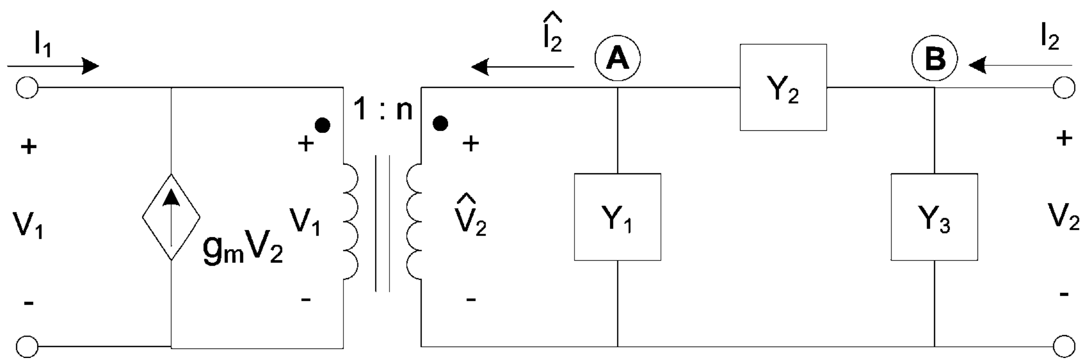
ECE 202 Linear Circuit Analysis II (Sp'09)

Homework #19 Solution (73–76)

Problem 73



$$g_m = 2\text{S}\ ,\ n = 2\ ,\ Y_1 = Y_2 = 0.2\ \text{S}\ ,Y_3 = 0.5s\ \text{S}$$

(a) Find Y-parameters

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}}_{[Y]} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

To find Y-parameters we need to express I_1 and I_2 in terms of V_1 and V_2. We will do this by writing node equations at nodes A and B as indicated in the diagram.

$$\text{node A:} \qquad 0 = \hat{I}_2 + \hat{V}_2 Y_1 + (\hat{V}_2 - V_2)Y_2 \tag{73.1}$$

$$\text{node B:} \qquad 0 = -I_2 + V_2 Y_3 + (V_2 - \hat{V}_2)Y_2 \tag{73.2}$$

Using voltage relationship of ideal transformer we get

$$\hat{V}_2 = nV_1 \tag{73.3}$$

$$\hat{I}_2 = -\frac{1}{n}\hat{I}_1 = -\frac{1}{n}(I_1 + g_m V_2) \tag{73.4}$$

Substituting $\hat{V}_2$ and $\hat{I}_1$ into (eq.73.1) and (eq.73.2),

$$0 = -\frac{1}{n}(I_1 + g_m V_2) + (nV_1)Y_1 + (nV_1 - V_2)Y_2 \quad \Rightarrow I_1 = \quad (n^2(Y_1 + Y_2))V_1 + \quad (-g_m - nY_2)V_2$$

$$0 = -I_2 + V_2 Y_3 + (V_2 - nV_1)Y_2 \quad \Rightarrow I_2 = \quad (-nY_2)V_1 + \quad (Y_2 + Y_3)V_2$$

Writing into a matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} n^2(Y_1+Y_2) & -(g_m+nY_2) \\ -nY_2 & Y_2+Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Plugging in the given values, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \boxed{\begin{bmatrix} 1.6 & -2.4 \\ -0.4 & 0.2+0.5s \end{bmatrix}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

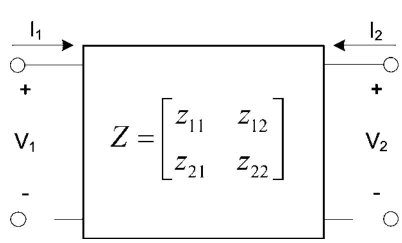
(b) Z-parameters

Z-paramter is simply the inverse of Y-paramter.

$$\begin{aligned} [I] &= [Y][V] \\ [Y]^{-1}[I] &= [V] \\ \Rightarrow [Z] &= [Y]^{-1} \\ &= \begin{bmatrix} 1.6 & -2.4 \\ -0.4 & 0.2+0.5s \end{bmatrix}^{-1} \\ &= \frac{1}{(1.6)(0.2+0.5s)-(-2.4)(-0.4)} \begin{bmatrix} 0.2+0.5s & 2.4 \\ 0.4 & 1.6 \end{bmatrix} \\ &= \boxed{\frac{1}{s-0.8} \begin{bmatrix} 0.625s+0.25 & 3 \\ 0.5 & 2 \end{bmatrix} [\Omega]} \end{aligned}$$

