ECE 202 - Linear Circuit Analysis II

Purdue University, Spring 2009

Homework Set 6 Solutions

Solution 57 - 58

(a)

$$\begin{aligned}\Omega_p &= 1\\ \Omega_s &= \frac{\omega_p}{\omega_s}\\ &= 5\end{aligned}$$

Equivalent NLP specs are given by ($\Omega_p = 1,\ A_{max} = 1\ dB$) and ($\Omega_s = 5,\ A_{min} = 35\ dB$)

(b)

The minimum filter order can be verified to be 3 in MATLAB using the command `buttord(1, 5, 1, 35, 's')`.

(c)

$$\begin{aligned}H_{3dBNLP}(s) &= \left(\frac{1}{s^2+s+1}\right)\left(\frac{1}{s+1}\right)\\ \Rightarrow H_{2ndorder}(s) &= \frac{1}{s^2+s+1}\\ H_{1storder}(s) &= \frac{1}{s+1}\end{aligned}$$

(d)

$$\Omega_c = \frac{1}{\varepsilon_{max}^{(1/n)}}$$

$$
\begin{aligned}
&= (10^{0.1A_{max}} - 1)^{-\frac{1}{2n}} \\
&= 1.2526 \\
H_{NLP}(s) &= H_{2NLP}(s)H_{1NLP}(s) \\
&= H_{2ndorder}\left(\frac{s}{1.2526}\right) H_{1storder}\left(\frac{s}{1.2526}\right) \\
&= \left(\frac{1.569}{s^2 + 1.2526s + 1.569}\right)\left(\frac{1.2526}{s + 1.2526}\right)
\end{aligned}
$$

(e)

$$
\begin{aligned}
H_{NHP}(s) &= H_{2NHP}(s)H_{1NHP}(s) \\
&= H_{2NLP}\left(\frac{1}{s}\right) H_{1NLP}\left(\frac{1}{s}\right) \\
&= \left(\frac{s^2}{s^2 + 0.7983s + 0.6373}\right)\left(\frac{s}{s + 0.7983}\right)
\end{aligned}
$$

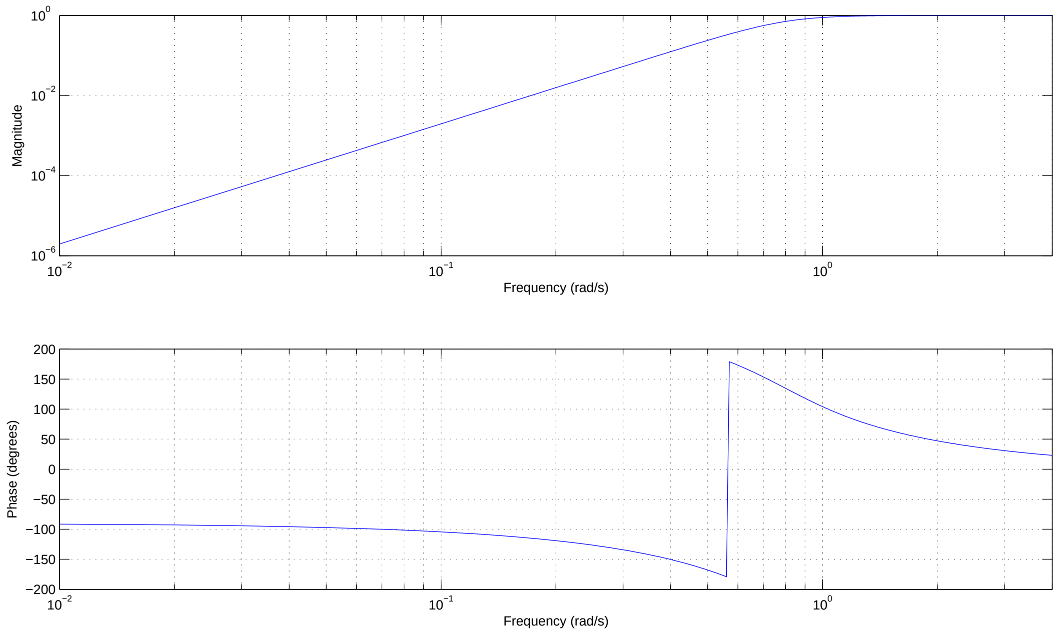


Figure 1: Frequency Response of $H_{NHP}(s)$

(f)

