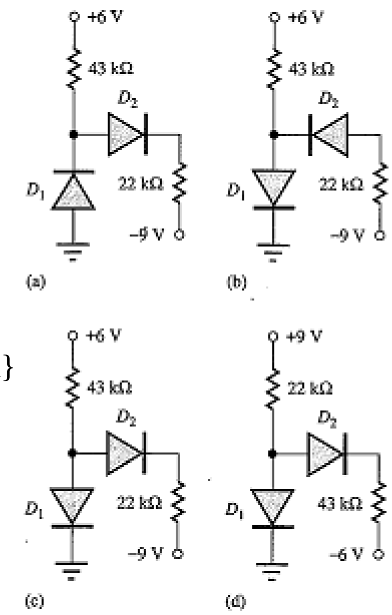
# ECE 255 ELECTRONIC ANALYSIS AND DESIGN Fall 2009

Homework 2 Solutions

**Problem 1 (Text 3.71)**

Find the *Q*-points for the diodes in the four circuits shown using:




(a) The ideal diode model

(a) Both diodes are forward biased

$$I_{D2} = \frac{[0-(-9)]\,V}{22\,k\Omega} = 409\,\mu A$$

$$I_{D1} = 409\,\mu A - \frac{(6-0)\,V}{43\,k\Omega} = 270\,\mu A$$

$\Rightarrow$ $Q_{D1}$: {$V_{D1} = 0$, $I_{D1} = 270\ \mu A$} and $Q_{D2}$: {$V_{D2} = 0$, $I_{D2} = 409\ \mu A$}

(b) Diode $D_1$ is forward biased and diode $D_2$ is reverse biased

$$I_{D1} = \frac{(6-0)\,V}{43\,k\Omega} = 140\,\mu A \quad \text{and} \quad V_{D2} = -9\ V$$

$\Rightarrow$ $Q_{D1}$: {$V_{D1} = 0$, $I_{D1} = 140\ \mu A$} and $Q_{D2}$: {$V_{D2} = -9\ V$, $I_{D2} = 0$}

(c) Diode $D_1$ is reverse biased and diode $D_2$ is forward biased

$$I_{D2} = \frac{[6-(-9)]\,V}{65\,k\Omega} = 231\,\mu A \quad \text{and} \quad V_{D1} = 6\,V - 43\,k\Omega \times I_{D2} = -3.92\,V$$

$\Rightarrow$ $Q_{D1}$: {$V_{D1} = -3.92\ V$, $I_{D1} = 0$} and $Q_{D2}$: {$V_{D2} = 0$, $I_{D2} = 231\ \mu A$}

(d) Both diodes are forward biased

$$I_{D2} = \frac{[0-(-6)]\,V}{43\,k\Omega} = 140\,\mu A$$

$$I_{D1} = \frac{(9-0)\,V}{22\,k\Omega} - 140\,\mu A = 270\,\mu A$$

$\Rightarrow$ $Q_{D1}$: {$V_{D1} = 0$, $I_{D1} = 270\ \mu A$} and $Q_{D2}$: {$V_{D2} = 0$, $I_{D2} = 140\ \mu A$}

(b) The constant voltage drop model with $V_{on} = .75$ V

(a) Both diodes are forward biased

$$I_{D2} = \frac{[-.75-.75-(-9)]\,V}{22\,k\Omega} = 341\,\mu A$$

$$I_{D1} = 341\,\mu A - \frac{6-(-.75)\,V}{43\,k\Omega} = 184\,\mu A$$

$\Rightarrow$ $Q_{D1}$: {$V_{D1} = .75\ V$, $I_{D1} = 184\ \mu A$} and $Q_{D2}$: {$V_{D2} = .75\ V$, $I_{D2} = 341\ \mu A$}

(b) Diode $D_1$ is forward biased and diode $D_2$ is reverse biased

$$I_{D1} = \frac{(6-.75)\,V}{43\,k\Omega} = 122\,\mu A \quad \text{and} \quad V_{D2} = -9 - .75 = -9.75\ V$$

