

## Homework 3 Solutions

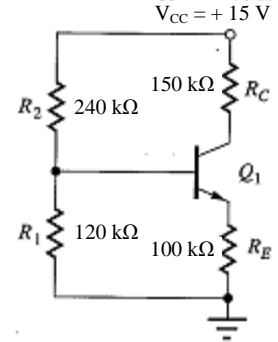
**Problem 1 (Text 5.86)**

Find the  $Q$ -point for the circuit shown below if  $R_1 = 120 \text{ k}\Omega$ ,  $R_2 = 240 \text{ k}\Omega$ ,  $R_E = 100 \text{ k}\Omega$ ,  $R_C = 150 \text{ k}\Omega$ ,  $\beta_F = 100$ ,  $V_{BE} = 0.7 \text{ V}$ , and the positive power supply voltage is  $15 \text{ V}$ .

$$V_{TH} = \frac{120 \text{ k}\Omega}{(120 \text{ k} + 240 \text{ k})\Omega} \times 15 \text{ V} = 5.0 \text{ V} \quad \text{and} \quad R_{TH} = 120 \text{ k} \parallel 240 \text{ k}\Omega = 80 \text{ k}\Omega$$

$$I_C = \frac{\beta(V_{TH} - V_{BE})}{R_{TH} + (\beta + 1)R_E} = \frac{100(5.0 - 0.7)}{80 \text{ k} + (101)100 \text{ k}} = \frac{.430}{10.18 \text{ k}} = 42.24 \mu\text{A}$$

$$V_{CE} = 15 - 42.24 \mu\text{A}(150 \text{ k}\Omega) - \frac{101}{100}(42.24 \mu\text{A})(100 \text{ k}\Omega) = 4.40 \text{ V}$$

**Problem 2 (Text 5.87)**

(a) Design a four-resistor bias network for an *npn* transistor to give  $I_C = 1 \text{ mA}$ ,  $V_{CE} = 5 \text{ V}$ , and  $V_E = 2 \text{ V}$  if  $V_{CC} = 12 \text{ V}$  and  $\beta_F = 100$ . Assume  $V_0 = 0.7 \text{ V}$ .

$$\text{Since } V_E = 2 \text{ V and } |I_E| = I_C + I_B \quad \Rightarrow \quad R_E = \frac{2 \text{ V}}{1 \text{ mA} + 0.01 \text{ mA}} = 1.98 \text{ k}\Omega$$

$$\text{Since } V_{R_C} = (12 - 5 - 2) \text{ V} = 5 \text{ V} \quad \Rightarrow \quad R_C = \frac{5 \text{ V}}{1 \text{ mA}} = 5.0 \text{ k}\Omega$$

$$\text{Set } I_{R_1} = 20 I_B = 0.2 \text{ mA, } V_B = (2.0 + 0.7) \text{ V} \quad \Rightarrow \quad R_2 = \frac{(12 - 2.7) \text{ V}}{0.2 \text{ mA}} = 46.5 \text{ k}\Omega$$

$$\text{Then } I_{R_2} = 19 I_B \text{ and } V_B = 2.7 \text{ V} \quad \Rightarrow \quad R_1 = \frac{2.7 \text{ V}}{0.19 \text{ mA}} = 14.2 \text{ k}\Omega$$

(b) Replace the exact values determined in part (a) with the nearest 5% values found in the resistor table and find the resulting  $Q$ -point.

$$R_E \rightarrow 2.0 \text{ k}\Omega, \quad R_C \rightarrow 5.1 \text{ k}\Omega, \quad R_1 \rightarrow 47 \text{ k}\Omega, \quad R_2 \rightarrow 15 \text{ k}\Omega$$

$$R_{TH} = R_1 \parallel R_2 = 47 \parallel 15 = 11.37 \text{ k}\Omega \quad \text{and} \quad V_{TH} = \frac{15}{15 + 47} \times 12 = 2.90 \text{ V}$$

$$I_C = \frac{100(2.90 - 0.7)}{11.37 + 101 \times 2.0} = 1.03 \text{ mA} \quad \text{and} \quad V_{CE} = 12 - I_C R_C - |I_E| R_E = 4.67 \text{ V}$$

### Problem 3 (Text 5.91)

Find the  $Q$ -point for the circuit shown using the graphical load line approach. Use the characteristics shown below.

