ECE 202 - Linear Circuit Analysis II

Purdue University, Spring 2009

Homework Set 5 Solutions

Solution 49

(a)

$$\left(\frac{\omega_s}{\omega_p}\right)^{2n} \geq \frac{10^{0.1A_{min}} - 1}{10^{0.1A_{max}} - 1}$$

```
>> wp=2*pi*550;
>> ws=2*pi*2200;
>> n=3;
>> Amax=0.75;
>> Amin=1;
>> limit=((ws/wp)^(2*n))*((10^(0.1*Amax))-1)

limit =

  772.1051

>> while(((10^(0.1*Amin))-1)<=limit)
Amin=Amin+1;
end
>> Amin=Amin-1

Amin =

    28
```

(b)

```
>> fp=550; fs=2200; Amax=0.75; Amin=28;
>> n=buttord(fp, fs, Amax, Amin, 's')
```

```
n =

     3

>> fcmin=fp/((10^(0.1*Amax)-1)^(1/(2*n)))

fcmin =

  726.3455

>> fcmax=fs/((10^(0.1*Amin)-1)^(1/(2*n)))

fcmax =

  751.3994

>> wcmin=2*pi*fcmin

wcmin =

  4.5638e+003

>> wcmax=2*pi*fcmax

wcmax =

  4.7212e+003
```

(c)

```
>> [z, p, k]=buttap(n)

z =

     []


p =
```

```
  -0.5000 + 0.8660i
  -0.5000 - 0.8660i
  -1.0000


k =

    1.0000

>> abs(p)

ans =

    1.0000
    1.0000
    1.0000
```

Thus poles lie on the unit circle.

(d)

```
>> firstorder=poly(p(3))

firstorder =

     1     1

>> secondorder=poly(p(1:2))

secondorder =

    1.0000    1.0000    1.0000
```

(e)

```
>> wc=wcmin;
>> fc=fcmin;
>> znew=z*wc
```

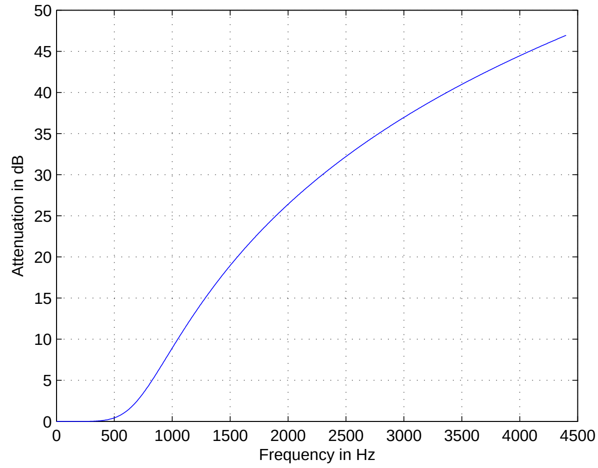
```
znew =

     []

>> pnew=p*wc

pnew =

  1.0e+003 *

  -2.2819 + 3.9523i
  -2.2819 - 3.9523i
  -4.5638

>> knew=k*wc^n

knew =

  9.5054e+010

>> abs(pnew)

ans =

  1.0e+003 *

    4.5638
    4.5638
    4.5638
```

Thus poles lie on a circle about origin.

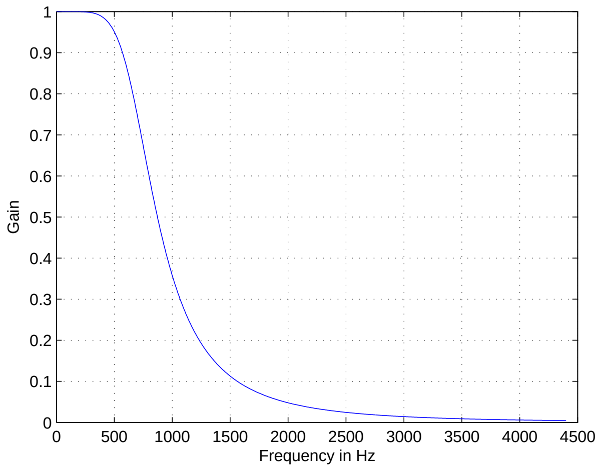
(f)

$$
\begin{aligned}
-20\log_{10}|H_{3dBNLP}(j1)| &= 3.0103\ dB \\
-20\log_{10}|H_{3dBNLP}\left(j\frac{\omega_s}{\omega_p}\right)| &= 36.1247\ dB
\end{aligned}
$$

(g)



(h)



At $f_s = 2200\ Hz$, the attenuation is 28.88 dB, which is greater than $A_{min} = 28\ dB$. At $f_p = 550\ Hz$, the attenuation is 0.75 dB, which is equal to $A_{max} = 0.75\ dB$. Thus filter response meets the given brickwall specifications.

