

## HW 11 Solution

8.2

$$V_{tp} = -0.8V, \quad K_p' \frac{W}{L} = 4 \text{ mA/V}^2$$

(a)  $V_{G1} = V_{G2} = 0.$

$V_{GS1} = V_{GS2}$  and the transistors are matched. So,  $I_1 = I_2 = I/2 = 0.25 \text{ mA}$

$$\frac{I}{2} = \frac{1}{2} K_p' \frac{W}{L} (V_{OV})^2$$

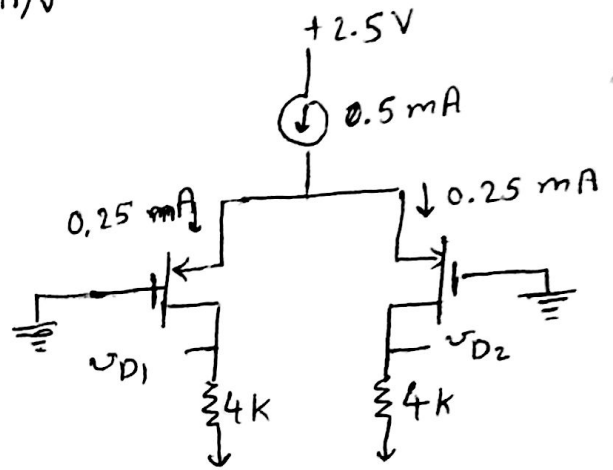
$$\Rightarrow 0.5 = 4 (V_{OV})^2 \Rightarrow |V_{OV}| = 0.355V$$

$$V_{OV} = -0.355V \quad -2.5V$$

$$|V_{OV}| = V_{SG} - |V_{tp}| = V_{SG} - 0.8 \Rightarrow V_{SG} = 1.155V \Rightarrow \underline{V_{GS} = -1.155V}$$

$$V_S = V_G - V_{GS} = 0 - 1.155 = \underline{1.155V}$$

$$-2.5 + I/2(4k) = -2.5 + 0.25(4) = \underline{-1.5V} = V_{D1} = V_{D2}$$



(b)  ~~$V_{GS} > V_{tp}$~~   $V_{DS} < V_{GS} - V_{tp}$  (To remain in sat<sup>n</sup> region),  
 $V_D < V_G - V_{tp} \Rightarrow V_{G_{min}} = V_D + V_{tp}$

$$\underline{V_{CM_{min}} = -1.5 - 0.8 = -2.3V}$$

$$|V_{OV}| = V_{SG} - |V_{tp}|$$

$$2.5 - V_{GS} - V_{SG} = V_{CM} \Rightarrow 2.5 - V_{GS} - (|V_{OV}| + |V_{tp}|) = V_{CM}$$

$$V_{GS} > 0.5V \Rightarrow 2.5 - (0.355 + 0.8) - V_{CM} > 0.5$$

$$\Rightarrow \underline{V_{CM_{max}} = 2 - (1.155) = 0.845V}$$

Input common mode range  $-2.3V < V_{CM} < 0.845V$

8.4  $V_{G2} = 0, \quad V_{G1} = v_{id}$

Let's find the value of  $v_{id}$  at which  $I_{Q1} = I$  and  $I_{Q2} = 0$ . This happens when  $V_{GS2}$  is ~~reduced~~ <sup>increased</sup> to  $V_t$ . (PMOS)

$$\Rightarrow V_{SG2} = V_S = |V_{tp}| = 0.8V$$

$$\text{Now } I_{Q1} = \frac{1}{2} \left( K_p' \frac{W}{L} \right) (V_{SG1} - |V_{tp}|)^2 = I$$

$$\Rightarrow V_{SG1} = |V_{tp}| + \sqrt{2} V_{OV} \quad (V_{OV} \text{ corresponds to } I_D = I/2)$$

$$\Rightarrow V_S - v_{id} = |V_{tp}| + \sqrt{2} V_{OV}$$

$$v_{id, \min} = -\sqrt{2} V_{OV} \quad (I_{Q1} = I \text{ and } I_{Q2} = 0)$$

even if  $v_{id}$  is reduced beyond  $-\sqrt{2} V_{OV}$ ,  $i_{Q1}$  remains  $I$ ,  
likewise current can be steered to  $Q2$  ( $I_{Q2} = I, I_{Q1} = 0$ )  
with  $v_{id, \max} = \sqrt{2} V_{OV}$