Load line definition:  $V_{OC} = 10 \text{ V}$ ,

$$I_{SC} = \frac{\beta \times V_{CC}}{\beta R_C + (\beta + 1) R_E} \approx \frac{V_{CC}}{R_C + R_E} = \frac{10}{2.02} = 4.95 \text{ mA}$$

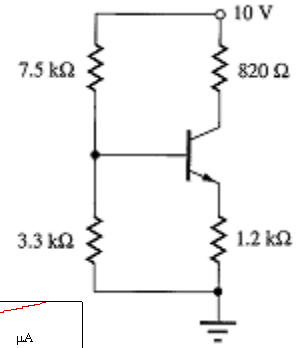
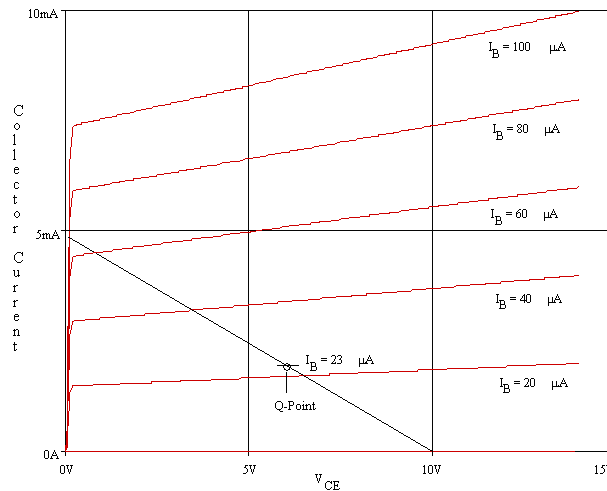
Estimate 
$$I_B \approx \frac{V_{TH} - V_0}{R_{TH} + (\beta + 1) R_E}$$

From plot, 
$$\beta \approx \frac{5 \text{ mA}}{60 \mu\text{A}} \Big|_{V_{CE} = 5 \text{ V}} \approx 83$$

$$R_{TH} = 7.5 \text{ k} \parallel 3.3 \text{ k} = 2.29 \text{ k}\Omega$$

$$V_{TH} = \frac{3.3 \text{ k}\Omega}{3.3 \text{ k}\Omega + 7.5 \text{ k}\Omega} \times 10 \text{ V} = 3.06 \text{ V}$$

$$I_B \approx \frac{(3.06 - 0.7) \text{ V}}{[2.29 \text{ k} + (84) 1.2 \text{ k}] \Omega} = 23 \mu\text{A}$$



### Problem 4

The transistor in the circuit shown below is modeled by  $V_0 = 0.7 \text{ V}$ ,  $\beta_{dc} = 100$ ,  $I_{CE0} = 5 \mu\text{A}$ , and  $R_{sat} = 60 \Omega$ . If  $V_{BB} = 4 \text{ V}$ ,  $R_B = 22 \text{ k}\Omega$ , and  $R_C = 600 \Omega$ , find  $I_C$  and  $V_{CE}$  if  $V_{CC}$  is:

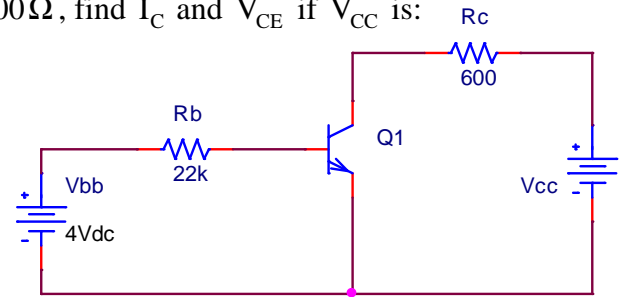
(a)  $12 \text{ V}$

Assume Active Region:

$$\text{Then } I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{(4 - 0.7) \text{ V}}{22 \text{ k}\Omega} = 0.15 \text{ mA}$$

$$\text{and } I_C = \beta_{dc} I_B + I_{CE0} = 100(0.15) + 0.001 = 15.001 \text{ mA}$$

$$V_{CE} = V_{CC} - I_C R_C = 12 - 15(0.6) = 12 - 9 = 3 \text{ V} \quad \therefore \quad \text{Active}$$