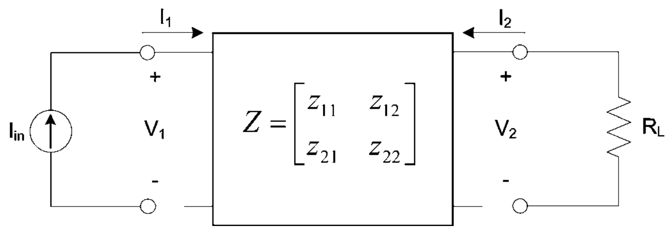
(c) impulse response $v_2(t)$

Before any connection the matrix equation is:



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

When port 1 is connected to a current source and port 2 is connected to a $R_L = 10\Omega$ resistor, our matrix equations changes in the following way:
Since $I_1 = I_{in}(s) = 1$ and $I_2 = -V_2 Y_L$,



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} 1 \\ -V_2/R_L \end{bmatrix}$$

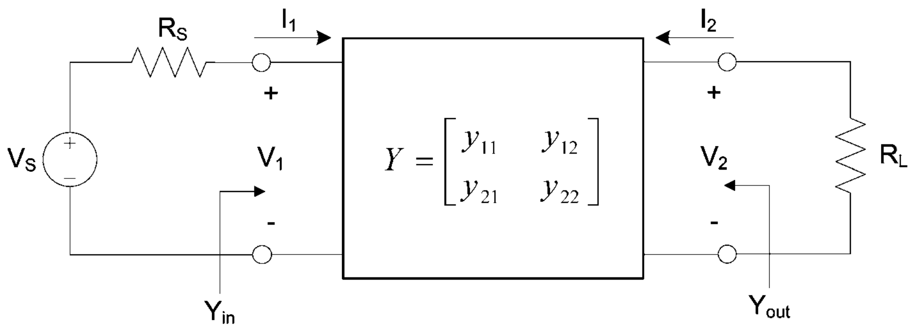
Writing out the second equation,

$$\begin{aligned} V_2 =& z_{21} - z_{22}\frac{V_2}{R_L} \\ V_2 + z_{22}\frac{V_2}{R_L} =& z_{21} \\ V_2 =& \frac{z_{21}R_L}{z_{22}+R_L} \\ =& \frac{\frac{1/2}{s-0.8}(10)}{\frac{2}{s-0.8}+10} \\ =& \frac{5/2}{5s-3} = \frac{1/2}{s-3/5} \\ v_2(t) =& \boxed{0.5e^{+0.6t}u(t)[\text{V}]} \end{aligned}$$

MATLAB codes

```
syms s I1 I2 V1 V2 y11 y12 y21 y22 Y Z z11 z12 z21 z22
gm=2; n=2; Y1=0.2; Y2=Y1; Y3=0.5*s;
y11=n^2*(Y1+Y2);
y12=-(gm+n*Y2);
y21=-n*Y2;
y22=Y2+Y3;
Y=[y11 y12;y21 y22]
% pretty(simplify(Y))
%
%                               [8/5         -12/5   ]
%                               [                    ]
%                               [-2/5    1/5 + 1/2 s]
Z=inv(Y);
z11=Z(1,1); z12=Z(1,2); z21=Z(2,1); z22=Z(2,2);
RL=10;
V2=RL*z21/(z22+RL)
%                                          1
%                                   5/2 --------
%                                       -3 + 5 s
```

