

EE648 (CC761-M) DSP II

Session 13 (live: 2/23/99)

Outline

- Data-adaptive Spatial Filtering using desired signal direction
- Eigenstructure Techniques for Spatial Spectrum Estimation

- previously derived optimum spatial filter when desired direction is known:

$$\text{Min } E\{|\underline{h}^H \underline{x}[n]|^2\} = \underline{h}^H \underline{R}_{xx} \underline{h}$$

$$\text{s.t. } \underline{h}^H \underline{a}(\theta_1) = 1$$

• where: $\underline{R}_{xx} = E\{\underline{x}[n] \underline{x}^H[n]\}$ is

$M \times M$ spatial correlation matrix

$$\underline{a}(\theta) = \left[1, e^{j \frac{2\pi}{\lambda} d \cos \theta}, \dots, e^{j \frac{(M-1)2\pi}{\lambda} d \cos \theta} \right]^T$$

• Sol'n: $\underline{h}^{opt} = \underline{R}_{xx}^{-1} \underline{a}_1 / \underline{a}_1^H \underline{R}_{xx}^{-1} \underline{a}_1$

• notation: $\underline{a}_1 \triangleq \frac{\underline{a}(\theta_1)}{\underline{a}^H(\theta_1) \underline{a}(\theta_1)} = \frac{\underline{a}(\theta_1)}{\|\underline{a}(\theta_1)\|^2}$

• this is a block estimate when using inverse of estimated spatial correlation matrix

$$\hat{\underline{R}}_{xx} = \frac{1}{N} \sum_{n=0}^{N-1} \underline{x}[n] \underline{x}^H[n]$$

- in practice :

- do not want to compute inverse of \hat{R}_{-xx} in real-time

- also: desire sol'n to adapt with time on a per snapshot basis

- to facilitate adaptive solution, convert constrained optimization problem to unconstrained one via

$$\underline{h} = \underline{a}_1 + \underline{B} \underline{w}$$

• where: \underline{B} is $M \times (M-1)$

\underline{w} is $(M-1) \times 1$

• each of the $M-1$ columns of \underline{B} is orthogonal to $\underline{a}_1 = \frac{\underline{a}(\theta_1)}{\underline{a}^H(\theta_1)\underline{a}(\theta_1)}$

• in matlab code: $\underline{B} = \text{null}(\underline{a}_1^H)$

• thus: $\underline{h}^H \underline{a}(\theta_1) = (\underline{a}_1^H + \underline{w}^H \underline{B}^H) \underline{a}(\theta_1)$
 $= \frac{\underline{a}^H(\theta_1)\underline{a}(\theta_1)}{\underline{a}^H(\theta_1)\underline{a}(\theta_1)} + \underline{w}^H \underline{B}^H \underline{a}(\theta_1) = 1 + 0$
 $= 1 !!$

- \underline{B} is referred to as blocking matrix \Rightarrow a "bank" of spatial filters

- recall model: $\underline{x}[n] = \sum_{k=1}^P s_k[n] \underline{a}(\theta_k) + \underline{v}[n]$

- $\underline{y}[n] = \underline{B}^H \underline{x}[n]$

$$= \sum_{k=1}^P s_k[n] \underline{B}^H \underline{a}(\theta_k) + \underline{B}^H \underline{v}[n]$$

$$= s_1[n] \underline{B}^H \underline{a}(\theta_1) + \sum_{k=2}^P s_k[n] \underline{B}^H \underline{a}(\theta_k) + \underline{B}^H \underline{v}[n]$$

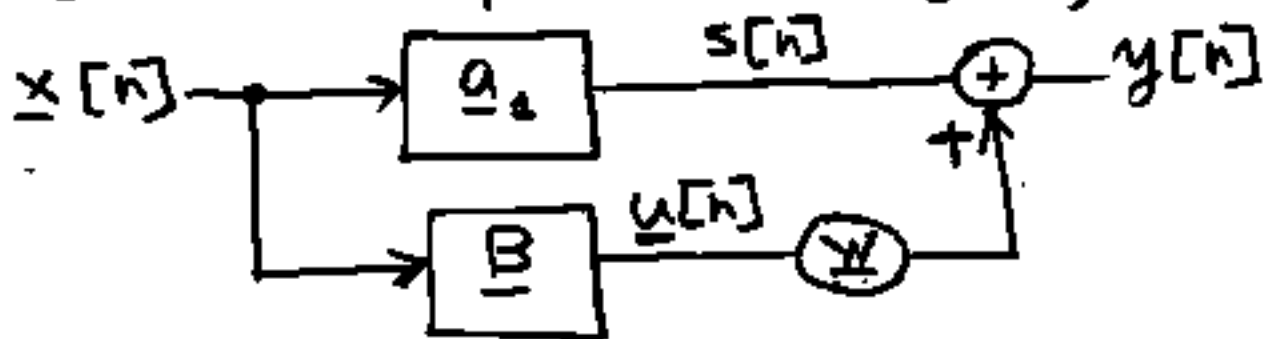
• constraint is automatically satisfied

• FIR spatial filter output:

$$y[n] = \underline{h}^H \underline{x}[n]$$

$$= \underline{a}_2^H \underline{x}[n] + \underline{w}^H \underline{B}^H \underline{x}[n]$$

• Schematically (block diagram):



• Since $\underline{h}^H \underline{a}(\theta_1) = 1$ is guaranteed,
find \underline{w} to minimize $E\{|y[n]|^2\}$

• problem is similar in concept
to adaptive noise cancellation

• note: there is undesired signal
contributions to $S[n] = \underline{a}_1^H \underline{x}[n]$

$$S[n] = s_1[n] + \sum_{k=2}^P s_k[n] \underbrace{(\underline{a}_1^H \underline{a}(\theta_k))}_{\neq 0} + \underline{a}_1^H \underline{v}[n]$$

- $y[n]$ has no desired signal and thus cannot subtract off desired signal
- Relative to LMS/RLS implementations:
 - $s[n]$ "plays" role of desired signal
 - $y[n]$ "plays" role of error signal
- note: $y[n]$ does not go to zero as $n \rightarrow \infty$, but rather approaches $s_1[n]$

• For either LMS or RLS:

$$s[n] = \underline{a}^H \underline{x}[n]$$

$$\underline{u}[n] = \underline{B}^H \underline{x}[n]$$

• LMS: $\underline{w}[n+1] = \underline{w}[n] + \mu [s[n] + \underline{w}^H \underline{u}[n]]^* \underline{u}[n]$

• RLS: $e[n, n-1] = s[n] + \underline{w}^H[n] \underline{u}[n]$

$$\underline{f}[n] = \hat{R}^{-1}[n] \underline{u}[n]$$

$$\mu[n] = \underline{f}^H[n] \underline{u}[n] + \alpha$$

$$\underline{K}[n] = \underline{f}[n] / \mu[n]$$

$$\underline{R}^{-1}[n+1] = (\underline{R}^{-1}[n] - \underline{K} \underline{f}^H[n]) / \alpha$$

$$\underline{w}[n+1] = \underline{w}[n] + e^*[n, n-1] \underline{K}[n]$$

- See Adapt AOA. m at
course web site