$\Rightarrow$ $Q_{D1}$: {$V_{D1} = .75\ V$, $I_{D1} = 122\ \mu A$} and $Q_{D2}$: {$V_{D2} = -9.75\ V$, $I_{D2} = 0$}

(c) Diode $D_1$ is reverse biased and diode $D_2$ is forward biased

$$I_{D2} = \frac{[6-.75-(-9)]\,\text{V}}{65\,\text{k}\Omega} = 219\,\mu\text{A} \quad \text{and} \quad V_{D1} = 6\,\text{V} - 43\,\text{k}\Omega \times I_{D2} = -3.43\,\text{V}$$

⇒ $Q_{D1}$: {$V_{D1}$ = –3.43 V, $I_{D1}$ = 0} and $Q_{D2}$: {$V_{D2}$ = .75 V, $I_{D2}$ = 219 μA}

(d) Both diodes are forward biased

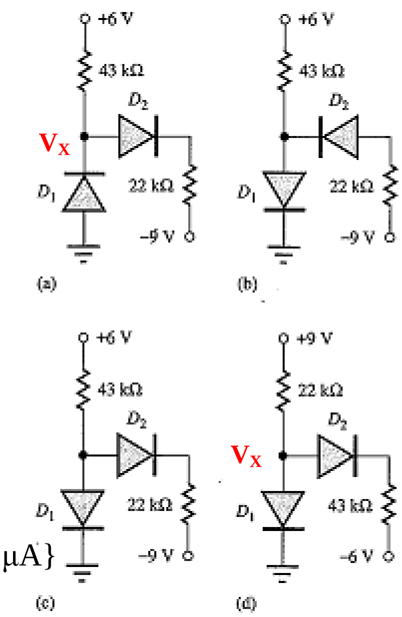
$$I_{D2} = \frac{[.75-.75-(-6)]\,\text{V}}{43\,\text{k}\Omega} = 140\,\mu\text{A}$$

$$I_{D1} = \frac{(9-.75)\,\text{V}}{22\,\text{k}\Omega} - 140\,\mu\text{A} = 235\,\mu\text{A}$$

⇒ $Q_{D1}$: {$V_{D1}$ = .75 V, $I_{D1}$ = 235 μA} and $Q_{D2}$: {$V_{D2}$ = .75 V, $I_{D2}$ = 140 μA}

**Problem 2 (Text 3.71 cont.)**

**(c)** A diode model containing an ideal diode, a voltage source, $V_0$ = .65 V, and resistor, $R_0$ = 200 Ω.




(a) Assume both diodes are forward biased
Define the voltage at the junction of the two diodes as $V_X$

$$\frac{6\,\text{V} - V_X}{43\text{k}\Omega} = \frac{V_X + .65\,\text{V}}{200\,\Omega} + \frac{V_X - .65\,\text{V} - (-9\,\text{V})}{22\,\text{k}\Omega + 200\,\Omega} \Rightarrow V_X = -688\,\text{mV}$$

$$I_{D1} = \frac{0 - .65\,\text{V} - V_X}{200\,\Omega} = 190\,\mu\text{A}$$

$$I_{D2} = \frac{V_X - .65\,\text{V} - (-9\,\text{V})}{22\,\text{k}\Omega + 200\,\Omega} = \frac{7.662\,\text{V}}{22.2\,\text{k}\Omega} = 345\,\mu\text{A}$$

$$V_{D2} = .65\,\text{V} + 200\,\Omega \times I_{D2} = 719\,\text{mV}$$

The results of the calculations are consistent with our initial assumptions regarding the regions of operation; therefore,

⇒ $Q_{D1}$: {$V_{D1}$ = .688 V, $I_{D1}$ = 190 μA} and $Q_{D2}$: {$V_{D2}$ = .719 V, $I_{D2}$ = 345 μA}