$$
\begin{aligned}
H_{2NHP}(s) = \frac{V_{out}}{V_{in}} &= \frac{s^2}{s^2 + 0.7983s + 0.6373} \\
\Rightarrow D^2 v_{out}(t) + 0.7983 D v_{out}(t) + 0.6373 v_{out}(t) &= D^2 v_{in}(t) \\
\Rightarrow v_{out} &= v_{in} + x_1 \ldots (1) \\
x_1 &= D^{-1}(-0.7983 v_{out}) + D^{-2}(-0.6373 v_{out}) \\
\Rightarrow \dot{x_1} = D x_1 &= -0.7983 v_{in} - 0.7983 x_1 + x_2 \\
x_2 &= D^{-1}(-0.6373 v_{out}) \\
\Rightarrow x_1 &= -0.7983 \int v_{in} - 0.7983 \int x_1 + \int x_2 \ldots (2) \\
\Rightarrow \dot{x_2} = D x_2 &= -0.6373 v_{out} \\
&= -0.6373 v_{in} - 0.6373 x_1 \\
\Rightarrow x_2 &= -0.6373 \int v_{in} - 0.6373 \int x_1 \ldots (3)
\end{aligned}
$$

Equation (1) can be realized using an op amp assuming x_1 is available. Similarly (2) can be realized assuming x_2 is available and (3) can be realized to obtain x_2. So we get the following realization for $H_{2NHP}(s)$.

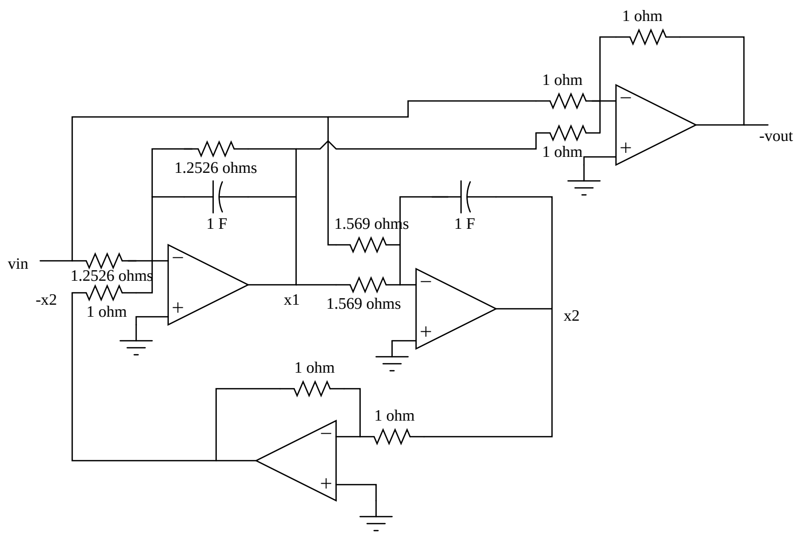


Figure 2: Observable canonical realization of $H_{2NHP}(s)$

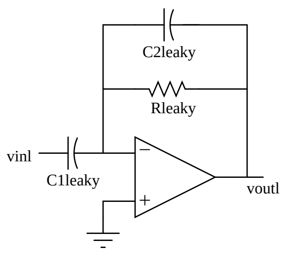


Figure 3: Leaky integrator

(g)

Writing KCL node equation at the inverting terminal of the op amp we get the following,

$$\begin{aligned} V_{inl}C_{1leaky}s &= -V_{outl}(C_{2leaky}s + G_{leaky}) \\ \Rightarrow H_{leaky}(s) &= -\frac{C_{1leaky}s}{C_{2leaky}s + G_{leaky}} \end{aligned}$$

Comparing the coefficients of $H_{1NHP}(s)$ and $H_{leaky}(s)$ [1], we get,

$$\begin{aligned} C_{1leaky} = C_{2leaky} &= 1\ F \\ G_{leaky} &= 0.7983\ \mho \end{aligned}$$

The new circuit is drawn on the next page.

