EE648 (CC761-M) DSP II

Session 12 (live: 2/18/99)

Outline

- FIR Spatial Filtering (Beam-forming)
- Non-data-adaptive - use of direction of desired signal
- Data-adaptive using direction of desired signal
- Data-adaptive with training sequence from desired user - direction not known

---

- Goal: to "pass" signal arriving at $\theta = \theta_1$ (with spatial freq. μ_1)
- in simplest case: $\underline{h} = \underline{s}(\mu_1)$
- or component-wise:

$$h[m] = e^{jm\mu_1}, \quad m = 0, 1, \ldots, M-1$$

- in terms of physical angle:

$$\underline{h} = \underline{a}(\theta)$$

- where:

$$\underline{a}(\theta) = \left[1, e^{j\frac{2\pi}{\lambda_c} d\cos\theta}, \ldots, e^{j(M-1)\frac{2\pi}{\lambda_c} d\cos\theta}\right]^T$$

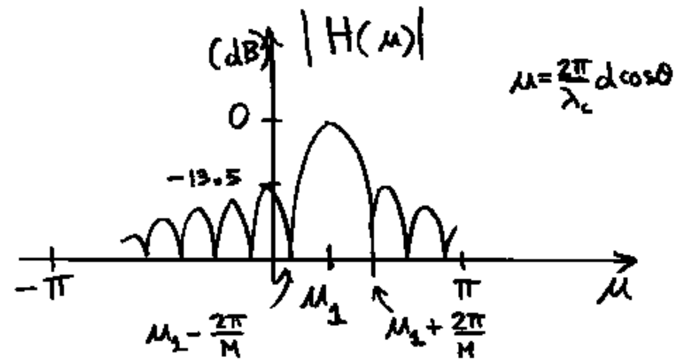
- Discrete-Space Fourier Transform

$$H(\mu) = \sum_{m=0}^{M-1} h[m]\, e^{-jm\mu}$$

$$\left(\mu = \frac{2\pi}{\lambda} d\cos\theta\right)$$

- Substitute: $h[m] = e^{jm\mu_1}$

$$H(\mu) = e^{-j\frac{M-1}{2}(\mu-\mu_1)} \frac{\sin\left[\frac{M}{2}(\mu-\mu_1)\right]}{\sin\left[\frac{1}{2}(\mu-\mu_1)\right]}$$



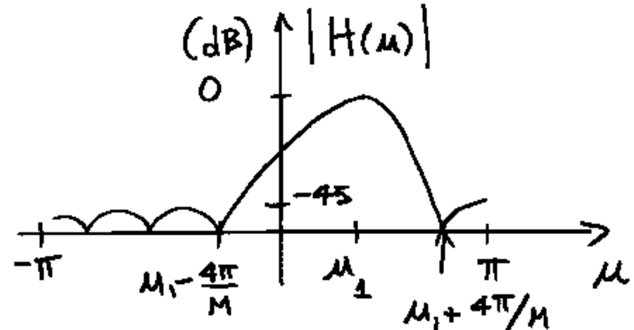
- "simple" spatial bandpass filter with center spatial frequency μ_1

- can reduce sidelobes at expense of wider mainlobe by employing a window (aperture taper)
- e.g., Hamming window

$$h[m] = w_{Hamm}[m]\, e^{jm\mu_1}$$

- where:

$$w_{Hamm}[m] = .54 - .46 \cos\left[\frac{2\pi}{M}\left(n+\tfrac{1}{2}\right)\right] \qquad m = 0, 1, \ldots, M-1$$



- due to nonlinear relationship between μ and θ, typically employ Chebyshev windows (or Taylor windows)

- relative to temporal FIR filtering
- M is fixed (can't vary)
- M is relatively small
  - each antenna and associated receiver electronics cost $$$
- thus, mainlobes are typically quite "wide"
- note: the mainlobe corresponds to a larger and large arc (sector) of space as one moves radially outward from array

- Data-Adaptive Spatial Filtering Using Known Direction of "Desired" Source
- developed in Session 10: the Minimum Variance Beamformer

$$\underline{h}_{MV} = \underline{R}_{xx}^{-1}\underline{s}(\mu_1) \Big/ \underline{s}^H(\mu_1)\underline{R}_{xx}^{-1}\underline{s}(\mu_1)$$

- where $\underline{R}_{xx}$ is $M \times M$ spatial correlation matrix $\underline{R}_{xx} = E\{\underline{x}[n]\,\underline{x}^H[n]\}$

estimated as: $\hat{\underline{R}}_{xx} = \frac{1}{N}\sum_{n=0}^{N-1} \underline{x}[n]\,\underline{x}^H[n]$

• Recall data snapshot model:

$$\underline{x}[n] = \sum_{k=1}^{P} s_k[n]\, \underline{s}(\mu_k) + \underline{v}[n]$$

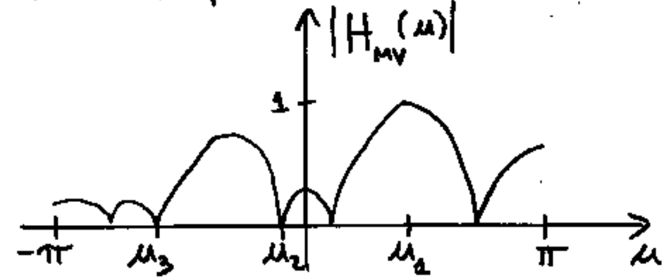
• can show that when SNR of each signal is moderately high $\underline{h}_{MV}$ satisfies:

$$\underline{h}_{MV}^{H} \underline{s}(\mu_1) = 1$$

$$\underline{h}_{MV}^{H} \underline{s}(\mu_k) \approx 0$$

for $k = 2, \ldots, P$

• mathematically places spatial nulls in directions of "interfering" signals w/o ever estimating their respective directions



• Data-Adaptive Spatial Filtering using training sequence from "desired" source-direction not known

• Use RLS to minimize:

$$\mathcal{E}[n] = \sum_{l=0}^{N_{tr}-1} w^{n-l} \Big| \underbrace{b[l]}_{\substack{\text{training}\\\text{symbols}}} - \underline{h}_{RLS}^{H} \underline{x}[l] \Big|^2$$

on a per snapshot basis

• can apply LMS similarly

• See SpatialAdapt.m