ECE 202 - Linear Circuit Analysis II

Purdue University, Spring 2009

Homework Set 3 Solutions

Solution 25 & 26

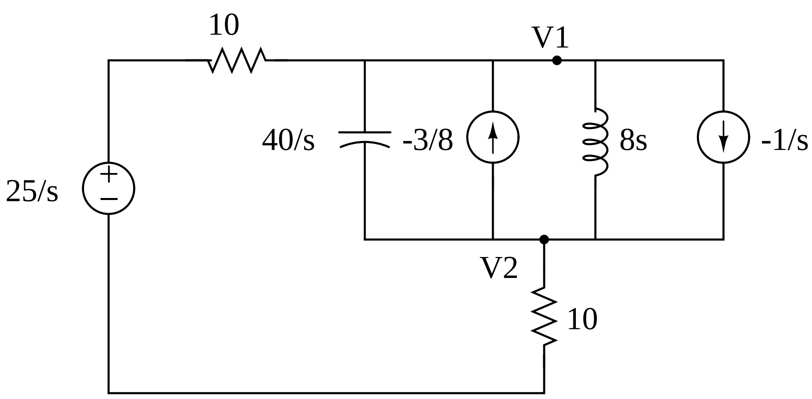
(a)

In steady state the capacitor is open circuit and the inductor is short circuit. Thus,

$$\begin{aligned} i_L(0-) = \frac{-25}{10+15} &= -1\ A \\ v_C(0-) = -25+10 &= -15\ V \end{aligned}$$

(b)

For t>0, the circuit looks like,



$$\begin{aligned} \frac{1}{10}\left(\frac{25}{s}-V_1\right)-\frac{3}{8} &= (V_1-V_2)\left(\frac{s}{40}+\frac{1}{8s}\right)-\frac{1}{s} \\ \frac{1}{10}\left(\frac{25}{s}-V_1\right) &= \frac{V_2}{10} \end{aligned}$$

Rearranging and simplifying, we get the following matrix equation,

$$\begin{pmatrix} \frac{s}{40}+\frac{1}{8s}+\frac{1}{10} & -\left(\frac{s}{40}+\frac{1}{8s}\right) \\ 1 & 1 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} \frac{3.5}{s}-\frac{3}{8} \\ \frac{25}{s} \end{pmatrix}$$

Solving using cramer's rule,

$$\begin{aligned} V_1 &= \frac{5(2s^2+28s+25}{2s(s^2+2s+5)} \\ V_2 &= \frac{5(8s^2-8s+25}{2s(s^2+2s+5)} \\ \Rightarrow V_C &= V_1-V_2 \\ &= \frac{15(6-s)}{s^2+2s+5} \end{aligned}$$

$$\Rightarrow v_C(t) = -15e^{-t}[\cos(2t)+3.5\sin(2t)]u(t),\ t>0$$

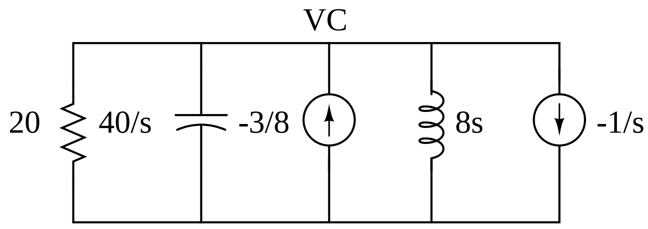
$$I_L = \frac{V_1-V_2}{8s}-\frac{1}{s}$$

$$= \frac{-(8s^2+31s-50)}{8s^3+16s^2+40s}$$

$$\Rightarrow i_L(t) = \frac{5}{4}-\frac{9}{4}e^{-t}\left[\cos(2t)+\frac{11}{12}\sin(2t)\right]u(t),\ t>0$$

(c)

The circuit only due to initial conditions is drawn below,



$$V_{C_{zi}} = \left(\frac{1}{s}-\frac{3}{8}\right)\left(20||8s||\frac{40}{s}\right)$$

$$= \frac{40-15s}{s^2+2s+5}$$

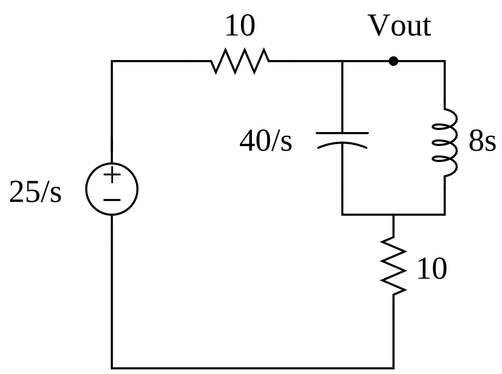
$$\Rightarrow v_{C_{zi}}(t) = -15e^{-t}\left[\cos(2t)-\frac{11}{6}\sin(2t)\right]u(t)$$

$$\Rightarrow v_{out_{zi}}(t) = \frac{v_{C_{zi}}(t)}{2}$$

$$= \frac{-15}{2}e^{-t}\left[\cos(2t)-\frac{11}{6}\sin(2t)\right]u(t)$$

(d)

