EE648 (CC761-M) DSP II

Session 13 (Live: 2/23/99)

Outline

- Data-adaptive Spatial Filtering using desired signal direction
- Eigenstructure Techniques for Spatial Spectrum Estimation

- previously derived optimum spatial filter when desired direction is known:

$$\text{Min } E\{|\underline{h}^H \underline{x}[n]|^2\} = \underline{h}^H \underline{R}_{xx} \underline{h}$$

$$\text{s.t. } \underline{h}^H \underline{a}(\theta_1) = 1$$

- where: $\underline{R}_{xx} = E\{\underline{x}[n]\,\underline{x}^H[n]\}$ is $M \times M$ spatial correlation matrix

$$\underline{a}(\theta) = \left[1, e^{j\frac{2\pi}{\lambda}d\cos\theta}, \ldots, e^{j\frac{(M-1)2\pi}{\lambda}d\cos\theta}\right]^T$$

- Sol'n.: $\underline{h}^{opt} = \underline{R}_{xx}^{-1}\underline{a}_1 / \underline{a}_1^H \underline{R}_{xx}^{-1}\underline{a}_1$
- notation: $\underline{a}_1 \triangleq \dfrac{\underline{a}(\theta_1)}{\underline{a}^H(\theta_1)\underline{a}(\theta_1)} = \dfrac{\underline{a}(\theta_1)}{\|\underline{a}(\theta_1)\|^2}$
- this is a block estimate when using inverse of estimated spatial correlation matrix

$$\hat{\underline{R}}_{xx} = \frac{1}{N}\sum_{n=0}^{N-1}\underline{x}[n]\,\underline{x}^H[n]$$

- in practice:
  - do not want to compute inverse of $\hat{\underline{R}}_{xx}$ in real-time
  - also: desire sol'n to adapt with time on a per snapshot basis
- to facilitate adaptive solution, convert constrained optimization problem to unconstrained one via

$$\underline{h} = \underline{a}_1 + \underline{B}\,\underline{w}$$

• where: $\underline{B}$ is $M \times (M-1)$

$\underline{w}$ is $(M-1) \times 1$

• each of the M-1 columns of $\underline{B}$ is orthogonal to $\underline{a}_1 = \dfrac{\underline{a}(\theta_1)}{\underline{a}^H(\theta_1)\,\underline{a}(\theta_1)}$

• in matlab code: $\underline{B} = \text{null}(\underline{a}_1^H)$

• thus: $\underline{h}^H \underline{a}(\theta_1) = (\underline{a}_1^H + \underline{w}^H \underline{B}^H)\,\underline{a}(\theta_1)$

$$= \frac{\underline{a}^H(\theta_1)\,\underline{a}(\theta_1)}{\underline{a}^H(\theta_1)\,\underline{a}(\theta_1)} + \underline{w}^H \underline{B}^H \underline{a}(\theta_1) = 1 + 0 = 1 \;!!$$

• $\underline{B}$ is referred to as blocking matrix $\Rightarrow$ a "bank" of spatial filters

• recall model: $\underline{x}[n] = \sum_{k=1}^{P} s_k[n]\,\underline{a}(\theta_k) + \underline{v}[n]$

• $\underline{y}[n] = \underline{B}^H \underline{x}[n]$

$$= \sum_{k=1}^{P} s_k[n]\,\underline{B}^H \underline{a}(\theta_k) + \underline{B}^H \underline{v}[n]$$

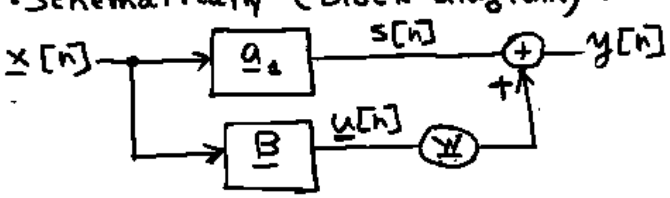
$$= \cancel{s_1[n]\,\underline{B}^H \underline{a}(\theta_1)}^{\,0} + \sum_{k=2}^{P} s_k[n]\,\underline{B}^H \underline{a}(\theta_k) + \underline{B}^H \underline{v}[n]$$

• constraint is automatically satisfied

• FIR spatial filter output:

$$y[n] = \underline{h}^H \underline{x}[n] = \underline{a}_1^H \underline{x}[n] + \underline{w}^H \underline{B}^H \underline{x}[n]$$

• Schematically (block diagram):



• Since $\underline{h}^H \underline{a}(\theta_1) = 1$ is guaranteed, find $\underline{w}$ to minimize $E\{|y[n]|^2\}$

• problem is similar in concept to adaptive noise cancellation

• note: there is undesired signal contributions to $s[n] = \underline{a}_1^H \underline{x}[n]$

$$s[n] = s_1[n] + \sum_{k=2}^{P} s_k[n]\,\underbrace{(\underline{a}_1^H \underline{a}(\theta_k))}_{\neq 0} + \underline{a}_1^H \underline{v}[n]$$

- $\underline{y}[n]$ has no desired signal and thus cannot subtract off desired signal
- Relative to LMS/RLS implementation:
  - $s[n]$ "plays" role of desired signal
  - $y[n]$ "plays" role of error signal
- <u>note</u>: $y[n]$ does not go to zero as $n \to \infty$, but rather approaches $s_1[n]$

- For either LMS or RLS:

$$s[n] = \underline{a}_1^H \underline{x}[n]$$

$$\underline{u}[n] = \underline{B}^H \underline{x}[n]$$

- LMS: $\underline{w}[n+1] = \underline{w}[n] + \mu\left[s[n] + \underline{w}^H \underline{u}[n]\right]^* \underline{u}[n]$
- RLS:

$$e[n, n-1] = s[n] + \underline{w}^H[n]\, \underline{u}[n]$$

$$\underline{f}[n] = \underline{\hat{R}}^{-1}[n]\, \underline{u}[n]$$

$$\mu[n] = \underline{f}^H[n]\, \underline{u}[n] + \alpha$$

$$\underline{k}[n] = \underline{f}[n] / \mu[n]$$

$$\underline{R}^{-1}[n+1] = \left(\underline{R}^{-1}[n] - \underline{k}\, \underline{f}^H[n]\right) / \alpha$$

$$\underline{w}[n+1] = \underline{w}[n] + e^*[n, n-1]\, \underline{k}[n]$$

- See AdaptAOA.m at course web site