

# Laplace Transform Properties

Property	$f(t)$	$\mathcal{L}\{f(t)\}=F(s)$
Linearity	$a_1f_1(t) + a_2f_2(t)$	$a_1F(s) + a_2F_2(s)$
Time Shift	$f(t - T)u(t - T)$	$e^{-sT}F(s)$
Multiplication by t	$tf(t)u(t)$	$-\frac{d}{ds}F(s)$
Multiplication by $t^n$	$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$
Frequency Shift	$e^{-at}f(t)$	$F(s + a)$
Time Differentiation	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
Second-order Differentiation	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - f^{(1)}(0^-)$
$n$ th-order Differentiation	$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1}f(0^-) - s^{n-2}f^{(1)}(0^-) - \dots - f^{(n-1)}(0^-)$
Time Integration	(a) $\int_{-\infty}^t f(q)dq$ (b) $\int_{0^-}^t f(q)dq$	$\frac{F(s)}{s} + \frac{\int_{-\infty}^{0^-} f(q)dq}{s}$ $\frac{F(s)}{s}$
Time or Frequency Scaling	$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$