(b) Assume diode $D_1$ is forward biased and diode $D_2$ is reverse biased

$$I_{D1} = \frac{(6 - .65)\,\text{V}}{43k\Omega + 200\,\Omega} = 124\,\mu\text{A} \quad \text{and} \quad V_{D1} = .65\,\text{V} + 200\,\Omega \times I_{D1} = 675\,\text{mV}$$

⇒ $Q_{D1}$: {$V_{D1}$ = .675 V, $I_{D1}$ = 124 μA} and $Q_{D2}$: {$V_{D2}$ = –9.68 V, $I_{D2}$ = 0}

(c) Assume diode $D_1$ is reverse biased and diode $D_2$ is forward biased

$$I_{D2} = \frac{[6 - .65 - (-9)]\,\text{V}}{[43\text{k} + 200 + 22\text{k}]\,\Omega} = \frac{14.35\,\text{V}}{65.2\,\text{k}\Omega} = 220\,\mu\text{A}$$

$$V_{D2} = .65\,\text{V} + 200\,\Omega \times I_{D2} = 694\,\text{mV}$$

$$V_{D1} = 6\,\text{V} - 43\,\text{k}\Omega \times I_{D2} = -3.46\,\text{V}$$

⇒ $Q_{D1}$: {$V_{D1}$ = –3.46 V, $I_{D1}$ = 0} and $Q_{D2}$: {$V_{D2}$ = .694 V, $I_{D2}$ = 220 μA}

(d) Assume both diodes are forward biased
Define the voltage at the junction of the two diodes as $V_X$

$$\frac{9\,V - V_X}{22\,k\Omega} = \frac{V_X - .65\,V}{200\,\Omega} + \frac{V_X - .65\,V - (-6\,V)}{43\,k\Omega + 200\,\Omega} \Rightarrow V_X = 697.5\,mV$$

$$I_{D1} = \frac{V_X - .65\,V}{200\,\Omega} = 237\,\mu A$$

$$I_{D2} = \frac{V_X - .65\,V - (-6\,V)}{43\,k\Omega + 200\,\Omega} = \frac{6.0475\,V}{43.2\,k\Omega} = 140\,\mu A$$

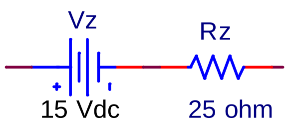
$$V_{D2} = .65\,V + 200\,\Omega \times I_{D2} = 678\,mV$$

The results of the calculations are consistent with our initial assumptions regarding the regions of operation; therefore,

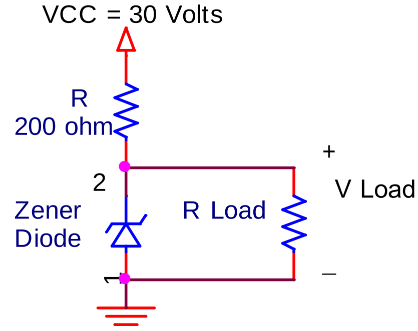
$\Rightarrow$ $Q_{D1}$: {$V_{D1}$ = .698 V, $I_{D1}$ = 237 μA} and $Q_{D2}$: {$V_{D2}$ = .678 V, $I_{D2}$ = 140 μA}

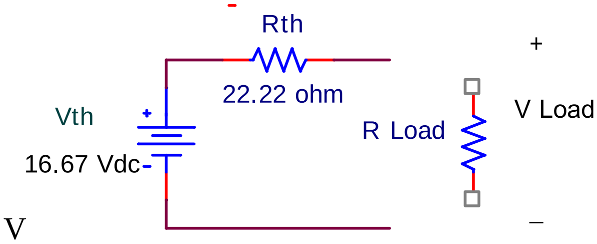
**Problem 3**

Using a Zener diode with $|V_{BR}| = V_Z = 15$ V and $R_Z = 25\ \Omega$ in the simple voltage regulator shown below, having $V_{CC} = 30$ V, calculate the load voltage, $V_{Load}$, when