Problem 74

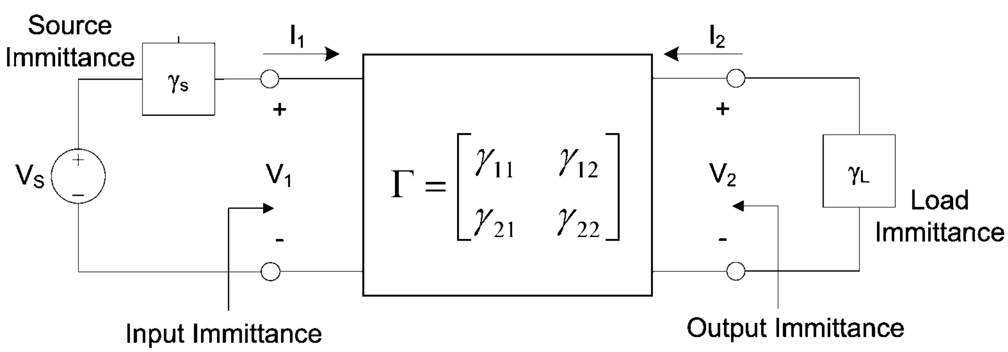


$$R_s = 1\Omega,\ y_{21} = -1\text{S},\ y_{12} = 0.75\text{S},\ y_{22} = 0.25\text{S},\ R_L = 8\Omega,\ \frac{V_1}{V_s} = \frac{2}{s+6}$$

(a)y_{11}

$$\frac{V_1}{V_s} = \frac{Z_{in}}{Z_{in}+Z_s} = \frac{Y_s}{Y_{in}+Y_s} = \frac{2}{s+6}$$
$$\Rightarrow Y_s(s+6) = 2(Y_{in}+Y_s)$$
$$Y_{in}(s) = \frac{s+4}{2}Y_s = \frac{s+4}{2}$$

<u>Input and Output Immittance</u>



$$\text{Input immittance} = \gamma_{11} - \frac{\gamma_{12}\gamma_{21}}{\gamma_{22} + (\text{load immittance})}$$
$$\text{Output immittance} = \gamma_{22} - \frac{\gamma_{12}\gamma_{21}}{\gamma_{11} + (\text{source immittance})}$$

Using above formula,

$$\begin{aligned}
Y_{in}(s) =& y_{11} - \frac{y_{12}y_{21}}{y_{22}+Y_L} \\
y_{11} =& Y_{in}(s) + \frac{y_{12}y_{21}}{y_{22}+Y_L} \\
y_{11} =& \frac{s+4}{2} + \frac{(0.75)(-1)}{0.25+1/8} \\
=& \boxed{\frac{s}{2}[\mathrm{S}]}
\end{aligned}$$

(b) Y_{out}

$$\begin{aligned}
Y_{out} =& y_{22} - \frac{y_{12}y_{21}}{y_{11}+Y_s} \\
=& (0.25) - \frac{(0.75)(-1)}{s/2+1} \\
=& \boxed{\frac{1}{4}\frac{s+8}{s+2}[\mathrm{S}]}
\end{aligned}$$

(c) Voltage Gain $G_v = \dfrac{V_2}{V_s}$

$$\begin{aligned}
G_v = \frac{V_2}{V_s} =& \frac{V_2}{V_1}\frac{V_1}{V_s} \\
=& \left(\frac{Y_s}{Y_{in}+Y_s}\right)\left(\frac{-y_{21}}{y_{22}+Y_L}\right) \\
=& \boxed{\frac{16/3}{s+6}}
\end{aligned}$$

(d) Power $p_L(t)$ absorbed by R_L

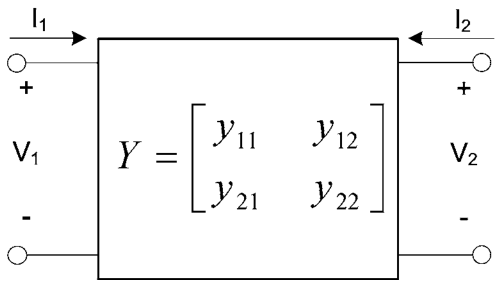
Given $v_s(t) = 30u(t)$ V, $V_s(s) = 30/s$

$$
\begin{aligned}
G_v = \frac{V_2}{V_s} \Rightarrow V_2 =& G_v V_s \\
=& \frac{16/3}{s+6}\frac{30}{s} \\
=& \frac{160}{s(s+6)} \\
\Rightarrow v_2(t) =& \frac{80}{3}\left(1 - e^{-6t}\right) [\mathrm{V}] \\
p_L(t) = \frac{(v_2(t))^2}{R_L} =& \frac{1}{8}\left(\frac{80}{3}(1 - e^{-6t})\right)^2 \\
=& \boxed{\frac{800}{9}(1 - e^{-6t})^2 [\mathrm{W}]}
\end{aligned}
$$