[1] Note that a negative sign only results in a phase change, the filter characteristics remain the same

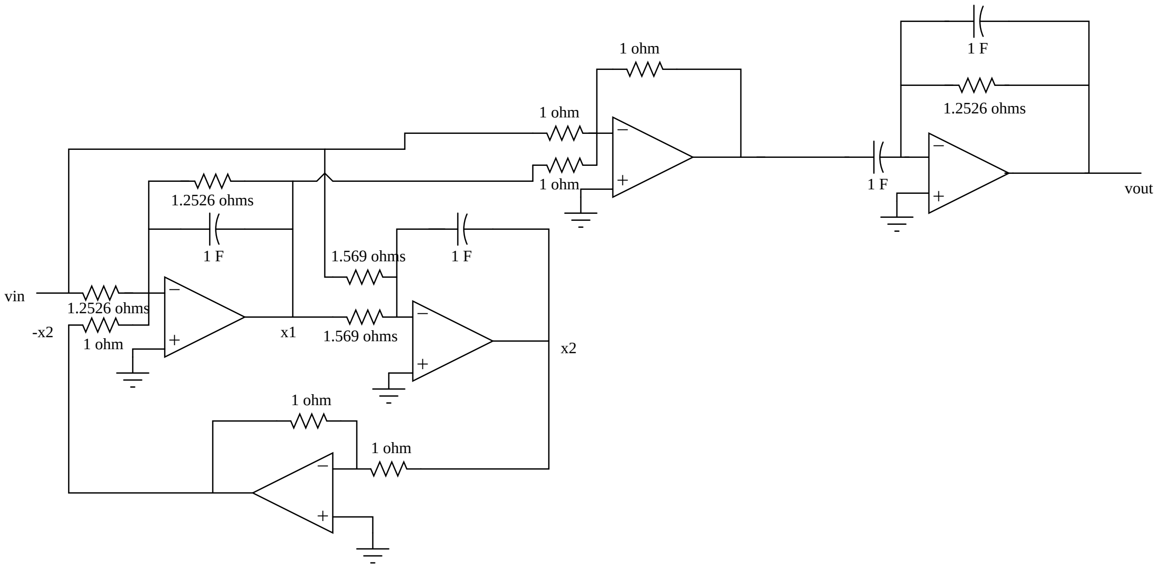


Figure 4: Observable canonical realization of $H_{NHP}(s)$

(h)

$$\begin{aligned} K_f = \omega_p &= 25.13k \\ \Rightarrow K_m = \frac{C_{old}}{C_{new}K_f} &= 7.96k \end{aligned}$$

The value for K_m is same for the modified leaky integrator circuit. The following table shows the element values before and after frequency and magnitude scaling.

Element	Before scaling	After scaling
C	$1\ F$	$5\ nF$
R	$1\ \Omega$	$7.96\ k\Omega$
R	$1.569\ \Omega$	$12.59\ k\Omega$
R	$1.2526\ \Omega$	$9.97\ k\Omega$

The final circuit is drawn as follows.

