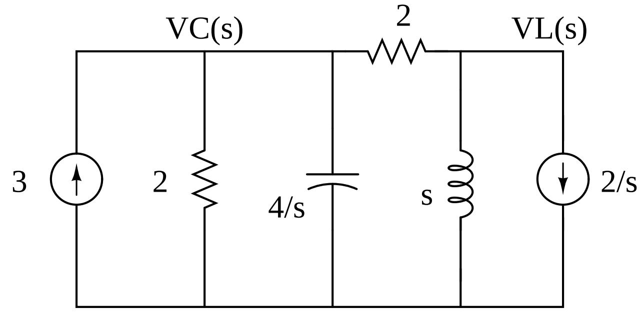
ECE 202 - Linear Circuit Analysis II

Purdue University, Spring 2009

Homework Set 2 Solutions

Solution 17



Note that the current sources $I_{in}(s)$ and $Cv_C(0-)$ have been combined into a single current source(2+1=3). From the above simplified circuit in s-domain, we can write [1],

$$\begin{aligned} \frac{V_C - V_L}{2} + \frac{(s+2)V_C}{4} &= 3 \\ \frac{V_L - V_C}{2} + \frac{2}{s} &= -\frac{V_L}{s} \end{aligned}$$

Solving using cramer's rule and **ilaplace** command in MATLAB,

$$\begin{aligned} V_C &= \frac{4(3s+4)}{s^2+4s+8} \\ \Rightarrow v_C(t) &= 12\left(\cos(2t) - \frac{1}{3}\sin(2t)\right)e^{-2t}u(t) \\ V_L &= \frac{8(s-2)}{s^2+4s+8} \\ \Rightarrow v_L(t) &= 8[\cos(2t) - 2\sin(2t)]e^{-2t}u(t) \end{aligned}$$

Solution 18

(a)

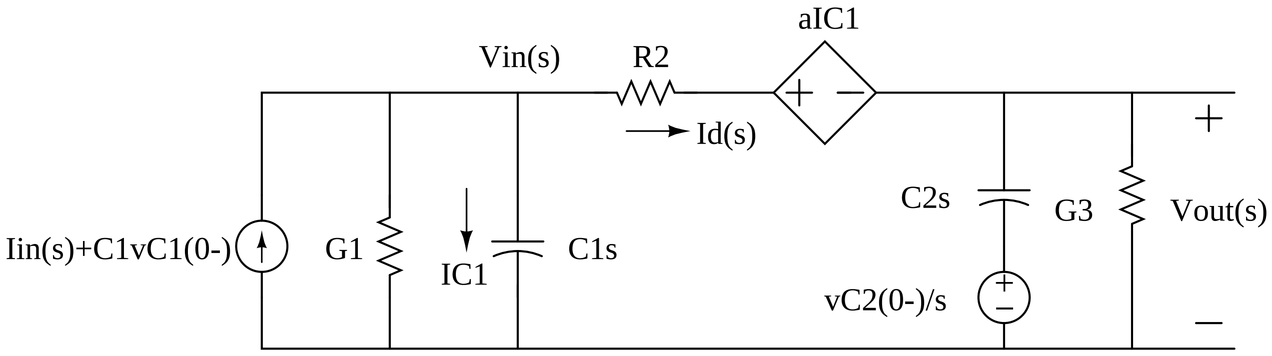
The equivalent frequency domain circuit taking into account initial conditions is drawn on the next page.