Solution 50

(a)

$$
\begin{aligned}
n &\geq \frac{\log_{10}\left(\sqrt{\frac{10^{0.1A_{min}}-1}{10^{0.1A_{max}}-1}}\right)}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)} \\
\Rightarrow n &\geq 1.8508 \\
\Rightarrow n &= 2 \\
\Rightarrow H_{3dBNLP} &= \frac{1}{s^2+\sqrt{2}s+1} \\
\omega_{c,min} &= \frac{\omega_p}{(10^{0.1A_{max}}-1)^{\frac{1}{2n}}} \\
&= 3592.4\ rad/s \\
\omega_{c,max} &= \frac{\omega_s}{(10^{0.1A_{min}}-1)^{\frac{1}{2n}}} \\
&= 3983.8\ rad/s
\end{aligned}
$$

(b)

(i)

$$
\begin{aligned}
H(s) &= \frac{1}{s^2LC+sCR_s+1} \\
&= \frac{1/LC}{s^2+sR_s/L+1/LC}
\end{aligned}
$$

(ii)

Realization: L=0.707 H, C=1.414 F, $R_s = 1\ \Omega$

(c)

$$
\begin{aligned}
K_f &= \omega_{c,min} \\
&= 3592.4 \\
\Rightarrow K_m &= \frac{C}{K_f C_{final}} \\
&= 3.9361 \times 10^4 \\
\Rightarrow L_{final} &= \frac{LK_m}{K_f} \\
&= 7.7464\ H
\end{aligned}
$$

$$\begin{aligned} \Rightarrow R_{final} &= K_m R \\ &= 39.361\ k\Omega \end{aligned}$$

Solution 51

(a)

$$\begin{aligned} H_1(s) &= \frac{R_L\|1/C_1s}{(R_L\|1/C_1s)+Ls} \\ &= \frac{1}{LC_1}\frac{1}{s^2+s/R_LC_1+1/LC_1} \\ H_2(s) = -\frac{Y_{in}(s)}{Y_{out}(s)} &= -\frac{G_1}{C_2s+G_2} \\ &= -\frac{1}{R_1C_2}\frac{1}{s+1/R_2C_2} \\ H(s) &= H_1(s)H_2(s) \end{aligned}$$

(b)

Realization: $R_1 = R_2 = R_L = 1\ \Omega,\ L = 1\ H,\ C_1 = 1\ F,\ C_2 = 1\ F$

(c)

$$\begin{aligned} K_f &= 4.5638k \\ K_{m1} &= 20k \\ K_{m2} &= 100 \\ \Rightarrow R_{2,new} &= 20\ k\Omega \\ \Rightarrow C_{1,new} &= \frac{C_1}{K_{m2}K_f} \\ &= 2.19\ \mu F \\ \Rightarrow C_{2,new} &= \frac{C_2}{K_{m1}K_f} \\ &= 10.96\ nF \\ \Rightarrow L_{new} &= \frac{LK_{m2}}{K_f} \\ &= 0.0219\ H \end{aligned}$$

Two separate magnitude scale factors are needed because the transfer function is realized as two sections, a second order section followed by an active first order section. So the two sections can be magnitude scaled separately. Thus the second order section is magnitude scaled by K_{m2} and the first order by K_{m1}.

Solution 52

(a)

The following KCL equations can be written,

$$\begin{aligned}
V_{in} - V_1 + \frac{V_{out} - V_1}{Ls} &= V_1 C_1 s \\
\Rightarrow V_1 \left(1 + C_1 s + \frac{1}{Ls}\right) &= V_{in} + \frac{V_{out}}{Ls} \\
\frac{V_1 - V_{out}}{Ls} &= V_{out}(1 + C_2 s) \\
\Rightarrow V_1 &= V_{out}[Ls(1 + C_2 s) + 1]
\end{aligned}$$

Eliminating V_1 from the above equations,

$$\begin{aligned}
V_{out}\left[\left\{1 + C_1 s + \frac{1}{Ls}\right\}\{1 + Ls(1 + C_2 s)\} - \frac{1}{Ls}\right] &= V_{in} \\
\Rightarrow \frac{V_{out}}{V_{in}} &= \frac{1}{s^3 LC_1C_2 + s^2(C_1 + C_2)L + s(C_1 + C_2 + L) + 2} \\
H_{cir}(s) = \frac{V_{out}(s)}{V_{in}(s)} &= \frac{\frac{1}{LC_1C_2}}{s^3 + \frac{C_1+C_2}{C_1C_2}s^2 + \frac{C_1+C_2+L}{LC_1C_2}s + \frac{2}{LC_1C_2}}
\end{aligned}$$