MATLAB code

```
syms s I1 I2 V1 V2 y11 y12 y21 y22 Y Yin Ys Yout Gv
Rs=1; Ys=1/Rs; y21=-1; y12=0.75; y22=0.25; RL=8; YL=1/RL;
Vs=30/s;
Yin=(s+4)/2;
y11=Yin+y12*y21/(y22+YL)
% 1/2*s
Yout=y22-(y12*y21)/(y11+Ys)
%                                          s + 8
%                                   1/4 -------
%                                          s + 2
Gv=(Ys/(Ys+Yin))*(-y21/(y22+YL))
%                                            1
%                                  16/3 -------
%                                          6 + s
V2=Gv*Vs
%                                        160
%                                   -----------
%                                    (6 + s) s
```

Problem 75 Admittance Parameters



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

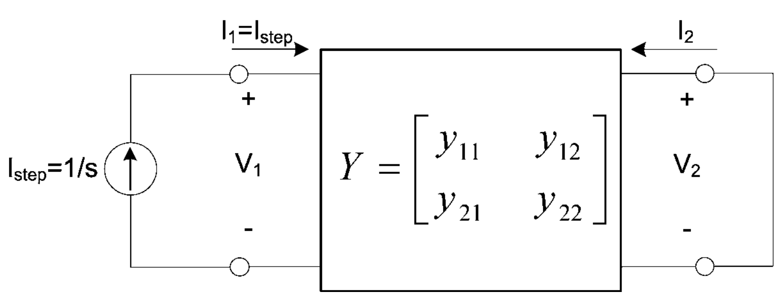
(a) Y-parameters

Short-ckt port-2 $\Rightarrow V_2 = 0$

Given

$$v_1(t) = (1 - e^{-4t})u(t) \qquad \Rightarrow V_1(s) = \frac{1}{s(s+4)}$$

$$v_2(t) = -e^{-3t}u(t) \qquad \Rightarrow V_2(s) = -\frac{1}{s+3}$$

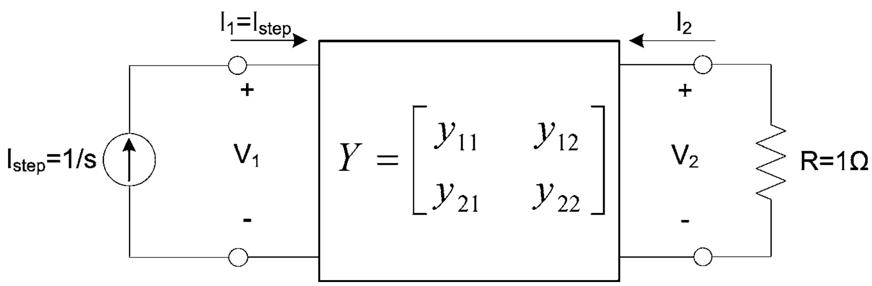


$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{1}{s+3}\frac{s(s+4)}{4} = \boxed{-\frac{1}{4}\frac{s(s+4)}{s+3}[\mathrm{S}]}$$
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \left(\frac{1}{s}\right)\frac{s(s+4)}{4} = \boxed{\frac{1}{4}(s+4)[\mathrm{S}]}$$

<u>1Ω resistor at port 2</u>

Given

$$v_1(t) = (1 - e^{-4t} + te^{-4t})u(t) \qquad \Rightarrow V_1(s) = \frac{5s+6}{s(s+4)^2}$$
$$v_2(t) = -e^{-7t}u(t) \qquad \Rightarrow V_2(s) = -\frac{1}{s+7}$$



$$\begin{bmatrix} I_1 \\ -V_2Y_L \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$\begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22}+Y_L \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{75.1}$$
$$I_1 = y_{11}V_1 + y_{12}V_2 \tag{75.2}$$
$$0 = y_{21}V_1 + (y_{22}+Y_L)V_2 \tag{75.3}$$

Rearranging (eq.75.2) we get

$$y_{12} = \frac{I_1 - y_{11}V_1}{V_2} = \frac{\frac{1}{s} + \left(\frac{1}{4}(s+4)\right)\left(\frac{5s+6}{s(s+4)^2}\right)}{-\frac{1}{s+7}} = \boxed{-\frac{1}{4}\frac{s+7}{s+4}[\mathrm{S}]}$$

and from (eq.75.3),

$$\begin{aligned}
V_2 =& \frac{-y_{21}}{y_{22}+Y_L}V_1 \\
\Rightarrow y_{22} =& -Y_L - y_{21}\frac{V_1}{V_2} \\
=& \boxed{\frac{1}{4}\frac{s^2+23s+64}{(s+3)(s+4)}[\text{S}]}
\end{aligned}$$

$$Y = \frac{1}{4}\begin{bmatrix} (s+4) & -\dfrac{s+7}{s+4} \\ -\dfrac{s(s+4)}{s+3} & \dfrac{s^2+23s+64}{(s+3)(s+4)} \end{bmatrix}$$