$$-\sqrt{2} V_{OV} \leq v_{id} \leq \sqrt{2} V_{OV}$$

$$|V_{OV}| = 0.355 \text{ V} \quad (\text{found in 8.2 for } I_{Q1} = I_{Q2} = I/2)$$

$$-0.5 \text{ V} \leq v_{id} \leq 0.5 \text{ V}$$

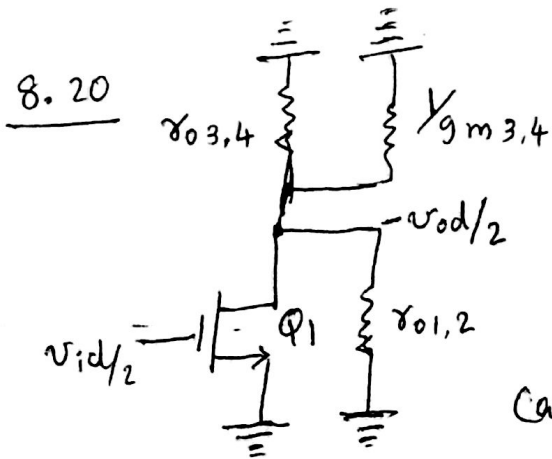
at  $v_{id} = -0.5 \text{ V}$       $v_{D1} = -2.5 + I(4) = -2.5 + 4(0.5) = -0.5 \text{ V}$

$$v_{D2} = -2.5 \text{ V}, \quad v_S = 0.8 \text{ V}$$

at  $v_{id} = 0.5 \text{ V}$ ,      $v_{D1} = -2.5 \text{ V}, \quad v_{D2} = -0.5 \text{ V}, \quad v_S = 0.8 \text{ V}$

~~$v_{D2} = -2.5$~~

$$v_S - v_{A1} = 0.8 \Rightarrow v_S = 1.3 \text{ V}$$



$$A_d = \frac{v_{od}}{v_{id}} = \frac{v_{od}/2}{v_{id}/2}$$

here  $-\frac{v_{od}/2}{v_{id}/2} = -g_{m1,2} (r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{g_{m3,4}})$

(a) so  $A_d = g_{m1,2} (r_{o1,2} \parallel r_{o3,4} \parallel \frac{1}{g_{m3,4}})$

(b) neglecting  $r_{o1,2}$  and  $r_{o3,4}$       $A_d = g_{m1,2} / g_{m3,4}$

$$g_m = \mu C_{ox} W/L V_{OV}$$

$$= \sqrt{2 \mu C_{ox} W/L I_D}$$

$$g_{m1,2} = \mu_n C_{ox} (W/L)_{1,2} V_{OV}$$

$$= \sqrt{2 \mu_n C_{ox} (W/L)_{1,2} I/2}$$

$$g_{m3,4} = \sqrt{2 \mu_p C_{ox} (W/L)_{3,4} I/2}$$

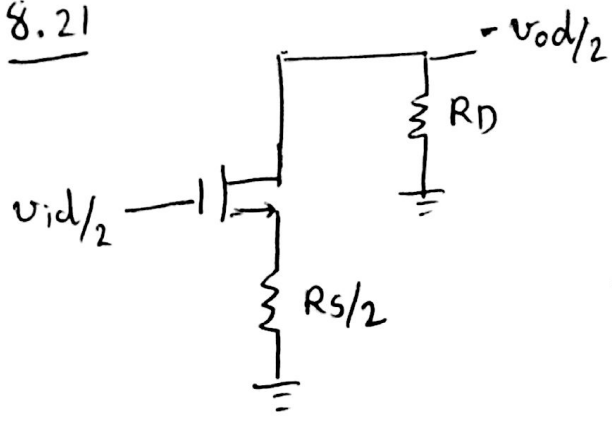
so  $A_d = \sqrt{\frac{\mu_n}{\mu_p} \frac{(W/L)_{1,2}}{(W/L)_{3,4}}}$