Zener Diode Model




Construct the Thévenin equivalent circuit

(a) $I_{Load} = 0$ $\quad$ $V_{Load} = V_{OC} = V_{TH} = 16.67$ V

(b) $I_{Load} = 1$ mA $\quad$ $V_{Load} = 16.67 - 22.22(0.001) = 16.65$ V

(c) $I_{Load} = 10$ mA $\quad$ $V_{Load} = 16.67 - 22.22(0.010) = 16.45$ V

(d) $I_{Load} = 70$ mA $\quad$ $V_{Load} = 16.67 - 22.22(0.070) = 15.11$ V

(e) What is the maximum possible current that can be drawn by the load before the Zener diode stops providing any regulation?

When the output voltage drops to 15 Volts, no current will be flowing through the Zener diode. If more current is drawn the Zener diode will appear to be an open circuit.

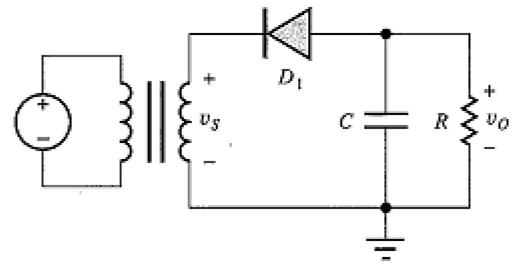
$$V_{Load} = 16.67 - 22.22(I_{max}) = 15.00\text{ V} \rightarrow \quad I_{max} = 75\text{ mA}$$

(f) If the 30 V supply, $V_{CC}$, has a ripple, $\Delta v_{CC} = 0.1\,V$, what is the ripple seen at the load when 50 mA is being drawn?

When $I_{Load}$ = 50 mA, $V_{Load}$ = 16.67 − 22.22(0.050) = 15.56 V
Therefore $R_{Load}$ = 15.56/0.05 = 311 Ω

$$\Delta v_{Load} = \frac{R_Z \| R_{Load}}{R_Z \| R_{Load} + R} \times \Delta v_{CC} = \frac{25\|311}{25\|311 + 200} \times 0.1 = \frac{23.14}{223.14} \times 0.1 = 0.104 \times 0.1 = 0.01\text{V}$$

**Problem 4 (Text 3.91)**
The half-wave rectifier shown below is operating at a frequency of 60 Hz, and the rms value of the transformer voltage is 6.3 V.




(a) What is the value of the dc output voltage, $V_O$, if the diode voltage drop is 1 V and assuming that R = ∞?

$$V_{dc} = -(V_P - V_0) = -\left(6.3\sqrt{2} - 1.0\right)\text{V} = -7.91\text{V}$$

(b) What is the minimum value of C required to ensure that the ripple voltage is less than 0.25 V if R = 50 Ω?

$$V_r \approx \frac{V_{max} \times T}{R \times C} \Rightarrow C \approx \frac{7.91}{.25 \times 50 \times 60} = 10.5\text{mF}$$

(c) What is the *minimum* PIV rating of the diode in this circuit?

$$\text{PIV} \geq 2 \times V_P - V_D = 2 \times 6.3\sqrt{2} - 1 = 16.82\text{V}$$

(d) What is the surge current when the power is first applied?

$$I_{surge} = \omega \times C \times V_P = 2\pi \times .0105 \times 6.3\sqrt{2} = 35.3\text{A}$$

(e) What is the *peak* amplitude of the repetitive current in the diode?
During the time interval ΔT the output voltage rises from ($V_{max}$ – $V_r$) to ($V_{max}$)