(b)

We can write the following nodal equations:-

$$V_{in}[G_1 + C_1 s] + I_d = I_{in} + C_1 v_{C1}(0-)$$

[1] from now onwards, we will not explicitly write $V_C(s)$ for a corresponding time function $v_C(t)$, upper case would mean s domain



$$\begin{aligned}
V_{in} - R_2 I_d - a I_{C1} &= V_{out} \\
V_{out} G_3 + \left[V_{out} - \frac{v_{C2}(0-)}{s} \right] C_2 s &= I_d \\
I_{C1} &= V_{in} C_1 s - C_1 v_{C1}(0-)
\end{aligned}$$

Rearranging and simplifying, we get the following matrix equation,

$$\begin{pmatrix} G_1 + C_1 s & 0 & 1 \\ 1 - a C_1 s & -1 & -R_2 \\ 0 & G_3 + C_2 s & -1 \end{pmatrix} \begin{pmatrix} V_{in}(s) \\ V_{out}(s) \\ I_d(s) \end{pmatrix} = \begin{pmatrix} I_{in}(s) + C_1 v_{C1}(0-) \\ -a C_1 v_{C1}(0-) \\ C_2 v_{C2}(0-) \end{pmatrix}$$

(c)

The transfer function is computed using MATLAB as follows:-

```
>> syms M MM Iin Vout s G1 C1 C2 G3 G2 R2 a dt;
>> M=[C1*s+G1 0 1; 1-a*C1*s -1 -R2; 0 G3+C2*s -1];
>> MM=[C1*s+G1 Iin 1; 1-a*C1*s 0 -R2; 0 0 -1];
>> dt=det(M)

dt =

G1 + G3 + C1*s + C2*s + G1*G3*R2 + C1*C2*R2*s^2 - C1*C2*a*s^2 + C2*G1*R2*s
+ C1*G3*R2*s - C1*G3*a*s

>> collect(dt)

ans =

(C1*C2*R2 - C1*C2*a)*s^2 + (C1 + C2 + C2*G1*R2 + C1*G3*R2 - C1*G3*a)*s
+ G1 + G3 + G1*G3*R2

>> Vout=det(MM)/det(M)

Vout =

(Iin - C1*Iin*a*s)/(G1 + G3 + C1*s + C2*s + G1*G3*R2 + C1*C2*R2*s^2
- C1*C2*a*s^2 + C2*G1*R2*s + C1*G3*R2*s - C1*G3*a*s)

% Numerical computation of transfer function:-

>> syms Iin s;
>> C1=1; C2=2; G1=1; R2=1; G3=2; a=0.5;
>> M=[C1*s+G1 0 1; 1-a*C1*s -1 -R2; 0 G3+C2*s -1]
```

```
M =

[   s + 1,       0,  1]
[ 1 - s/2,      -1, -1]
[       0, 2*s + 2, -1]

>> dt=det(M)

dt =

s^2 + 6*s + 5

>> factor(dt)

ans =

(s + 5)*(s + 1)

>> MM=[C1*s+G1 Iin 1; 1-a*C1*s 0 -R2; 0 0 -1]

MM =

[   s + 1, Iin,  1]
[ 1 - s/2,   0, -1]
[       0,   0, -1]

>> syms Vout;
>> Vout=det(MM)/det(M)

Vout =

(Iin - (Iin*s)/2)/(s^2 + 6*s + 5)
```

Hence the transfer function is given by:-

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} \quad = \quad \frac{1 - 0.5s}{s^2 + 6s + 5}$$

(d)

The impulse response is calculated as follows:-

```
>> syms s t H h;
>> H=-(1/2*s-1)/(s^2+6*s+5);
>> h=ilaplace(H)

h =

3/(8*exp(t)) - 7/(8*exp(5*t))
```

(e)

The step response is computed as follows:-

```
>> syms s t Vin vout H;
>> H=(1-0.5*s)/(s^2+6*s+5);
>> Vin=1/s;
>> vout=ilaplace(Vin*H)
```

```
vout =

7/(40*exp(5*t)) - 3/(8*exp(t)) + 1/5

>>
```

(f)

