Fall 2008

EE 612: Nanoscale Transistors

HW5 SOLUTION

Ballistic FETs and V_T

1) For a p-MOSFET with $N_D = 2.7\times10^{18}$ cm^{-3}, a p$^+$ polysilicon gate with ($E_V - E_F$) = 0.0, an EOT_{elec} of 1.1nm, and charge at the oxide-Silicon interface of $Q_F / q = 5\times10^{10}$ cm^{-2}, compute the threshold voltage at $V_{SB} = 0, -0.5, +0.5$ V.

Solution 1)

$V_T = V_{FB} - 2\psi_B - \dfrac{Q_S(2\psi_B + V_{SB})}{C_{OX}}$ in the p-MOSFET,

(i) The flatband voltage,

$\phi_{ms} = +1.06V$, $\dfrac{Q_F}{C_{OX}} = 0.0025V$ where $C_{OX} = \dfrac{\varepsilon_o \varepsilon_{SiO_2}}{EOT_{elec}}$

$$V_{FB} = \phi_{ms} - \frac{Q_F}{C_{OX}} = +1.058V$$

(ii) $2\psi_B = 2(\phi_{ms} - \dfrac{E_g/q}{2}) = 1.0V$

(iii) $\dfrac{Q_S}{C_{OX}} \approx \dfrac{Q_D}{C_{OX}} = +\dfrac{\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V_{SB})}}{C_{OX}} = 0.3024(V_{SB} = 0), 0.2138(V_{SB} = -0.5), 0.3704(V_{SB} = +0.5)$

Therefore,

$V_T = -0.2444(V_{SB} = 0), -0.1588(V_{SB} = -0.5), -0.3124(V_{SB} = +0.5)$

(It is ok that you properly consider V_T increase by poly depletion effect)

2) Assume that the off-current of a MOSFET at T = 27ºC and V_{SB} = 0.0 is $0.1\,\mu$A/μm. Estimate the off current at T = 100ºC.

Solution 2)

In the subthreshold region,

$$I_D = \mu_{eff} C_{ox}\left(\frac{W}{L}\right)(m-1)\left(\frac{k_B T}{q}\right)^2 e^{q(V_{GS}-V_T)/mk_BT}\left(1 - e^{-qV_{DS}/k_BT}\right)$$

Assuming that the mobility is approximately independent of temperature, the off current at 100C can be approximated as,

$$I_{off}(373K) = I_{off}(300K)\left(\frac{373}{300}\right)^2 e^{\frac{-q}{mk_B}\left(\frac{V_T(373)}{373}-\frac{V_T(300)}{300}\right)}$$

Temperature coefficient can be obtained from eqn 3.45.

Using m=1+3t$_{ox}$/W$_{dm}$ =1.15 from prob 1, dV$_t$/dT = 0.5 mV/K.

Vt (100C) = -0.210V, Vt(27C)= - 0.244V.

Therefore,

$$I_{off}(373K) = 1.68\mu A/\mu m$$

If we assume that both the mobility and threshold voltage are approximately independent of temperature, the off current can be approximated as,

$$I_{off} = I_{DS}(V_{GS}=0, V_{DS}=V_{DD}) \propto T^2 e^{-qV_T/mk_BT}$$

From the results of problem 1, V$_T$ =0.244V, m = 1+3t$_{ox}$/W$_{dm}$ =1.15.

I$_{off}$ at 100C is,

$$I_{off}(373K) = I_{off}(300K)\left(\frac{373}{300}\right)^2 e^{\frac{-qV_T}{mk_B}\left(\frac{1}{373}-\frac{1}{300}\right)} = 0.75\mu A/\mu m$$

3) As derived in Taur and Ning,

$$I_D = \mu_{eff} C_{ox}\left(\frac{W}{L}\right)(m-1)\left(\frac{k_BT}{q}\right)^2 e^{q(V_{GS}-V_T)/mk_BT}\left(1-e^{-qV_{DS}/k_BT}\right)$$

describes the I-V characteristic of a MOSFET in the subthreshold region. Explain how to modify this expression to describe the subthreshold region of a ballistic MOSFET.