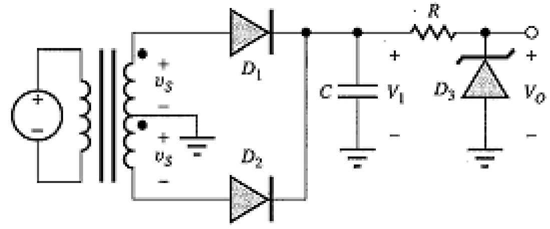
$$V_P \cos(-\omega \Delta T) = V_P - V_r \quad \text{expanding} \quad \cos(x) = 1 - \frac{x^2}{2!} + \cdots \quad \text{yields} \quad \frac{(\omega \Delta T)^2}{2} = \frac{V_r}{V_P}$$

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{2\pi 60}\sqrt{\frac{2 \times 0.25}{6.3\sqrt{2}}} = 0.628\text{ms}$$

$$I_P \approx I_{dc}\frac{2T}{\Delta T} \quad \text{where} \quad I_{dc} = \frac{V_P - V_{on} - 0.5V_r}{R} = \frac{7.91 - .125}{50} = 158\text{mA}$$

$$\text{Therefore} \quad I_P = .158 \times \frac{2\text{k}}{60 \times .628} = 8.28\text{A}$$

**Problem 5 (Text 3.95)**
For the Zener regulated power supply shown below the rms value of $v_s$ is 15 V, the operating frequency is 60 Hz, R = 100 Ω, C = 1000 μF, the on-voltage of diodes $D_1$ and $D_2$ is .75 V, and the Zener voltage of diode $D_3$ is 15 V.



(a) What is the type of rectifier used in this power supply circuit?
Full-wave rectifier.

(b) What is the dc voltage $V_1$?

$$V_{1(max)} = 15\sqrt{2} - .75 = 20.463\,V$$

$$V_{1(min)} = \left(V_{1(max)} - 15\right)e^{\frac{-T}{RC}} + 15 \approx \left(V_{1(max)} - 15\,V\right)\left(1 - \frac{T}{RC}\right) + 15\,V$$

$$V_{1(min)} = 5.463\left(1 - \frac{1}{120\times100\times.001}\right) + 15 = 5.463(1 - .0833) + 15 = 20.008\,V$$

$$V_{dc} = \frac{V_{1(max)} + V_{1(min)}}{2} = 20.24\,V$$

(c) What is the dc output voltage $V_O$? 15 V

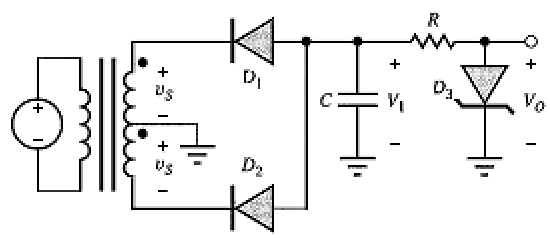
(d) What is the magnitude of the ripple voltage at $V_1$?

$$V_r = V_{1(max)} - V_{1(min)} = 0.455\,V$$

(e) What is the minimum PIV rating for the rectifier diodes?

$$PIV \geq 2\times V_P - V_D = 2\times15\sqrt{2} - .75 = 41.7\,V$$

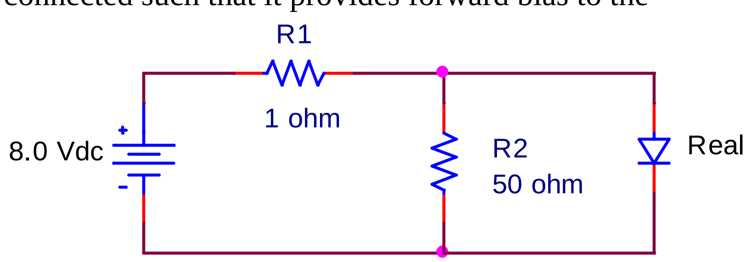
(f) Draw a new version of the circuit that will produce an output voltage of –15 V.



