

# EE648 (CC761-M) DSP II

## Session 12 (live: 2/18/99)

### Outline

- FIR Spatial Filtering (Beam-forming)
- Non-data-adaptive - use of direction of desired signal
- Data-adaptive using direction of desired signal
- Data-adaptive with training sequence from desired user - direction not known

• Goal: to "pass" signal arriving at  $\theta = \theta_1$  (with spatial freq.  $\mu_1$ )

• in simplest case:  $\underline{h} = \underline{s}(\mu_1)$

• or component-wise:

$$h[m] = e^{j m \mu_1}, \quad m=0, 1, \dots, M-1$$

• in terms of physical angle:

$$\underline{h} = \underline{a}(\theta)$$

• where:

$$\underline{a}(\theta) = \left[ 1, e^{j \frac{2\pi}{\lambda_c} d \cos \theta}, \dots, e^{j \frac{(M-1) 2\pi}{\lambda_c} d \cos \theta} \right]^T$$

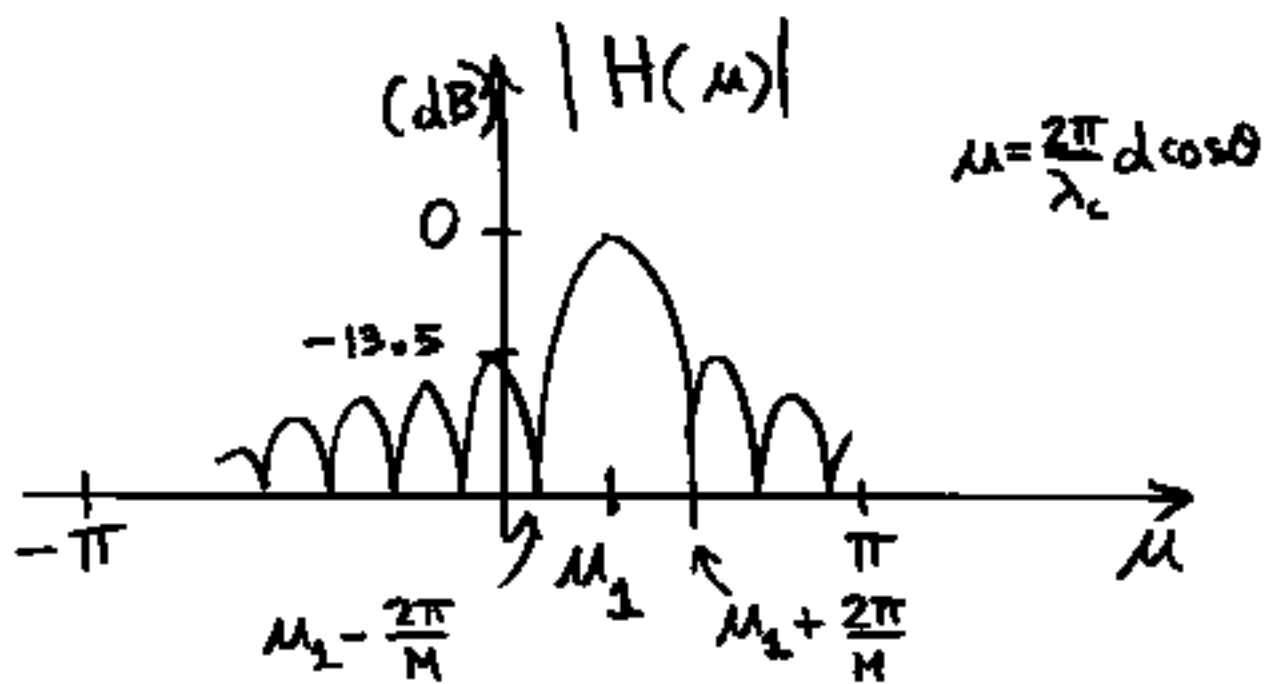
• Discrete-Space Fourier Transform

$$H(\mu) = \sum_{m=0}^{M-1} h[m] e^{-j m \mu}$$

$$\left( \mu = \frac{2\pi}{\lambda} d \cos \theta \right)$$

• Substitute:  $h[m] = e^{j m \mu_1}$

$$H(\mu) = e^{-j \frac{M-1}{2} (\mu - \mu_1)} \frac{\sin \left[ \frac{M}{2} (\mu - \mu_1) \right]}{\sin \left[ \frac{1}{2} (\mu - \mu_1) \right]}$$