(b) $Z_{in}(s)$

$$\begin{aligned}
Y_{in}(s) =& y_{11} - \frac{y_{12}y_{21}}{y_{22}+Y_L} \\
=& \frac{s^2+8s+16}{5s+16} \\
Z_{in}(s) =& \frac{1}{Y_{in}(s)} \\
=& \boxed{\frac{5s+16}{s^2+8s+16}[\Omega]}
\end{aligned}$$

(c) Steady state magnitude of the gain

$$i_1(t) = cos(4t)u(t) \Rightarrow I_1(s) = \frac{s}{s^2+16}$$

Using the Y-matrix obtained from part (a) and (eq.75.1),

$$\begin{bmatrix} \dfrac{s}{s^2+16} \\ 0 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} (s+4) & -\dfrac{s+7}{s+4} \\ -\dfrac{s(s+4)}{s+3} & \dfrac{s^2+23s+64}{(s+3)(s+4)} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The values of V_1 and V_2 can be easily calculated using MATLAB. Using MATLAB,

$$\begin{aligned} V_2 =& \frac{1}{4}\frac{s^2}{s^2+16} \\ \Rightarrow G(s) =& \frac{V_2}{I_1} = \frac{s}{4} \\ \Rightarrow G(j4) =& \left|\frac{s}{4}\right|_{s=j4} \\ =& \boxed{1[\Omega]} \end{aligned}$$

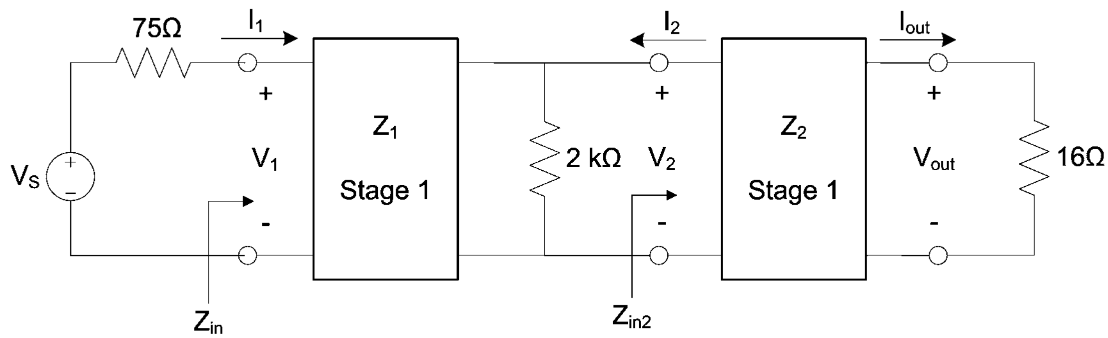
MATLAB code

```
syms s y11 y12 y21 y22 v1 v2 V1 V2 Y I1 I2 RL YL Yin Zin I V G w
%Part a)
V1=laplace(1-exp(-4*t));
I2=laplace(-exp(-3*t));
V2=0;
I1=1/s;
y11=I1/V1
%                                    1/4 s + 1
y21=I2/V1
%                                       s (s + 4)
%                                 - 1/4 ---------
%                                         s + 3

%Part b)
RL=1; YL=1/RL;
V1=laplace(1-exp(-4*t)+t*exp(-4*t));
I2=laplace(-exp(-7*t));
V2=-I2*1;
y12=(I1-y11*V1)/V2
%                                         s + 7
%                                   - 1/4 -----
%                                         s + 4
y22=-YL-y21*V1/V2
%                                    2
%                                   s  + 23 s + 64
%                               1/4 ---------------
%                                   (s + 3) (s + 4)
Yin=y11-(y12*y21)/(y22+YL)
%                                   2
%                                  s  + 8 s + 16
%                                  -------------
%                                    5 s + 16
%Part c)
I=[s/(s^2+16); 0];
Y=[y11, y12; y21, y22];
V=inv(Y)*I
```

```
%
%                              [       2               ]
%                              [     (s  + 23 s + 64) s]
%               V1  =          [1/4 -------------------]
%                              [            2   2      ]
%                              [     (s + 4)  (s  + 16)]
%
%                              [             2         ]
%                              [            s          ]
%                V2  =         [      1/4 -------      ]
%                              [           2           ]
%                              [          s  + 16      ]
V2=V(2,1);
G=V2/I(1)
%                                        1/4 s
s=i*4;
abs(1/4*s)
% ans =
%
%      1
```