Here $I_{in} = 0$, $v_{C1}(0-) = 16$ and $v_{C2}(0-) = 0$
The following MATLAB code computes the response:-

```
>> syms s;
>> C1=1; C2=2; G1=1; R2=1; G3=2; a=0.5; vC10=16;
>> M=[C1*s+G1 0 1; 1-a*C1*s -1 -R2; 0 G3+C2*s -1]

M =

[   s + 1,       0,  1]
[ 1 - s/2,      -1, -1]
[       0, 2*s + 2, -1]

>> MM=[C1*s+G1 C1*vC10 1; 1-a*C1*s -a*C1*vC10 -R2; 0 0 -1]

MM =

[   s + 1, 16,  1]
[ 1 - s/2, -8, -1]
[       0,  0, -1]

>> syms Vout vout;
Vout=det(MM)/det(M)

Vout =

24/(s^2 + 6*s + 5)

>> vout=ilaplace(Vout)

vout =

6/exp(t) - 6/exp(5*t)
```

(g)

Here $I_{in} = 0$, $v_{C1}(0-) = 0$ and $v_{C2}(0-) = -8$
The following MATLAB code computes the response:-

```
>> C1=1; C2=2; G1=1; R2=1; G3=2; a=0.5; vC20=-8;
>> MM=[C1*s+G1 0 1; 1-a*C1*s 0 -R2; 0 C2*vC20 -1]

MM =

[   s + 1,   0,  1]
[ 1 - s/2,   0, -1]
[       0, -16, -1]

>> Vout=det(MM)/det(M)
```

```
Vout =

-(8*s + 32)/(s^2 + 6*s + 5)

>> vout=ilaplace(Vout)

vout =

- 6/exp(t) - 2/exp(5*t)
```

(h)

Complete response=sum of the responses (e), (f) and (g)

$$v_{complete}(t) = \left[\frac{-233}{40}e^{-5t} - \frac{3}{8}e^{-t} + \frac{1}{5}\right]u(t)$$

Solution 19

The following loop equations can be written in s-domain,

$$\begin{aligned} V_{in} - 2(I_1 - I_3) - \frac{4}{s}(I_1 - I_2) &= 0 \\ -(I_2 - I_1)\frac{4}{s} - 2(I_2 - I_3) - 2I_2 &= 0 \\ -sI_3 - 2(I_3 - I_2) - 2(I_3 - I_1) &= 0 \end{aligned}$$

Simplifying and expressing them in matrix form,

$$\begin{pmatrix} 2+4/s & -4/s & -2 \\ 4/s & -(4+4/s) & 2 \\ 2 & 2 & -(4+s) \end{pmatrix} \begin{pmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{pmatrix} = \begin{pmatrix} V_{in}(s) \\ 0 \\ 0 \end{pmatrix}$$

This system can be solved using MATLAB by the following commands,

```
» syms s I1 I2 I3 Vin;
» A=[(2+4/s) -4/s -2; 4/s -(4+4/s) 2; 2 2 -(4+s)];
» b=[Vin; 0; 0];
» x=[I1; I2; I3];
» x=(inv(A))*b
```

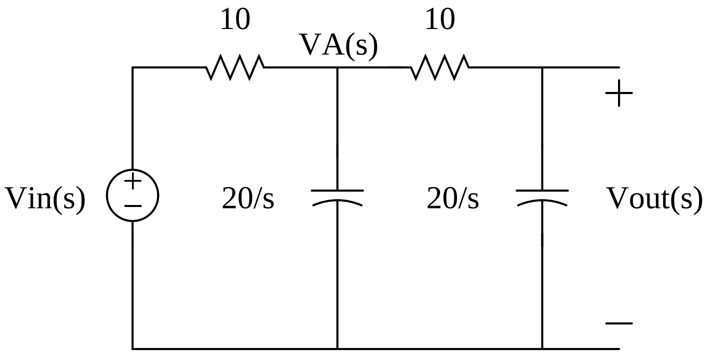
We get the following,

$$\begin{aligned} I_1(s) &= \frac{V_{in}(s)}{2} \\ &= 5\left(\frac{1}{s} - \frac{1}{s+2}\right) \\ &= \frac{10}{s(s+2)} \\ I_2(s) = I_3(s) &= \frac{V_{in}(s)}{s+2} \\ &= \frac{20}{s(s+2)^2} \\ \Rightarrow V_C(s) &= [I_1(s) - I_2(s)](4/s) \end{aligned}$$

$$
\begin{aligned}
&= \frac{40}{s(s+2)^2} \\
\Rightarrow v_C(t) &= [10 - 20te^{-2t} - 10e^{-2t}]u(t)
\end{aligned}
$$