(b)  $6 \text{ V}$

Assume Active Region: Then  $I_B = 0.15 \text{ mA}$  and  $I_C = 15.0 \text{ mA}$  as in part (a)

$$V_{CE} = V_{CC} - I_C R_C = 6 - 15(0.6) = 6 - 9 = -3 \text{ V} \quad \therefore \quad \text{Not Active}$$

Assume Saturation Region:

$I_B = 0.15 \text{ mA}$  as above

$$I_C = \frac{V_{CC}}{R_C + R_{sat}} = \frac{6 \text{ V}}{(0.6 + 0.06) \Omega} = 9.09 \text{ mA}$$

$$\text{and } V_{CE} = V_{CC} - I_C R_C = 6 - 9.09(0.6) = .546 \text{ V} \quad \therefore \quad \text{The transistor is in saturation}$$

### Problem 5

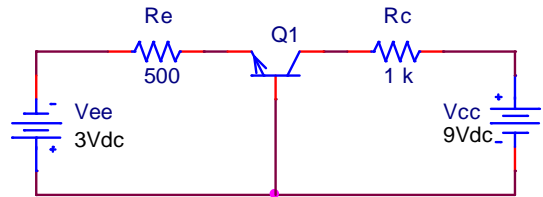
An *npn* transistor is modeled in its active region by  $\beta_{dc} = 75$ ,  $V_0 = 0.6\text{ V}$ , and  $I_{CE0} = 1\mu\text{ A}$ . A  $1\text{ k}\Omega$  resistor and a  $9\text{ V}$  dc source are connected in series between the base and collector (positive source terminal to collector), and a  $500\ \Omega$  resistor and a  $3\text{ V}$  dc source in series are connected between the base and emitter (negative source terminal to emitter). Find  $I_B$ ,  $I_C$ ,  $I_E$ ,  $V_{BE}$ , and  $V_{CE}$ .

$$I_E = -\frac{(3.0-0.6)\text{ V}}{500\ \Omega} = -4.8\text{ mA}$$

$$I_C = \beta I_B + I_{CE0} = 75 I_B + 0.001 = |I_E| - I_B = 4.8 - I_B$$

$$\Rightarrow I_B = 63.17\ \mu\text{ A}$$

$$\therefore I_C = 4.8 - I_B = 4.74\text{ mA}$$



$$V_{BE} = V_0 = 0.6\text{ V (given)} \quad \text{and} \quad V_{CE} = 9 + 3 - (1)I_C + (0.5)I_E = 4.86\text{ V}$$

### Problem 6

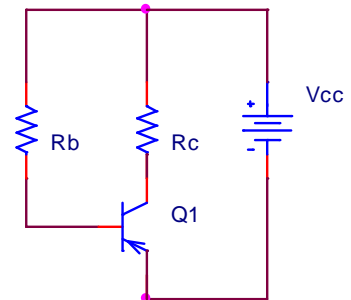
A *npn* transistor is described by  $V_0 = \frac{2}{3}\text{ V}$ ,  $I_{CE0} = 3\mu\text{ A}$ , and  $\beta_{dc} = 80$ . Design a circuit (using a single power supply) that will give an operating point of  $|I_C| = 6\text{ mA}$  and  $|V_{CE}| = 6\text{ V}$ .

