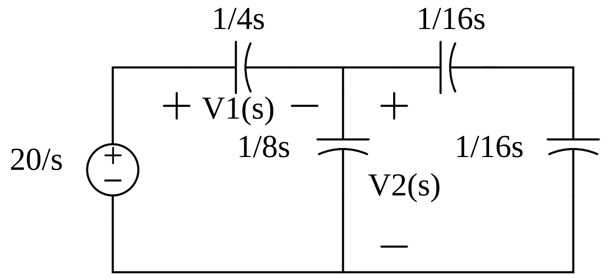
ECE 202 - Linear Circuit Analysis II

Purdue University, Spring 2009

Homework Set 3 Solutions

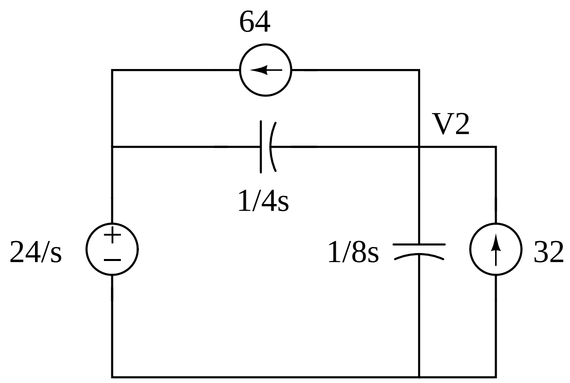
Solution 21

When the switches are moved to position A, the circuit looks like the following:-



$$
\begin{aligned}
V_1 &= \left[\frac{20}{s}\left(\frac{1}{4s}+\frac{1}{16s}\right)^{-1}\right]\frac{1}{4s} \\
&= \frac{16}{s} \\
\Rightarrow v_1(t) &= 16u(t) \\
V_2 &= \left[\frac{20}{s}\left(\frac{1}{4s}+\frac{1}{16s}\right)^{-1}\right]\frac{1}{16s} \\
&= \frac{4}{s} \\
\Rightarrow v_2(t) &= 4u(t) \\
\Rightarrow v_1(0+) = v_1(2-) &= 16\ V \\
v_2(0+) = v_2(2-) &= 4\ V
\end{aligned}
$$

At t=2, when the switch is moved to position B, we get the following circuit,
Thus is the new time frame we have,



$$
\left(\frac{24}{s}-V_2\right)4s+32 = V_2(8s)+64
$$

$$\Rightarrow v_2 = \frac{16}{3}u(t)$$
$$\Rightarrow v_2(2+) = \frac{16}{3}\ V$$
$$v_1(2+) = 24 - \frac{16}{3} = \frac{56}{3}\ V$$

Solution 22

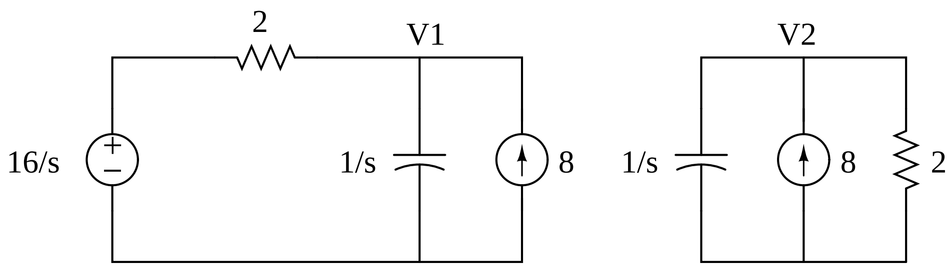
(a)

Before t=0, the circuit reaches steady state so that capacitors can be assumed to be in open circuit. Thus we get,

$$v_1(0-) = v_2(0-) = \frac{16}{2R} \times R = 8\ V$$

(b)

After S_2 is opened the circuit looks like the following,



$$\frac{1}{2}\left(\frac{16}{s} - V_1\right) + 8 = sV_1$$
$$V_1 = \frac{8(s+1)}{s(s+0.5)}$$
$$\Rightarrow v_1(t) = [16 - 8e^{-0.5t}]u(t),\ 0 \le t < 1$$
$$sV_2 + 0.5V_2 = 8$$
$$V_2 = = \frac{8}{s+0.5}$$
$$\Rightarrow v_2(t) = 8e^{-0.5t}u(t),\ 0 \le t < 1$$
$$\Rightarrow v_1(0+) = v_2(0+) = 8\ V$$

Check: The voltage across a capacitor cannot change instantaneously once it is in steady state, so our solution is correct.