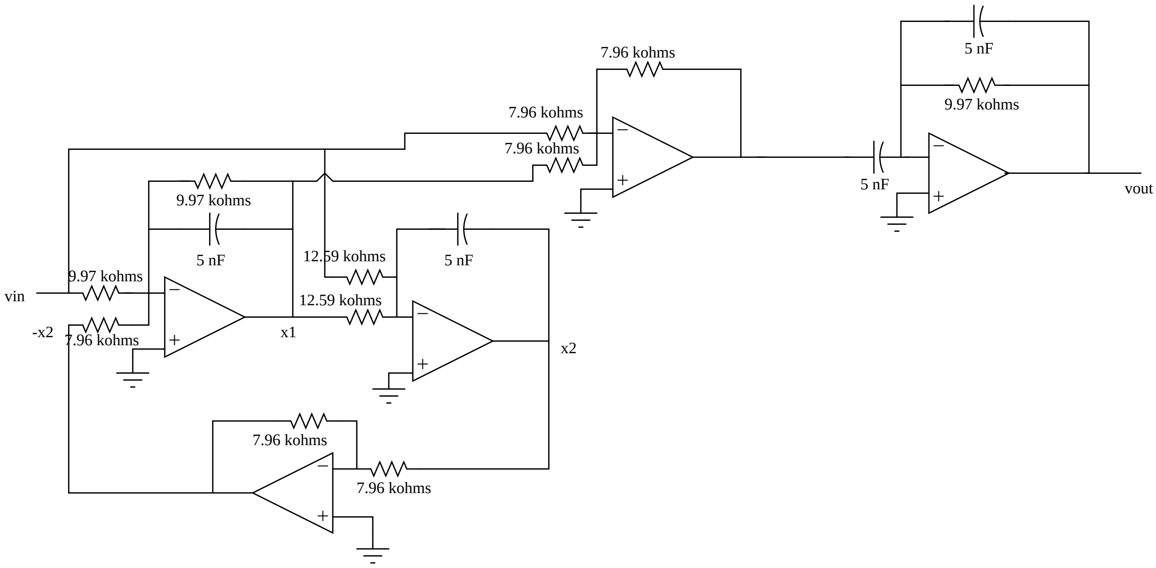


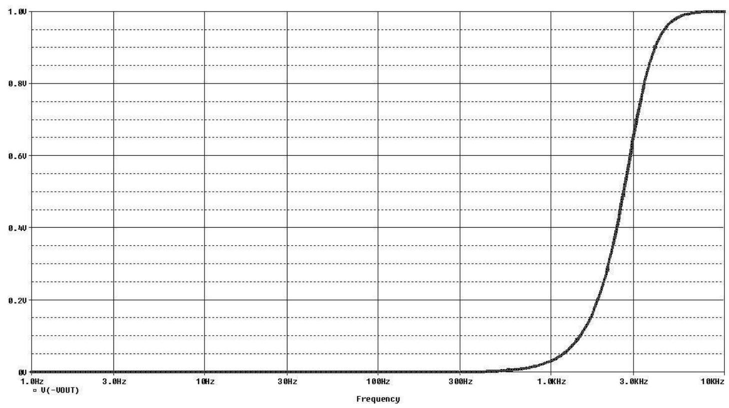
Figure 5: Observable canonical realization of $H_{HP}(s)$

(i)

Since the input resistor in our design is 9.97 $k\Omega$, an input source resistance of 100 Ω would modify our input resistor to 9.87 $k\Omega$, which is a change of 1.00%.

(j)

The frequency response simulated in spice is shown below,



Solution 59 - 60

Task 1

Clearly, the center frequency is the frequency where $|H_{BP}(j\omega)|$ is maximum, which is $\omega_m/2\pi$ here. Thus $\omega_m = (40 \times 10^3) \times 2\pi = 251.33 \times 10^3$. The 3dB bandwidth is B_ω. Thus $B_\omega = (10 \times 10^3) \times 2\pi = 62.83 \times 10^3$. Gain at the channel center frequency is k/B_ω. Thus $k = 10 \times 62.83 \times 10^3 = 628.3 \times 10^3$.

