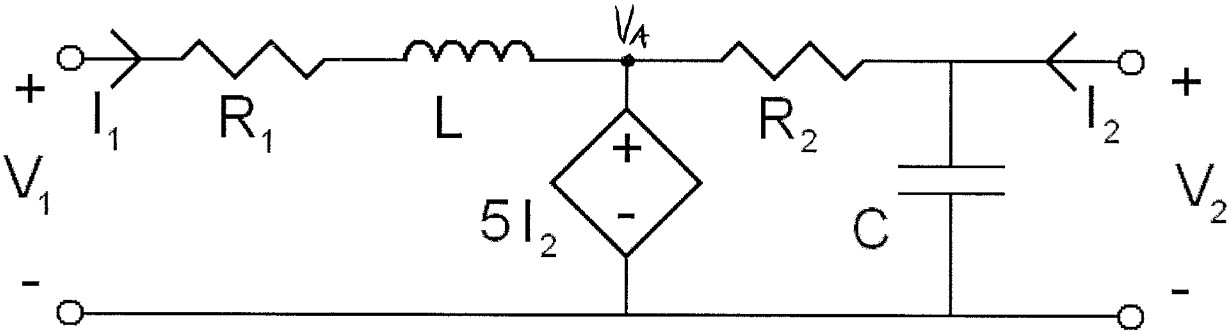
Find the admittance parameters of the circuit below.

R_1=1/12Ω

R_2=1Ω

C=6F

L=1/2H



Admittance → Nodal Equations

@ V_A ⇒ $V_A = 5I_2$

@ Port 1 $I_1 = \frac{V_1 - 5I_2}{R_1 + sL}$

$I_1 = \frac{V_1 - 5(V_2(s + \frac{1}{6}))}{R_1 + sL}$

$I_1 = \frac{V_1 - V_2\ 5(s + \frac{1}{6})}{\frac{1}{12} + \frac{s}{2}}$

$I_1 = \frac{2V_1}{s + 1/6} - 10V_2 \frac{s + 1/6}{s + 1/6}$

$= V_1 \frac{2}{s + 1/6} - V_2\ 10$

@ Port 2 $I_2 = V_2 sC + \frac{V_2 - 5I_2}{R_2}$

$I_2(1 + \frac{5}{R_2}) = V_2(sC + \frac{1}{R_2})$

$I_2 = V_2 \frac{sC + \frac{1}{R_2}}{1 + \frac{5}{R_2}}$

$I_2 = V_2 \frac{6s + 1}{1 + 5} = V_2(s + \frac{1}{6})$

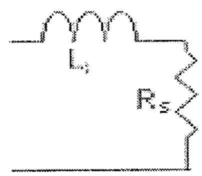
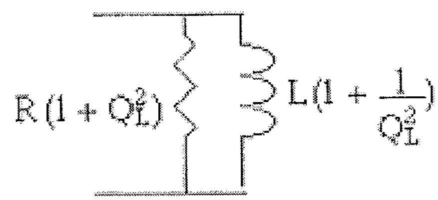
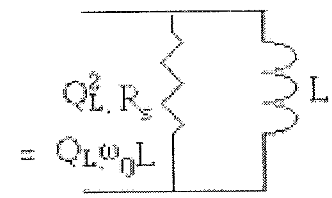
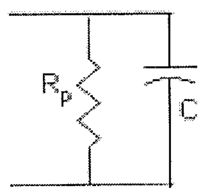
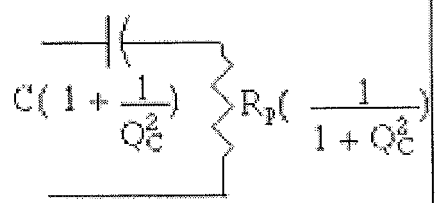
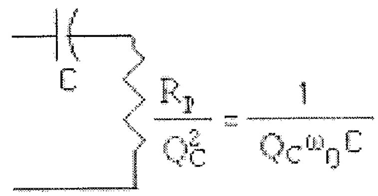
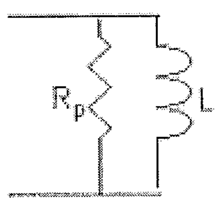
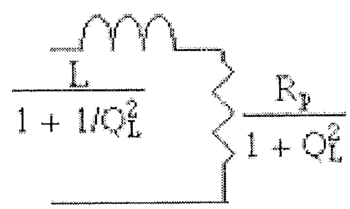
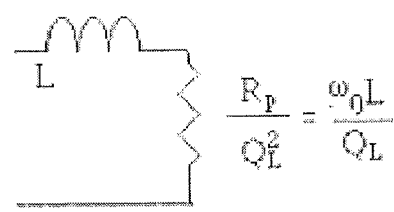
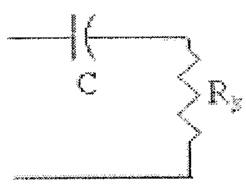
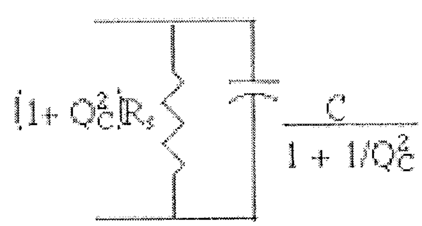
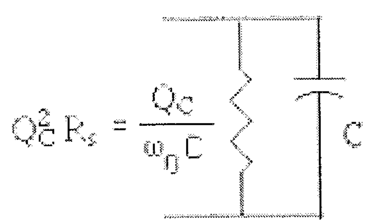
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{s + 1/6} & -10 \\ 0 & s + 1/6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Laplace Transform Pairs