Problem 76

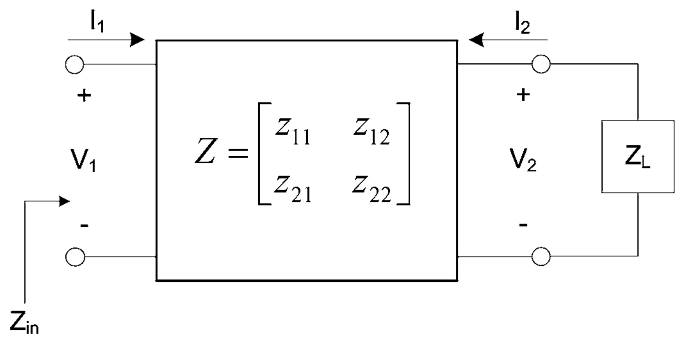


$$Z_1 = \begin{bmatrix} 2 & 0 \\ -10^3 & 20 \end{bmatrix} [\text{k}\Omega] \qquad \text{and} \qquad Z_2 = \begin{bmatrix} 62.582 & 1.2075 \\ 63.75 & 1.25 \end{bmatrix} [\text{k}\Omega]$$

(a) $Z_{in2}(s)$ and $Z_{in}(s)$

$$\begin{aligned} Z_{in2}(s) =& z_{2,11} - \frac{z_{2,12} z_{2,21}}{z_{2,22} + Z_L} \\ =& (62.582) - \frac{(1.2075)(63.75)}{1.25 + 0.016} \\ =& \boxed{1.7778[\text{k}\Omega]} \\ Z_{in}(s) =& z_{1,11} - \frac{z_{1,12} z_{1,21}}{z_{1,22} + (2k\Omega || Z_{in2}(s))} \\ =& (2) - \frac{(0)(-10^3)}{z_{1,22} + (2k\Omega || Z_{in2}(s))} \\ =& \boxed{2[\text{k}\Omega]} \end{aligned}$$

(b) $G_v = \frac{V_{out}}{V_s}$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \text{and} \qquad V_2 = -I_2 Z_L$$

$$\Rightarrow \begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} + Z_L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow I_2 = \frac{-z_{21}}{z_{22} + Z_L} I_1$$

$$\Rightarrow V_2 = \frac{z_{21} Z_L}{z_{22} + Z_L} I_1 \tag{76.1}$$

$$G_v = \frac{V_{out}}{V_s} = \left(\frac{V_{out}}{V_2}\right)\left(\frac{V_2}{V_1}\right)\left(\frac{V_1}{V_s}\right)$$

$$= G_{v3} G_{v2} G_{v1}$$

$$G_{v1} = \frac{V_1}{V_s} = \frac{Z_{in}}{Z_{in} + Z_s} = \frac{2}{2 + 0.075} = 0.9639$$

$$G_{v2} = \frac{V_2}{V_1} = \frac{V_2}{I_1}\frac{I_1}{V_1} = \frac{z_{1,21}(2||Z_{in2})}{z_{1,22} + (2||Z_{in2})}\frac{1}{Z_{in}}$$

$$= -22.4720$$

$$G_{v3} = \frac{V_{out}}{V_2} = \frac{V_{out}}{I_2}\frac{I_2}{V_2} = \frac{z_{2,21} Z_L}{z_{2,22} + Z_L}\frac{1}{Z_{in2}}$$