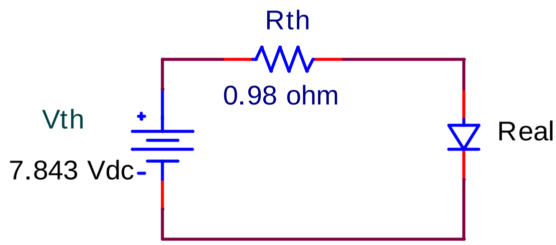

**Problem 6**

A diode is represented by a dc model with $V_0 = 0.60\,V$ and $R_0 = 10\,\Omega$, and an ac model having $r = 2\,\Omega$. If the diode is in parallel with a 50 Ω resistor and this combination is in series with a 1 Ω resistor and a voltage source, $v_S = 8 + 0.1\cos(1000t)$ volts, find the total voltage across the diode as a function of time. Assume that the source is connected such that it provides forward bias to the diode.

Consider the D.C. bias first



Thévenin equivalent: $R_{th} = 50\|1 = \frac{50}{51} = .98\ \Omega$ and $V_{th} = \frac{50}{51}(8) = \frac{400}{51} = 7.843\,V$



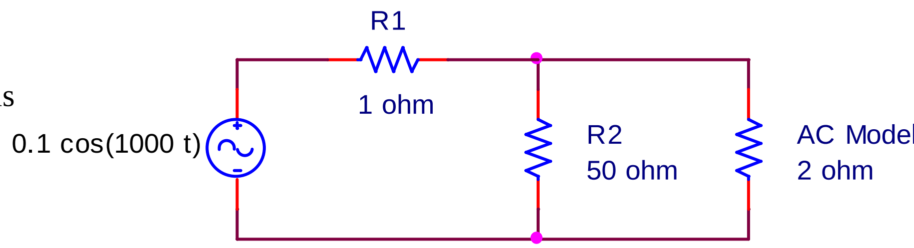

D.C. Analysis of the operating point using the given model

$$I_D = \frac{V_{th} - 0.6\,V}{R_{th} + 10\,\Omega} = \frac{7.843 - 0.6}{0.98 + 10} = \frac{7.243}{10.98} = 650\,mA$$

$$V_D = (10\,\Omega)\,I_D + 0.6V = 10 \times 1.211 + 0.6 = 7.20\,V$$

Next consider the A.C. Analysis

A.C. Equivalent circuit




$$50\,\Omega \,\|\, 2\,\Omega = \frac{100}{52} = 1.923\,\Omega$$

Therefore, $v_d = \frac{1.923}{1+1.923}\left[0.1\cos(1000t)\right] = 0.0658\cos(1000t)$

Combining the D.C. solution (Operating Point, Q) and the A.C. solution yields the total solution

$$v_D = V_D + v_d = 7.20 + 0.0658\cos(1000t)$$

**Problem 7**

For each of the following statements, determine whether it applies to (normal) active-region operation, of an *npn* transistor, a *pnp* transistor, both, or neither.

(a) $V_{EB} = 0.6\,V$ PNP

(b) $I_C > I_E$ NPN (Since $I_C < 0$ for PNP)

(c) $|I_C| > |I_E|$ Neither (Since $|I_E| = |I_C| + |I_B|$)

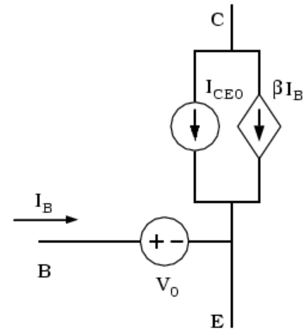
(d) $I_E > I_B$ PNP (Since $I_E < 0$ for NPN)

(e) $V_{CE} > V_{CB}$ NPN

(f) $|V_{CE}| > |V_{CB}|$ Both

**Problem 8**

The active region model for an *npn* transistor contains $V_0 = 0.55\,V$, $\beta_{dc} = 125$, and $I_{CE0} = 2\mu A$.
Find $I_C$ when