Solution 20

The circuit for computing $v_{out}(t)$ for $0 \le t < 2$ is shown below:-
Writing node equations at nodes V_A and V_{out} and solving using **ilaplace** command in MATLAB,



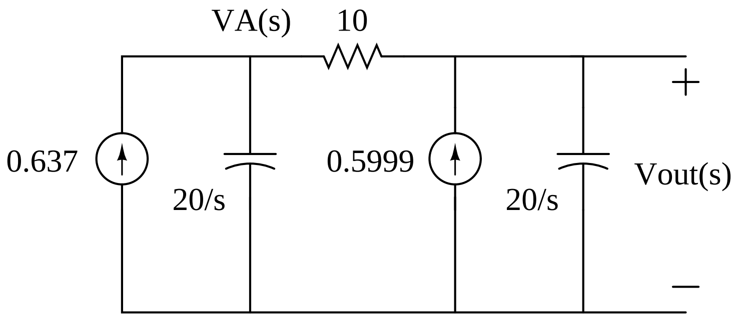
$$
\begin{aligned}
\frac{V_{in} - V_A}{10} + \frac{V_{out} - V_A}{10} &= \frac{sV_A}{20} \\
\frac{V_A - V_{out}}{10} &= \frac{sV_{out}}{20} \\
\Rightarrow V_A &= V_{out}\left(1 + \frac{s}{2}\right) \\
V_{in} &= \frac{20}{s+0.2} \\
V_{out} &= \frac{80}{(s+0.2)(s^2+6s+4)} \\
V_A &= \frac{40(s+2)}{(s+0.2)(s^2+6s+4)} \\
\Rightarrow v_{out}(t) &= \frac{2000}{71}\left[e^{-0.2t} - e^{-3t}\left\{\cosh(\sqrt{5}t) + \frac{14\sqrt{5}}{25}\sinh(\sqrt{5}t)\right\}\right]u(t),\ 0 \le t < 2 \\
\Rightarrow v_A(t) &= \frac{1800}{71}\left[e^{-0.2t} - e^{-3t}\left\{\cosh(\sqrt{5}t) + \frac{11\sqrt{5}}{45}\sinh(\sqrt{5}t)\right\}\right]u(t),\ 0 \le t < 2
\end{aligned}
$$

Now to find initial conditions on the capacitors when the switch is thrown to position B, we have,

$$
\begin{aligned}
v_{out}(2-) &= 11.9989\ V \\
v_A(2-) &= 12.7397\ V
\end{aligned}
$$

Redefining the time frame as t=0 when switch is thrown to position B, the equivalent circuit in s-domain is drawn on the next page.
Again, writing node equations and solving using MATLAB,

$$
\begin{aligned}
\frac{V_A - V_{out}}{10} + \frac{sV_A}{20} &= 0.637 \\
\frac{sV_{out}}{20} + \frac{V_{out} - V_A}{10} &= 0.5999
\end{aligned}
$$



$$\Rightarrow V_{out} = \frac{5.999s + 24.738}{0.5s^2 + 2s}$$
$$\Rightarrow v_{out}(t) = [12.369 - 0.371e^{-4t}]u(t)$$

Combining the two solutions in the original time frame we have,

$$v_{out}(t) = \frac{2000}{71}\left[e^{-0.2t} - e^{-3t}\left\{\cosh(\sqrt{5}t) + \frac{14\sqrt{5}}{25}\sinh(\sqrt{5}t)\right\}\right]u(t),\ 0 \le t < 2$$
$$= [12.369 - 0.371e^{-4(t-2)}]u(t-2),\ t \ge 2$$