Pick  $V_{CC}$  to be sufficiently larger than  $6\text{ V}$  to provide for  $V_{RC}$

Pick  $V_{CC} = -12\text{ V}$ , then  $V_{RC} = 6\text{ V}$  and  $R_C = 1\text{ k}\Omega$

$$|I_B| = \frac{|I_C - I_{CE0}|}{\beta_{dc}} = \frac{(6 - .003)\text{ mA}}{80} = 75\ \mu\text{ A}$$

$$\text{and} \quad R_B = \frac{(12 - \frac{2}{3})\text{ V}}{0.075\text{ mA}} = 151\text{ k}\Omega$$



### Problem 7 (Text 13.49)

(a) What collector current is required for a bipolar transistor to achieve a transconductance of  $30\text{ m}\Omega^{-1}$ ?

$$g_m = 38.92 I_C \Rightarrow I_C = \frac{30}{38.92} = .771\text{ mA}$$

(b) Repeat for a transconductance of  $250\text{ m}\Omega^{-1}$ .

$$I_C = \frac{250}{38.92} = 6.42\text{ mA}$$

(c) Repeat for a transconductance of  $50\text{ m}\Omega^{-1}$ .

$$I_C = \frac{50}{38.92} = 1.28\text{ mA}$$

**Problem 8 (Text 13.50)** Assume an ideality constant of unity and  $T = 25^\circ\text{C}$ .

a) At what Q-point current will  $r_\pi = 10\text{ k}\Omega$  for a bipolar transistor with  $\beta_0 = 75$ ?

$$r_\pi = \frac{\beta_0}{g_m} = \frac{\beta_0}{39.92|I_C|} \Rightarrow I_C = \frac{\beta_0}{38.92 r_\pi} = \frac{75}{38.92 \times 10^4} = 192.7\ \mu\text{A} \quad | \text{ Q-point: } (193\ \mu\text{A}, V_{CE} \geq 0.7\ \text{V})$$

What are the approximate values of  $g_m$  and  $r_o$  if  $V_A = 100\text{ V}$ ?

$$g_m = \frac{q}{n k T} |I_C| = 38.92 \times 193\ \mu = 75.1\ \text{mS} \quad \text{and} \quad r_o = r_d = \frac{V_A + V_{CE}}{I_C} \cong \frac{V_A}{I_C} = \frac{100\ \text{V}}{193\ \mu\text{A}} = 518\ \text{k}\Omega$$

**Problem 9 (Text 13.54) part 1** Assume an ideality constant of unity and  $T = 25^\circ\text{C}$ .

What are the values of  $\beta_{dc}$  and  $V_A$ ?

$$r_o = r_d = \frac{V_A + V_{CE}}{I_C} \Rightarrow V_A = I_C r_o - V_{CE}$$

Using the values from Row 1:  $V_A = 0.002(40000) - 10 = 70\ \text{V}$

Using the values from Row 2:  $\beta_0 = g_m r_\pi = 0.12(500) = 60 \approx \beta_F = \beta_{dc} = h_{FE}$

**Problem 10 (Text 13.54) part 2** Fill in the table.

Row 1:  $g_m = 38.92 I_C = 38.92 \times 0.002 = 0.0778\ \text{S} = 77.8\ \text{mS}$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{60}{77.8\ \text{m}} = 771\ \Omega \quad \text{and} \quad \mu_f = g_m r_o = 0.0778(40000) = 3.11\ \text{k}$$

Row 2:  $I_C = \frac{g_m}{38.92} = \frac{0.12}{38.92} = 3.08\ \text{mA}$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{80}{3.08\ \text{m}} = 26.0\ \text{k}\Omega \quad \text{and} \quad \mu_f = g_m r_o = 0.12(26.0\ \text{k}) = 3.12\ \text{k}$$

Row 3:  $g_m = \frac{\beta_0}{r_\pi} = \frac{60}{4.8 \times 10^5} = 1.25 \times 10^{-4}\ \text{S}; \quad I_C = \frac{g_m}{38.92} = \frac{1.25 \times 10^{-4}}{38.92} = 3.21\ \mu\text{A}$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{80}{3.21 \times 10^{-6}} = 24.9\ \text{M}\Omega \quad \text{and} \quad \mu_f = g_m r_o = 0.12(24.9\ \text{k}) = 3.11\ \text{k}$$