Solution 3)

$$\begin{aligned}
I_D &= \left(\mu_{eff}\frac{k_BT}{q}\right)\frac{1}{L}C_{ox}W(m-1)\left(\frac{k_BT}{q}\right)e^{q(V_{GS}-V_T)/mk_BT}\left(1-e^{-qV_{DS}/k_BT}\right)\\
&= \frac{D_{eff}}{L}C_{ox}W(m-1)\left(\frac{k_BT}{q}\right)e^{q(V_{GS}-V_T)/mk_BT}\left(1-e^{-qV_{DS}/k_BT}\right)\\
&= \upsilon_T C_{ox}W(m-1)\left(\frac{k_BT}{q}\right)e^{q(V_{GS}-V_T)/mk_BT}\left(1-e^{-qV_{DS}/k_BT}\right)
\end{aligned}$$

4) Electrons moving in the x-y plane of a MOSFET channel have a +x-directed velocity of $\upsilon_x = \upsilon \cos\theta$ where $m^* \upsilon^2 / 2 = \left(E - \varepsilon_1\right)$, where ε_1 is the energy of the first subband (think of this like the bottom of the conduction band).

a) Show that at a given energy, the average +x-directed velocity, $\left\langle \upsilon_x \right\rangle = 2/\pi$.

b) Show by integrating over energy, that the overall average +x-directed velocity is

$$\left\langle \upsilon_x \right\rangle = \int_{\varepsilon_1}^{\infty} \frac{2}{\pi} \upsilon e^{-(E-E_F)/k_B T} dE \Bigg/ \int_{E_C}^{\infty} e^{-(E-E_F)/k_B T} dE = \sqrt{\frac{2k_B T}{\pi m^*}} \equiv \upsilon_T$$

Solution 4 a)

Electrons are free to move in the *x-y* plane, and they enter the device with a spread of velocities, so we need to use the average velocity in the *x*-direction (or average cos θ)

$$\left\langle \upsilon_x \right\rangle = \frac{\int_{-\pi/2}^{+\pi/2} \upsilon \cos\theta d\theta}{\pi} = \frac{2}{\pi}\upsilon$$

Solution 4 b)

$$\left\langle \upsilon_x \right\rangle = \frac{\int_{\varepsilon_1}^{\infty} \frac{2}{\pi} \sqrt{\frac{2\left(E-\varepsilon_1\right)}{m^*}} e^{-(E-E_F)/k_B T} dE}{\int_{\varepsilon_1}^{\infty} e^{-(E-E_F)/k_B T} dE}, \qquad x \equiv \frac{E-\varepsilon_1}{k_B T}, \eta_F \equiv \frac{E_F - \varepsilon_1}{k_B T}$$

$$\left\langle \upsilon_x \right\rangle = \frac{2}{\pi} \sqrt{\frac{2k_B T}{m^*}} \frac{\int_0^{\infty} x^{1/2} e^{-(x-\eta_F)} dx}{\int_0^{\infty} e^{-(x-\eta_F)} dx} = \frac{2}{\pi} \sqrt{\frac{2k_B T}{m^*}} \Gamma(\frac{3}{2}) = \sqrt{\frac{2k_B T}{\pi m^*}}$$

Where gamma function $\Gamma(z) = \int_0^{\infty} x^{z-1} e^{-x} dx$

5) Assume Boltzmann statistics and compute the ballistic mobility for an L = 50nm MOSFET with silicon channel (m* = 0.19m$_0$) vs. GaAs channel (m* =0.063m$_0$). <u>Compare</u> the two numbers and <u>discuss</u> the significance.

Solution 5)

Assuming Boltzmann statistics,

$$\mu_B = \frac{\upsilon_T L}{\left(2k_B T/q\right)} = \frac{L}{\left(2k_B T/q\right)} \sqrt{\frac{2k_B T}{\pi m^*}} = 1.19\times10^3 (Si),\ 2.07\times10^3 (GaAS)\ [cm^2/V\cdot\sec]$$

It is clear the light effective mass helps boost mobility.