Task 2

$$
\begin{aligned}
H_{NBP}(s) = H_{BP}(\omega_m s) &= \frac{k\omega_m s}{\omega_m^2 s^2 + B_\omega \omega_m s + \omega_m^2} \\
&= \frac{(k/\omega_m)s}{s^2 + (B_\omega/\omega_m)s + 1} = \frac{as}{s^2 + bs + 1} \\
\Rightarrow a = \frac{k}{\omega_m} &= 2.5,\ b = \frac{B_\omega}{\omega_m} = 0.25
\end{aligned}
$$

Task 3

We develop the following equations based on the transfer function,

$$
\begin{aligned}
V_{out} &= \frac{V_{in}}{s^2 + bs + 1}(as) \\
&= as\hat{V_{out}} \\
\Rightarrow v_{out} &= a\dot{\hat{v_{out}}} \ldots (1) \\
\Rightarrow s^2\hat{V_{out}} + bs\hat{V_{out}} + \hat{V_{out}} &= V_{in} \\
\Rightarrow \ddot{\hat{v_{out}}} &= v_{in} - b\dot{\hat{v_{out}}} - \hat{v_{out}} \\
\Rightarrow -\dot{\hat{v_{out}}} &= -\int v_{in} - b\int -\dot{\hat{v_{out}}} - \int -\hat{v_{out}} \ldots (2)
\end{aligned}
$$

Equation (1) can be realized using an inverting amplifier and (2) can be realized using an integrator with output feedback. This leads to the realization of $H_{NBP}(s)$ which is shown below.

Task 4

$$
K_f = \omega_m = 251.33k
$$

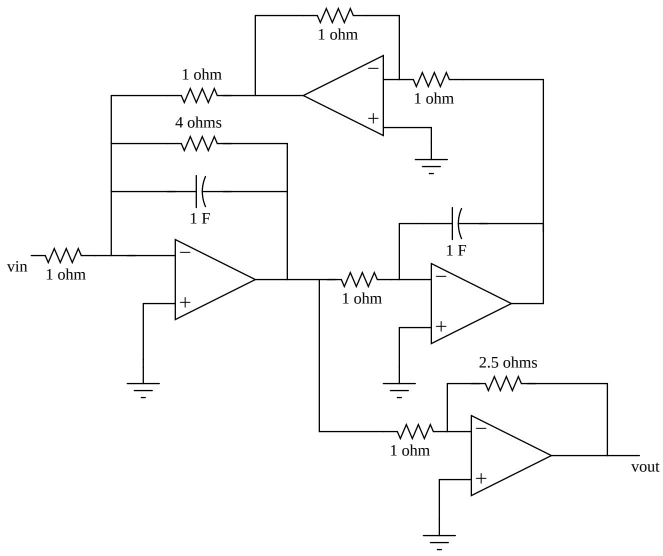


Figure 6: Realization of $H_{NBP}(s)$

$$\Rightarrow K_m = \frac{C_{old}}{C_{new}K_f} \quad = \quad 3.98k$$

Task 5

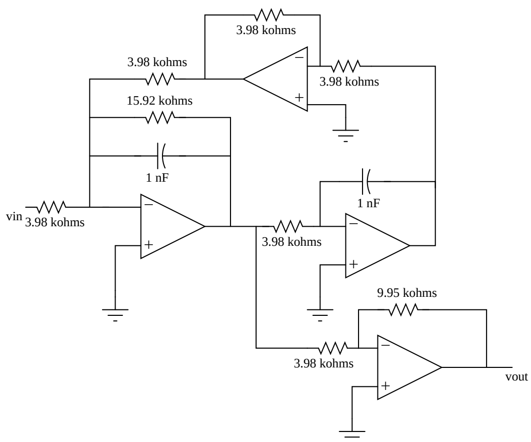


Figure 7: Final biquad realization of $H_{BP}(s)$

Task 6

The final circuit of $H_{BP}(s)$ used resistors of values 3.98 $k\Omega$, 15.92 $k\Omega$, and 9.95 $k\Omega$. As the practical design requires resistors with 5% tolerance, we can use E24 series of standard resistors of values 3.9 $k\Omega$, 16 $k\Omega$, and 10 $k\Omega$ to replace the ideal resistors.

Task 7

The frequency response simulated in spice is shown below,