(c)

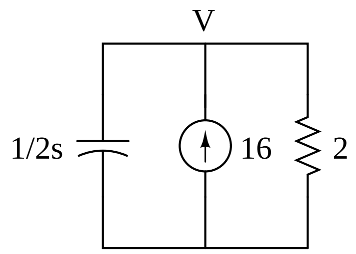
See part (b)

(d)

$$
\begin{aligned}
v_1(1-) = 16 - 8e^{-0.5} &= 11.1478\ V \\
v_1(1-) = 8e^{-0.5} &= 4.8522\ V
\end{aligned}
$$

(e)

Redefining the time frame when switch S_2 is closed and S_1 is opened, we get the following circuit. Parallel capacitances are clubbed together and also the current sources due to initial conditions.

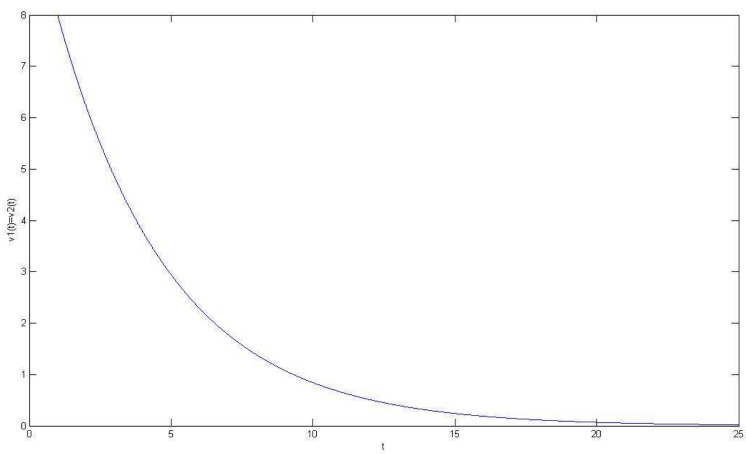


$$
\begin{aligned}
V(2s+0.5) &= 16 \\
\Rightarrow V &= \frac{8}{s+0.25} \\
\Rightarrow v(t) &= 8e^{-0.25t}u(t) \\
\Rightarrow v_1(1+) = v_2(1+) &= 8\ V
\end{aligned}
$$

Thus in the original time frame,

$$
v_1(t) = v_2(t) = 8e^{-0.25(t-1)}u(t-1),\ t > 1
$$

(f)



Solution 23

(a)

From the pole-zero diagram,

$$\begin{aligned} H(s) &= \frac{K(s^2-16)}{(s+2)(s^2+16)} \\ H(0) &= -10 \\ \Rightarrow K &= 20 \end{aligned}$$

(b)

Impulse response is found using [1] **ilaplace** command in MATLAB,

$$h(t) = [32\cos(4t) - 12e^{-2t} - 16\sin(4t)]u(t)$$

(c)

$$\begin{aligned} Step\ response &= L^{-1}[H(s)/s] \\ &= [4\cos(4t) + 6e^{-2t} + 8\sin(4t) - 10]u(t) \end{aligned}$$

(d)

$$V_{zs} = \frac{200(s^2-16)}{(s+a)(s+2)(s^2+16)}$$

To get rid of the term of the form $Ke^{-at}u(t)$ in $v_{zs}(t)$, we should get rid of the term $K/(s+a)$ in the partial fraction expansion of V_{zs}. Also, $a > 0$, so by seeing the form of V_{zs}, this is possible if a=4. Thus V_{zs} becomes,

$$\begin{aligned} V_{zs} &= \frac{200(s-4)}{(s+2)(s^2+16)} \\ \Rightarrow v_{zs}(t) &= [60\cos(4t) - 60e^{-2t} + 20\sin(4t)]u(t) \end{aligned}$$

Solution 24

(a)

The following s-domain equations can be written,

$$\begin{aligned} \frac{V_{in} - V_{out}}{R} + g_m V_{out} &= V_{out}(Cs) \\ \Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} &= \frac{1}{1 + R(Cs - g_m)} \\ &= \frac{1}{s - (4g_m - 1)} \end{aligned}$$

[1] Note that we will be using ‘ilaplace’ frequently whenever calculating inverse laplace transform

For BIBO stability, poles should be in the left half plane, which is possible if $g_m < 0.25$.

(b)

The following s-domain equations can be written,

$$\begin{aligned}
\frac{V_{in} - V_{out}}{4} &= (V_{out} - r_m V_{out})\frac{s}{4} \\
\Rightarrow H(s) = \frac{V_{out}(s)}{V_{in}(s)} &= \frac{1}{1 + s(1 - r_m)} \\
&= \frac{1/(1 - r_m)}{s + 1/(1 - r_m)}
\end{aligned}$$

For BIBO stability, poles should be in the left half plane, which is possible if $r_m < 1$.