

EE648 (CC761-M) Sp'99
Session 1 (live: 1/12/99)

Outline:

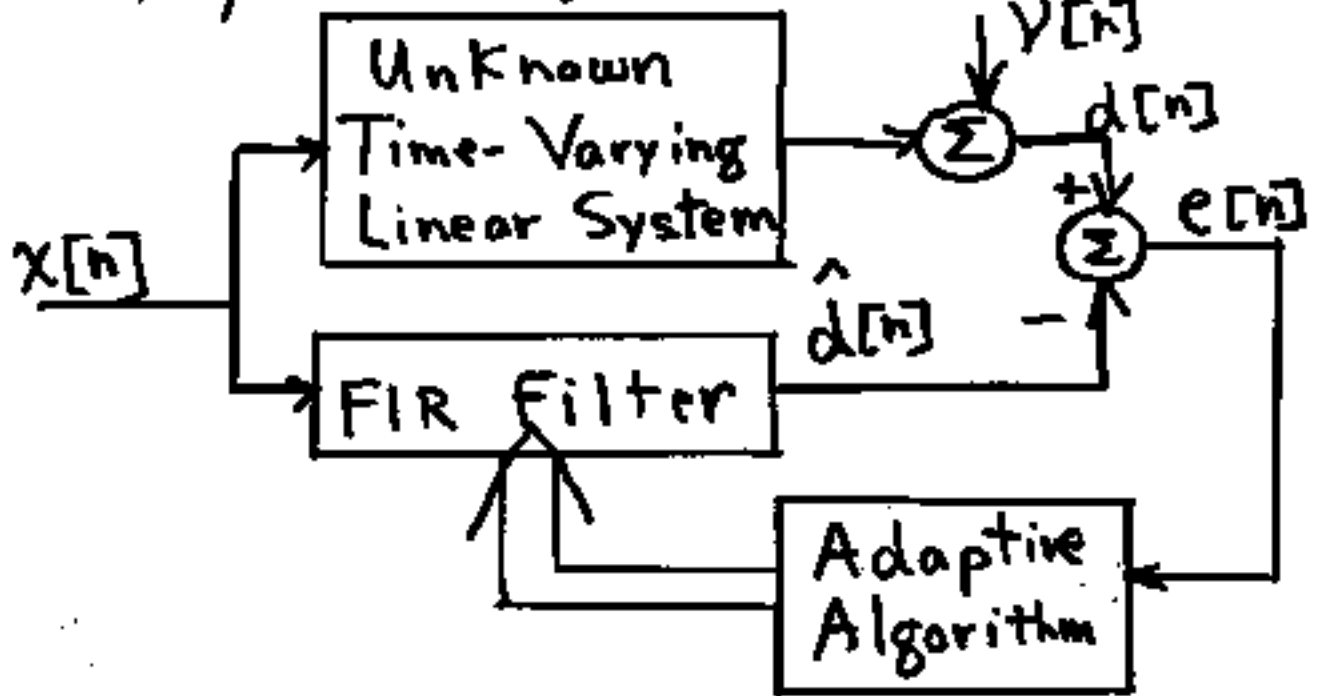
- Intro. to Adaptive Filtering
 - System Identification Application
 - Minimum Mean Square Error (MMSE) Criterion
- Main Reference: Chap. 12 of 1st Edition of Reference Text by Proakis & Manolakis

• Other references:

Advanced Digital Signal Processing, Authors: Proakis, Rader, Ling, and NIKIAS.
Publishers Macmillan. 1992.
ISBN: 0-02-396841-9
Chap. 6 Sects. 6.1 and 6.2

Adaptive Filter Theory,
Author: Simon Haykin,
Publisher: Prentice-Hall.
Chaps. 1, 3, and 5

System Identification



- Given $d[n]$ and $x[n]$
- determine $h[k], k=0, 1, \dots, M-1$
to minimize

$$E\{e^2[n]\} = E\left\{\left(d[n] - \hat{d}[n]\right)^2\right\}$$

$$= E\left\{\left(d[n] - \sum_{k=0}^{M-1} h[k] x[n-k]\right)^2\right\}$$

- vector notation:

$$\underline{h}_M = [h[0], h[1], \dots, h[M-1]]^T$$

$$\underline{x}[n] = [x[n], x[n-1], \dots, x[n-(M-1)]]^T$$

$$\begin{aligned} \cdot \hat{d}[n] &= \underline{h}_M^T \underline{x}[n] = \underline{x}^T[n] \underline{h}_M \\ &= \sum_{k=0}^{M-1} h[k] x[n-k] \end{aligned}$$

$$\begin{aligned} \cdot E\{e^2[n]\} &= E\left\{ \left(d[n] - \underline{h}_M^T \underline{x}[n] \right)^2 \right\} \\ &= E\{d^2[n]\} - 2 \underline{h}_M^T E\{d[n] \underline{x}[n]\} \\ &\quad + \underline{h}_M^T E\{\underline{x}[n] \underline{x}^T[n]\} \underline{h}_M \\ &= E\{d^2[n]\} - 2 \underline{h}_M^T r_{dx} + \underline{h}_M^T R_{xx} \underline{h}_M \end{aligned}$$

$$\underline{r}_{dx} = E\{d[n] \underline{x}[n]\} = \begin{bmatrix} E\{d[n] x[n]\} \\ E\{d[n] x[n-1]\} \\ \vdots \\ E\{d[n] x[n-(M-1)]\} \end{bmatrix}$$

$M \times 1$ vector

$$\underline{R}_{xx} = \begin{bmatrix} r_{xx}[0] & r_{xx}[1] & \dots & r_{xx}[M-1] \\ r_{xx}[1] & r_{xx}[0] & \dots & r_{xx}[M-2] \\ r_{xx}[2] & r_{xx}[1] & \dots & r_{xx}[M-3] \\ \vdots & \vdots & \dots & \vdots \\ r_{xx}[M-1] & r_{xx}[M-2] & \dots & r_{xx}[0] \end{bmatrix}$$

Toeplitz-Symmetric $M \times M$ matrix

• where: $r_{xx}[l] = E\{x[n]x[n-l]\}$
autocorrelation sequence for $x[n]$
assuming stationarity

• recall: completing the square of
a quadratic polynomial

$$\begin{aligned} f(x) &= ax^2 + bx + c \quad (a > 0) \\ &= ax^2 + bx + \left(\frac{b}{2a}\right)^2 + c - \left(\frac{b}{2a}\right)^2 \\ &= a\left(x + \frac{b}{2a}\right)^2 + c - \left(\frac{b}{2a}\right)^2 \end{aligned}$$

• since $a > 0$, minimum ^{of $f(x)$} occurs
with $x = -\frac{b}{2a}$

$$\cdot E\{e^2[n]\} = E\{d^2[n]\} - 2 \underline{h}_M^T \underline{r}_{dx}$$

$$+ \underline{h}_M^T \underline{R}_{xx} \underline{h}_M + \underline{r}_{dx}^T \underline{R}_{xx}^{-1} \underline{r}_{dx} - \underline{r}_{dx}^T \underline{R}_{xx}^{-1} \underline{r}_{dx}$$

$$= E\{d^2[n]\} - \underline{r}_{dx}^T \underline{R}_{xx}^{-1} \underline{r}_{dx}$$

$$+ (\underline{R}_{xx} \underline{h}_M - \underline{r}_{dx})^T \underline{R}_{xx}^{-1} (\underline{R}_{xx} \underline{h}_M - \underline{r}_{dx})$$

Minimized
when: $\underline{h}_M = \underline{R}_{xx}^{-1} \underline{r}_{dx}$