Item Number	$f(t)$	$\mathcal{L}\{f(t)\}$=F(s)
1	$K\delta(t)$	K
2	$Ku(t) or K$	$\frac{K}{s}$
3	$r(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{-at}u(t)$	$\frac{1}{s+a}$
6	$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$
7	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$
8	$\sin(\omega t)\, u(t)$	$\frac{\omega}{s^2+\omega^2}$
9	$\cos(\omega t)\, u(t)$	$\frac{s}{s^2+\omega^2}$
10	$e^{-at}\sin(\omega t)\, u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
11	$e^{-at}\cos(\omega t)\, u(t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
12	$t\sin(\omega t)\, u(t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
13	$t\cos(\omega t)\, u(t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
14	$\sin(\omega t+\varphi)\, u(t)$	$\frac{s\sin\varphi+\omega\cos\varphi}{s^2+\omega^2}$
15	$\cos(\omega t+\varphi)\, u(t)$	$\frac{s\cos\varphi-\omega\sin\varphi}{s^2+\omega^2}$
16	$e^{-at}[\sin(\omega t)-\omega t\cos(\omega t)]u(t)$	$\frac{2\omega^3}{[(s+a)^2+\omega^2]^2}$
17	$te^{-at}\sin(\omega t)u(t)$	$2\omega\frac{s+a}{[(s+a)^2+\omega^2]^2}$
18	$e^{-at}\left[C_1\cos(\omega t)+\left(\frac{C_2-C_1a}{\omega}\right)\sin(\omega t)\right]u(t)$	$\frac{C_1s+C_2}{(s+a)^2+\omega^2}$
19	$2\sqrt{A^2+B^2}e^{-at}\cos\left[\omega t-\tan^{-1}\left(\frac{B}{A}\right)\right]u(t)$	$\frac{A+jB}{s+a+j\omega}+\frac{A-jB}{s+a-j\omega}$
20	$2\sqrt{A^2+B^2}te^{-at}\cos\left[\omega t-\tan^{-1}\left(\frac{B}{A}\right)\right]u(t)$	$\frac{A+jB}{(s+a+j\omega)^2}+\frac{A-jB}{(s+a-j\omega)^2}$

Laplace Properties

Property	$f(t)$	$\mathcal{L}\{f(t)\}$=F(s)
Linearity	$a_1f_1(t) + a_2f_2(t)$	$a_1F(s) + a_2F_2(s)$
Time Shift	$f(t-T)u(t-T)$	$e^{-sT}F(s)$
Multiplication by t	$tf(t)u(t)$	$-\frac{d}{ds}F(s)$
Multiplication by t^n	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
Frequency Shift	$e^{-at}f(t)$	$F(s+a)$
Time Differentiation	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
Second-order Differentiation	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf\ 0 — f^{(1)}(0^-)$
*n*th-order Differentiation	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f^{(1)}(0^-) - \cdots - f^{n-t}(0^-)$
Time Integration	(a) $\int_{-\infty}^{t} f(q)dq$ (b) $\int_{0^-}^{t} f(q)dq$	$\frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(q)dq}{s}$ $\frac{F(s)}{s}$
Time or Frequency Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$

Original Circuit	Exact Equivalent Circuit at ω_0	Approximate Equivalent circuit, for high Q, ($Q_L > 6$ and $Q_C > 6$) and ω within $(1 \pm 0.05)\omega_0$
 L_s, R_s; $Q_L(\omega_0) = \frac{\omega_0 L}{R_s}$	 $R(1 + Q_L^2)$, $L(1 + \frac{1}{Q_L^2})$	 $Q_L^2 . R_s = Q_L \omega_0 L$, L
 R_p, C; $Q_c(\omega_0) = \omega_0 R_p C$	 $C(1 + \frac{1}{Q_C^2})$, $R_p(\frac{1}{1 + Q_C^2})$	 C, $\frac{R_p}{Q_C^2} = \frac{1}{Q_C \omega_0 C}$
 R_p, L; $Q_L(\omega_0) = \frac{R_p}{\omega_0 L}$	 $\frac{L}{1 + 1/Q_L^2}$, $\frac{R_p}{1 + Q_L^2}$	 L, $\frac{R_p}{Q_L^2} = \frac{\omega_0 L}{Q_L}$
 C, R_s; $Q_C(\omega_0) = \frac{1}{\omega_0 R_s C}$	 $[1 + Q_C^2]R_s$, $\frac{C}{1 + 1/Q_C^2}$	 $Q_C^2 R_s = \frac{Q_C}{\omega_0 C}$, C