• "Simple" spatial bandpass filter  
with center spatial frequency  $u_1$

• can reduce sidelobes at expense of wider mainlobe by employing a window (aperture taper)

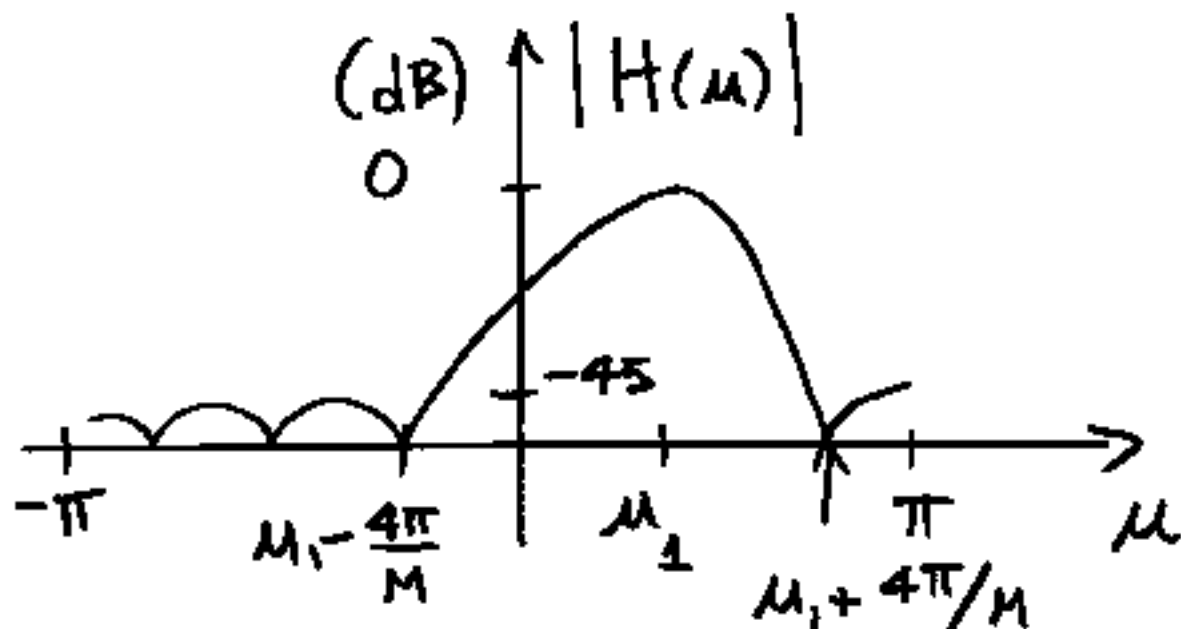
• e.g., Hamming window

$$h[m] = w_{\text{Hamm}}[m] e^{j m \omega_1}$$

• where:

$$w_{\text{Hamm}}[m] = .54 - .46 \cos \left[ \frac{2\pi}{M} \left( m + \frac{1}{2} \right) \right]$$

$$m = 0, 1, \dots, M-1$$



- due to nonlinear relationship between  $\mu$  and  $\theta$ , typically employ Chebyshev windows (or Taylor windows)

- relative to temporal FIR filtering
- $M$  is fixed (can't vary)
- $M$  is relatively small
  - each antenna and associated receiver electronics cost \$\$\$
- thus, mainlobes are typically quite "wide"
- note: the mainlobe corresponds to a larger and larger arc (sector) of space as one moves radially outward from array