DC bias is same for PMOS & NMOS

(c)  $10 = \sqrt{4 \left( \frac{W_{1,2}}{W_{3,4}} \right)} \Rightarrow \boxed{\frac{W_{1,2}}{W_{3,4}} = 25}$

2)

8.21



$$\frac{-v_{od}/2}{v_{id}/2} = \frac{-R_D}{1/g_m + R_s/2}$$

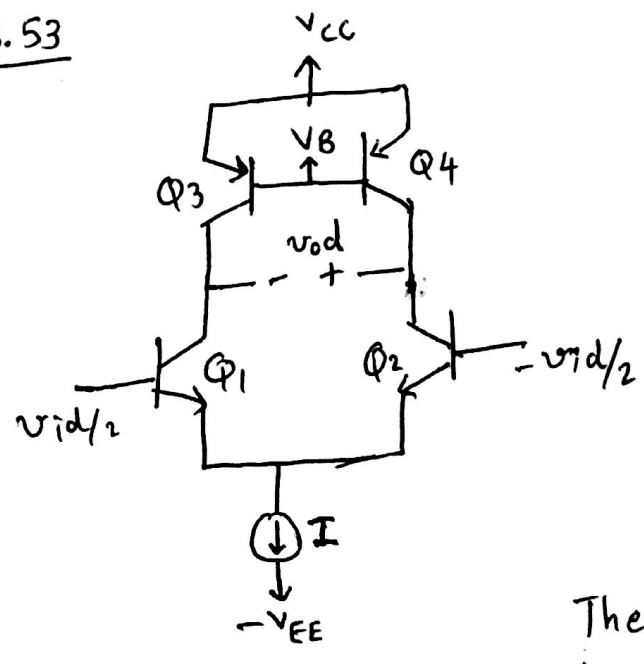
$$\Rightarrow \frac{v_{od}}{v_{id}} = \frac{R_D}{1/g_m + R_s/2} = A_d$$

with  $R_s = 0$ ,  $A_d = g_m R_D$

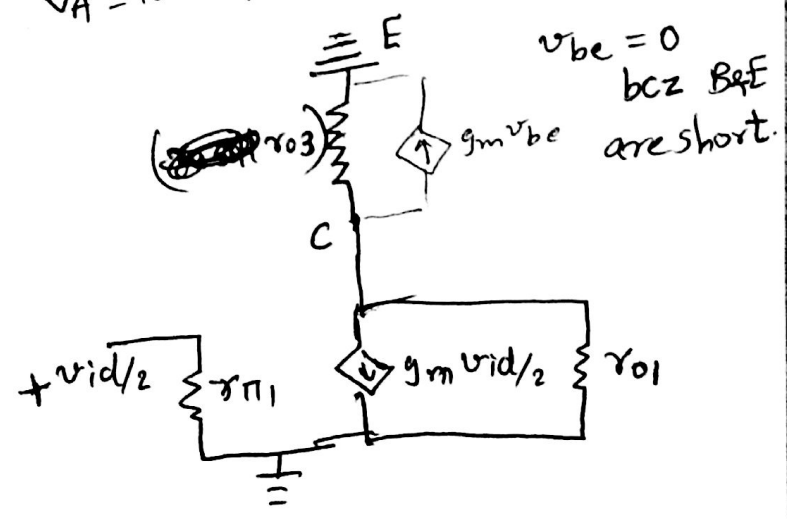
$$gain = \frac{1}{2} g_m R_D = \frac{R_D}{1/g_m + R_s/2} = \frac{R_D g_m}{1 + g_m R_s/2}$$

$$\Rightarrow 2 = 1 + g_m R_s/2 \Rightarrow \boxed{R_s = 2/g_m}$$

8.53



$V_A = 10V$  for all transistors



The pnp transistors are connected to  $v_B$  at the base.

$$gain = \frac{-v_{od}/2}{v_{id}/2} = -g_{m1} (r_{o1} \parallel r_{o3}) \Rightarrow \frac{v_{od}}{v_{id}} = g_{m1} (r_{o1}/2)$$

$$r_o = \frac{|V_A|}{I_C}, \quad g_m = \frac{I_C}{V_T}$$

$$= \frac{I/2}{V_T} \frac{|V_A|}{I/2} (2)$$

$$= \frac{10}{2(0.025)} = 200$$