(b)

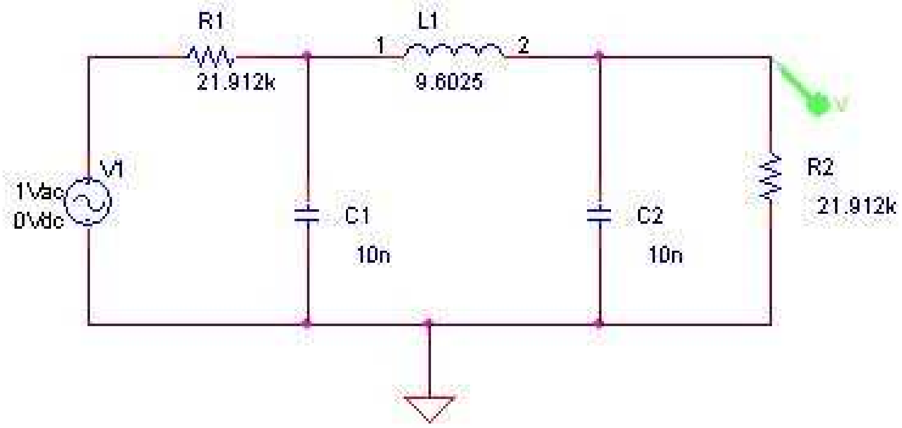
$$\begin{aligned}
H_{3dBNLP}(s) &= \frac{K}{s^3 + 2s^2 + 2s + 1} \\
\Rightarrow \frac{C_1 + C_2}{C_1C_2} &= 2 \\
LC_1C_2 &= 2 \\
\frac{C_1 + C_2 + L}{LC_1C_2} &= 2 \\
\Rightarrow C_1 = C_2 &= 1\ F \\
L &= 2\ H
\end{aligned}$$

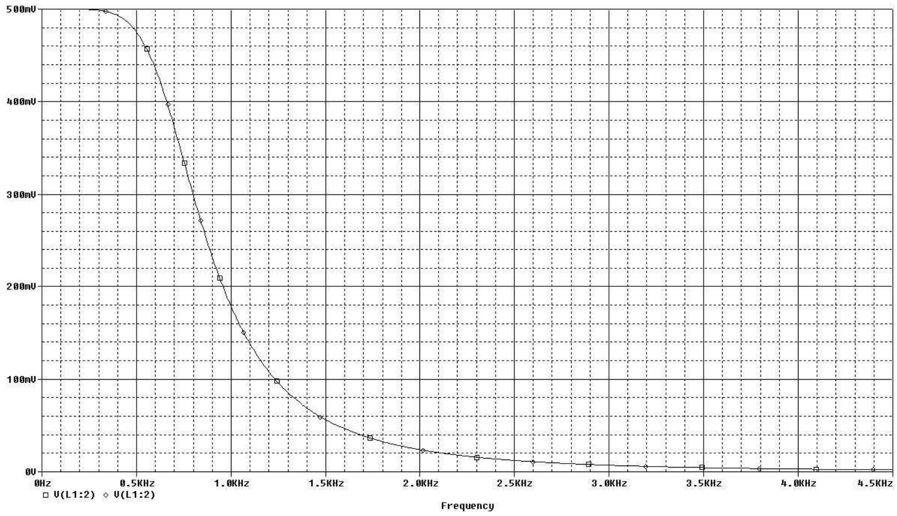
(c)

$$\begin{aligned} K_f &= 4.5638k \\ \Rightarrow K_m &= \frac{C_2}{C_{2final} K_f} \\ &= 21.912k \\ \Rightarrow C_{1final} &= 10\ nF \\ L_{final} &= \frac{L K_m}{K_f} \\ &= 9.6025\ H \end{aligned}$$

(d)

The frequency response can be verified in SPICE. The circuit schematic and the simulated response are shown below.





500mV
400mV
300mV
200mV
100mV
0V
0Hz
0.5KHz
1.0KHz
1.5KHz
2.0KHz
2.5KHz
3.0KHz
3.5KHz
4.0KHz
4.5KHz
□ V(L1:2) ◇ V(L1:2)
Frequency