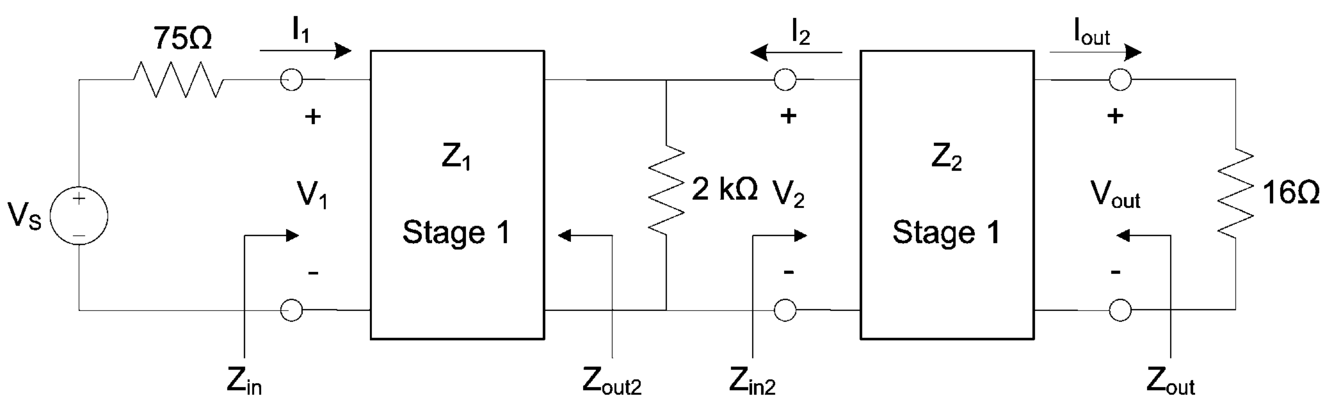
$$= 0.4532$$

$$\Rightarrow G_v = (0.9639)(-22.4720)(0.4532) = \boxed{-9.8161}$$

(c) $P_{gain} = \dfrac{v_{out}i_{out}}{v_1 i_1}$

$$\begin{aligned}P_{gain} = \frac{v_{out}i_{out}}{v_1 i_1} &= \frac{v_{out}}{v_1}\frac{i_{out}}{i_1}\\ &= \frac{v_{out}}{v_1}\frac{v_{out}/0.016}{v_1/Z_{in}}\\ &= (G_{v2}G_{v3})^2\frac{2}{0.016}\\ &= \boxed{12.965\times 10^3}\end{aligned}$$

(d) Check the impedance matching



Output Impedance

$$\begin{aligned}Z_{out2} &= z_{1,22} - \frac{z_{1,12}z_{1,21}}{z_{1,11} + Zs}\\ &= 20[\mathrm{k}\Omega]\\ Z_{out} &= z_{2,22} - \frac{z_{2,12}z_{2,21}}{z_{2,11} + (Z_{out2}\|2)}\\ &= \boxed{54.7[\Omega]}\end{aligned}$$

The load impedance is 16Ω and the output impedance is 54.7Ω. So the impedances do not match well.

MATLAB code

```
Zs=75e-3; ZL=16e-3;
Z1=[2, 0; -10^3, 20];
    z1_11=Z1(1,1);
    z1_12=Z1(1,2);
    z1_21=Z1(2,1);
    z1_22=Z1(2,2);

Z2=[62.582, 1.2075; 63.75, 1.25];
    z2_11=Z2(1,1);
    z2_12=Z2(1,2);
    z2_21=Z2(2,1);
    z2_22=Z2(2,2);

Zin2= z2_11-(z2_12*z2_21)/(z2_22+ZL)
% Zin2 =
%
%     1.7778
Zin= z1_11-(z1_12*z1_21)/(z1_22+(Zin2*2)/(Zin2+2))
% Zin =
%
%      2
Gv1=Zin/(Zin+Zs)
% Gv1 =
%
%     0.9639
Z2=(Zin2*2)/(Zin2+2);
Gv2=(z1_21*Z2)/(z1_22+Z2)/Zin
% Gv2 =
%
%   -22.4720
Gv3=(z2_21*ZL)/(z2_22+ZL)/Zin2
% Gv3 =
%
%     0.4532
Gv=Gv1*Gv2*Gv3
% Gv =
%
%    -9.8161
P=(Gv2*Gv3)^2*Zin/ZL
% P =
%
%   1.2965e+004
Zout2=z1_22-(z1_12*z1_21)/(z1_11+Zs)
% Zout2 =
%
%     20

Zout=z2_22-(z2_12*z2_21)/(z2_11+(Zout2*2)/(Zout2+2))
% Zout =
%
%     0.0547
```