- Data-Adaptive Spatial Filtering Using Known Direction of "Desired" Source

- developed in Session 10: the Minimum Variance Beamformer

$$\underline{h}_{MV} = \underline{R}_{xx}^{-1} \underline{s}(\mu_1) / \underline{s}^H(\mu_1) \underline{R}_{xx}^{-1} \underline{s}(\mu_1)$$

- where  $\underline{R}_{xx}$  is  $M \times M$  spatial correlation matrix  $\underline{R}_{xx} = E\{\underline{x}[n] \underline{x}^H[n]\}$

estimated as:  $\hat{\underline{R}}_{xx} = \frac{1}{N} \sum_{n=0}^{N-1} \underline{x}[n] \underline{x}^H[n]$

- Recall data snapshot model:

$$x[n] = \sum_{k=1}^P s_k[n] s(\mu_k) + v[n]$$

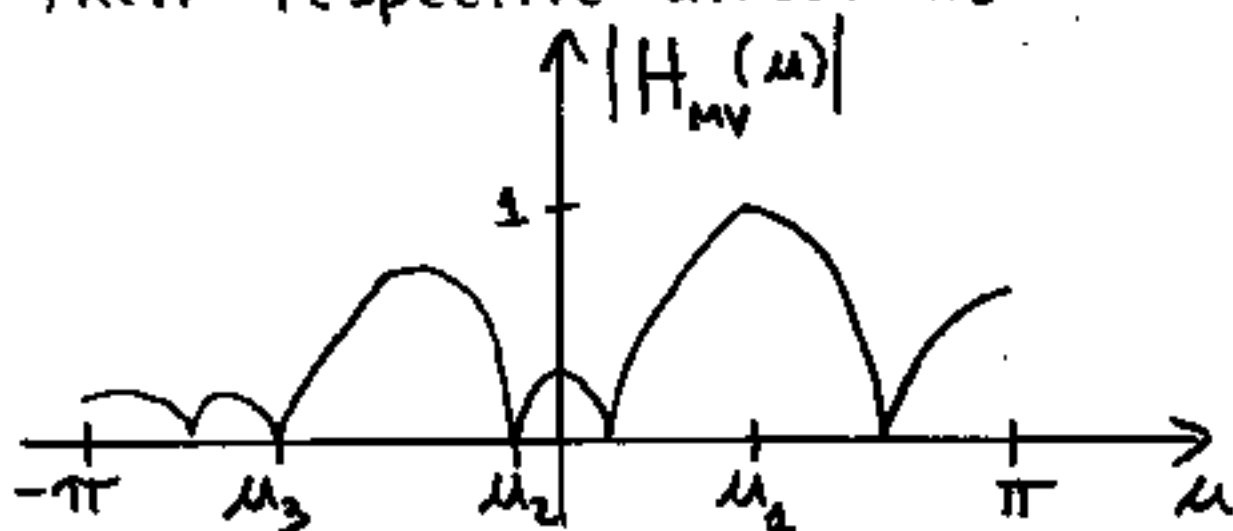
- can show that when SNR of each signal is moderately high

$\underline{h}_{MV}$  satisfies:  $\underline{h}_{MV}^H s(\mu_1) = 1$

$$\underline{h}_{MV}^H s(\mu_k) \approx 0$$

for  $k=2, \dots, P$

- mathematically places spatial nulls in directions of "interfering" signals w/o ever estimating their respective directions



- Data-Adaptive Spatial Filtering using training sequence from "desired" source-direction not known

- Use RLS to minimize:

$$J[n] = \sum_{l=0}^{N_{tr}-1} w^{n-l} \left| \underbrace{b[l]}_{\text{training symbols}} - \underbrace{h}_{\text{RLS}} x[l] \right|^2$$

- on a per snapshot basis
- can apply LMS similarly
- See Spatial Adapt. n