(a) $I_B = 200\mu A$

$$I_C = \beta I_B + I_{CE0} = 125(0.2) + 0.002\,mA = 25.002\,mA$$

(b) $|I_E| = 3\,mA$

$$I_B + I_C = -I_E = |I_E| = 3\,mA$$

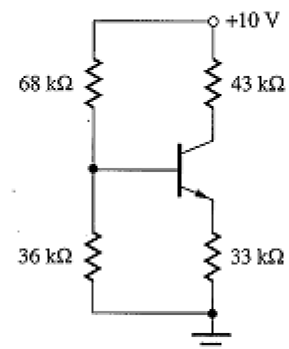
$$I_C = \beta I_B + I_{CE0} = 125(3 - I_C) + 0.002 \quad \Rightarrow \quad I_C = 2.976\,mA$$

(c) Find $V_{BE}$ when $I_C = 2\,mA$

$$V_{BE} = V_0 = 0.55\,V \quad \text{(given)}$$

**Problem 9 (Text 5.82)**

(a) Find the *Q*-point for the circuit shown below. Assume that $\beta_F = 50$ and $V_{BE} = 0.7$ V.




$$V_{TH} = \frac{36\,k\Omega}{(68\,k + 36\,k)\Omega} \times 10\,V = 3.462\,V \quad \text{and} \quad R_{TH} = 36\,k \| 68\,k\,\Omega = 23.54\,k\Omega$$

$$V_{TH} = R_{TH} \times I_B + V_{BE} + R_E \times (I_C + I_B) \quad \Rightarrow \quad I_B = \frac{V_{TH} - V_{BE}}{R_{TH} + (\beta + 1) R_E}$$

$$I_B = \frac{2.762}{23.54\,k + (51)\,33\,k} = 1.618\mu A \quad \therefore \quad I_C = \beta_F I_B = 80.9\ \mu A$$

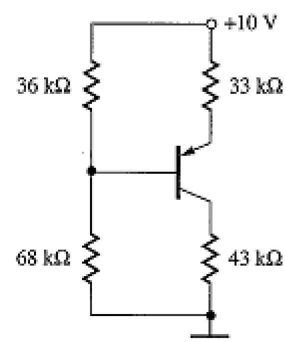
and $$V_{CE} = V_{CC} - I_C R_C - |I_E| R_E = 3.80\ V$$

(b) Repeat the calculation if all of the resistor values are decreased by a factor of 5.

$V_{TH}$ is unchanged and $R_{TH}$(new) = 0.2 x $R_{TH}$(old) = 4.708 kΩ $\Rightarrow$ $I_B$(new) = 5 x $I_B$(old) = 8.09 μA
$I_C$(new) = 5 x $I_C$(old) = 405 μA and $V_{CE}$ is unchanged, $V_{CE}$ = 3.80 V

**Problem 10 (Text 5.82)**

(c) Repeat (a) of text problem 5.82 for the second circuit shown.




$$V_{TH} = \frac{68\,k\Omega}{(68\,k + 36\,k)\Omega} \times 10\,V = 6.538\,V \quad \text{and} \quad R_{TH} = 36\,k \| 68\,k\,\Omega = 23.54\,k\Omega$$

$$V_{TH} = R_{TH} \times I_B - 0.7\,V + R_E \times (I_C + I_B) + 10\,V \quad \Rightarrow \quad -I_B = \frac{10\,V - V_{TH} - 0.7\,V}{R_{TH} + (\beta + 1) R_E}$$

$$I_B = \frac{-2.762}{23.54\,k + (51)\,33\,k} = -1.618\mu A \quad \therefore \quad I_C = -80.9\ \mu A \quad \text{and} \quad V_{CE} = -3.80\ V$$

(d) Repeat (b) of text problem 5.83 for the second circuit shown.

$V_{TH}$ is unchanged and $R_{TH}$(new) = 0.2 x $R_{TH}$(old) = 4.708 kΩ $\Rightarrow$ $I_B$(new) = 5 x $I_B$(old) = – 8.09 μA
$I_C$(new) = 5 x $I_C$(old) = – 405 μA and $V_{CE}$ is unchanged, $V_{CE}$ = –3.80 V