The circuit with zero initial conditions is drawn below,



$$I = \frac{25}{s}\left(\frac{40s}{s^2+5}+20\right)^{-1}$$

$$V_{out_{zs}} = \frac{25}{s}-10I$$

$$= \frac{25s^2+100s+125}{2s^3+4s^2+10s}$$

$$\Rightarrow v_{out_{zs}}(t) = 12.5[1+e^{-t}\sin(2t)]u(t)$$

(e)

$$\begin{aligned}
v_{out_{complete}}(t) &= v_{out_{zs}}(t)+v_{out_{zi}}(t) \\
&= 12.5+e^{-t}[26.25\sin(2t)-7.5\cos(2t)]u(t) \\
\Rightarrow v_{out_{ss}}(t) &= 12.5 \\
v_{out_{tr}}(t) &= e^{-t}[26.25\sin(2t)-7.5\cos(2t)]u(t)
\end{aligned}$$

Solution 27

(a)

$$\begin{aligned}
H(j4) &= \frac{64j+96}{32j+48} \\
&= 2\angle 0^\circ \\
\Rightarrow v_{out}(t) &= 8\cos(4t+45^\circ)
\end{aligned}$$

(b)

Clearly, $\omega = 3$.

$$\begin{aligned}
H(j3) &= \frac{4\sqrt{5}(-9-4)}{11+12j} \\
&= 7.1427\angle 132.5103^\circ \\
\Rightarrow K = \frac{25}{7.1427} &= 3.5 \\
\theta = 100-132.5103 &= -32.5103^\circ \\
\Rightarrow v_{in}(t) &= 3.5\cos(3t-32.5103^\circ)
\end{aligned}$$

(c)

$$\begin{aligned}
H(s) = Z_{in}(s) &= (R_1+Ls)||\left(R_2+\frac{1}{Cs}\right) \\
&= (15+s)||\left(5+\frac{100}{s}\right) \\
&= \frac{5(s^2+35s+300)}{s^2+20s+100} \\
\Rightarrow H(j10) &= 10.0778\angle -29.7422^\circ \\
\Rightarrow v_{out}(t) &= 1007.78\cos(10t-29.7422^\circ)
\end{aligned}$$

Solution 28

$$Z_1 = R_1||\frac{1}{C_1 s} = \frac{R_1}{R_1C_1s+1}$$

$$
\begin{aligned}
Z_2 = R_2 || \frac{1}{C_2 s} &= \frac{R_2}{R_2 C_2 s + 1} \\
H(s) = -\frac{Z_2}{Z_1} &= -\frac{R_2}{R_1}\left(\frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}\right) \\
&= -\frac{C_1}{C_2}\left(\frac{s + 1/R_1 C_1}{s + 1/R_2 C_2}\right) \\
&= \frac{-5(s+0.2)}{s+4} \\
C_2 &= 1\,F \\
\Rightarrow C_1 &= 5\,F \\
R_1 &= 1\,\Omega \\
R_2 &= 0.25\,\Omega
\end{aligned}
$$

(a)

$$
\begin{aligned}
H(j\omega) &= \frac{-5(j\omega + 0.2)}{j\omega + 4} \\
&= \frac{-5(0.2 + j\omega)(4 - j\omega)}{16 + \omega^2} \\
&= \frac{-5(\omega^2 + 0.8 + 3.8 j\omega)}{16 + \omega^2} \\
&= \rho e^{\phi + \pi} \\
\rho &= \frac{5\sqrt{\omega^4 + 16.04\omega^2 + 0.64}}{\omega^2 + 16} \\
\phi &= \tan^{-1}\left(\frac{3.8\omega}{\omega^2 + 0.8}\right) \\
v_{in}(t) &= 2\cos(\omega t + \frac{\pi}{2})u(t) \\
\Rightarrow v_{out}(t) &= 2\rho\cos(\omega t + \frac{3\pi}{2} + \phi) = K\cos(\omega t + \theta) \\
\Rightarrow K &= 2\rho \\
\theta &= \frac{3\pi}{2} + \phi
\end{aligned}
$$

(b)

$$
\begin{aligned}
V_{in} &= \frac{2\omega}{\omega^2 + s^2} \\
V_{out} = V_{in} H(s) &= \frac{-10\omega(s + 0.2)}{(s^2 + \omega^2)(s + 4)} \\
&= \frac{A}{s+4} + \frac{Bs + C}{s^2 + \omega^2}
\end{aligned}
$$

If we solve the above partial fraction expansion,

$$
\begin{aligned}
A &= \frac{38\omega}{\omega^2 + 16} \\
B &= \frac{-38\omega}{\omega^2 + 16} \\
C &= \frac{-10\omega^3 - 8\omega}{\omega^2 + 16}
\end{aligned}
$$

Employing the well known expression,

$$
\begin{aligned}
-a\cos(\omega t) - b\sin(\omega t) &= \sqrt{a^2+b^2}\sin(\omega t + \pi + \Theta),\ a,b>0 \\
&= \sqrt{a^2+b^2}\cos(\omega t + \frac{3\pi}{2} + \Theta) \\
\Theta &= \tan^{-1}\left(\frac{a}{b}\right)
\end{aligned}
$$

We can get after taking inverse laplace transform,

$$
v_{out}(t) = Ae^{-4t} + B\cos(\omega t) + \frac{C}{\omega}\sin(\omega t)
$$

Clearly,

$$
\begin{aligned}
v_{out_{tr}}(t) &= Ae^{-4t} \\
&= \frac{38\omega}{\omega^2+16}e^{-4t} \\
v_{out_{ss}}(t) &= K\cos(\omega t + \theta) \\
K &= \sqrt{B^2 + (C/\omega)^2} \\
\theta &= \frac{3\pi}{2} + \tan^{-1}(B\omega/C)
\end{aligned}
$$

(c)

Clearly, steady state value part (a) and steady state part of (b) coincide.