6) At T = 0K in a ballistic MOSFET, the density of inversion layer electrons with positive velocities is given by

$$n_S^+ = \frac{m^*}{2\pi\hbar^2}\left(E_F - \varepsilon_1\right),$$

where ε_1 is the energy of the first subband (think of this like the bottom of the conduction band). At T = 0K in the ballistic MOSFET, the average velocity of the electrons with positive velocities is

$$\tilde{\upsilon}_T = \frac{4}{3\pi}\upsilon_F = \frac{4}{3\pi}\sqrt{\frac{2\left(E_F - \varepsilon_1\right)}{m^*}}.$$

a) Use the two equations above to develop an expression for the on-current of a ballistic MOSFET in the fully degenerate limit.

b) Assume $n_S^+ = 10^{13}$ per cm^2 and $m^* = 0.19 m_0$ (silicon) and compare the ballistic injection velocities under non-degenerate and fully degenerate conditions.

Solution 6 a)

The on-current of a ballistic MOSFET in the fully degenerate limit consists of I$^+$ alone.

$$n_S = n_S^+ = \frac{m^*}{2\pi\hbar^2}\left(E_F - \varepsilon_1\right) = \frac{C_{ox}(V_G - V_t)}{q}$$

$$I_D(on) = Wqn_S^+\upsilon^+ = WC_{ox}(V_G - V_t)\left[\frac{8\hbar}{3m^*}\sqrt{\frac{C_{ox}(V_G - V_t)}{q\pi}}\right]$$

Solution 6 b)

In the non-degenerate condition

$$\tilde{\upsilon}_T = \sqrt{\frac{2k_BT}{\pi m^*}} = 1.2\times10^7 cm/\sec$$

In the non-degenerate condition, from results of part (a)

$$\tilde{\upsilon}_T = \frac{8\hbar}{3m^*}\sqrt{\frac{C_{ox}(V_G - V_t)}{q\pi}} = \frac{8\hbar}{3m^*}\sqrt{\frac{n_s^+}{\pi}} = 2.9\times10^7 cm/\sec$$

7) The transconductance of a MOSFET in the saturation region of operation is frequently used to estimate the velocity in the channel according to

$$\langle \upsilon \rangle = \frac{g_m}{WC_{ox}}$$

a) Show that this expression gives the velocity at the top of the barrier for a ballistic MOSFET in the non-degenerate limit.

b) Show that under fully degenerate conditions ($T = 0\text{K}$), this expression does not give the velocity at the top of the barrier.

c) Compare a) and b) to the result from the complete velocity saturation model.

Solution 7 a)
From current expression, intrinsic transconductance can be written as,

$$g_{mi} = \frac{\partial (I_{ds}/W)}{\partial V_{gs}} = C_{eff} < \upsilon > + C_{eff}(V_{gs} - V_t)\frac{\partial < \upsilon >}{\partial V_{gs}}$$

And $\langle \upsilon \rangle = \sqrt{\frac{2k_B T}{\pi m^*}} \frac{\mathcal{F}_{1/2}(\eta_{F1})}{\mathcal{F}_0(\eta_{F1})}$

In the non-degenerate limit,

Because of $\langle \upsilon \rangle = \sqrt{\frac{2k_B T}{\pi m^*}}$, $\frac{\partial < \upsilon >}{\partial V_{gs}} = 0$ and $\langle \upsilon \rangle = \frac{g_m}{C_{ox}}$

Solution 7 b)

In the degenerate limit, using a result of 6 (b),

$$\frac{g_{mi}}{C_{eff}} = < \upsilon > + \frac{1}{2}\left(\frac{8\hbar}{3m^*} \sqrt{\frac{C_{eff}(V_G - V_t)}{q\pi}} \right) = \frac{3}{2} < \upsilon >$$

$$< \upsilon > = \frac{2}{3}\frac{g_{mi}}{C_{eff}}$$

Solution 7 c)

The complete velocity saturation model is $I_D(on) = WC_{ox}(V_G - V_t)\upsilon_{sat}$

,

$$\frac{g_m}{WC_{ox}} = \upsilon_{sat}$$

8) Explain in words (no more than three sentences) why the subthreshold current increases exponentially with gate voltage.

Solution 7 c)

The probability of thermionic emission over the barrier is exp(-E_b/k_BT) and the barrier height decreases linearly with gate voltage. Therefore, the subthreshold current exponentially increase with the gate voltage.