EE648 (CC761-M) DSP II

Session 14 (Live: 2/25/99)

Outline

- Data-adaptive Spatial Filtering
  - switch to decision-directed mode
- Eigenstructure Techniques for Spatial Spectrum Estimation

- In the matlab demo DirectAOA.m the spatial filter is first adapted using known direction of desired signal - no training sequence
- done for first N_s symbols
- then switches over to decision-directed mode, i.e., error term uses symbol decision rather than "raw" output of spatial filter

- in decision-directed mode, we update spatial filter $\underline{h}$ directly not $\underline{w}$ as in $\underline{h} = \underline{a}_1 + \underline{B}\,\underline{w}$

$$\underline{h} = \underline{a}_1 + \underbrace{\underline{B}}_{M\times(M-1)}\,\underbrace{\underline{w}}_{(M-1)\times 1}$$

- just prior to switching to decision-directed (DD) mode, RLS has a current estimate of

$$\hat{\underline{R}}_{uu}^{-1}[n] = \left(\underline{B}^H \hat{\underline{R}}_{xx}[n]\,\underline{B}\right)^{-1}$$

• Recall: $$\hat{\underline{R}}_{xx}[n] = \sum_{n=0}^{N-1} \underline{x}[n]\,\underline{x}^H[n]$$

$$\hat{\underline{R}}_{uu}[n] = \sum_{n=0}^{N-1} \underline{u}[n]\,\underline{u}^H[n]$$

where: $\underline{u}[n] = \underline{B}^H \underline{x}[n]$

$$= \sum_{n=0}^{N-1} \underline{B}^H \underline{x}[n]\,\underline{x}^H[n]\,\underline{B}$$

$$\hat{\underline{R}}_{uu}[n] = \underline{B}^H\,\hat{\underline{R}}_{xx}[n]\,\underline{B}$$

- DD mode requires initial estimate of $\underline{\hat{R}}_{xx}^{-1}[n]$

- How do we compute this from $\underline{\hat{R}}_{uu}^{-1}[n] = \left(\underline{B}^H \underline{\hat{R}}_{xx}^{-1}[n]\, \underline{B}\right)^{-1}$

- Use block matrix inversion lemma

$$\begin{bmatrix} \underline{A} & \underline{C} \\ \underline{B} & \underline{D} \end{bmatrix}^{-1} = \begin{bmatrix} \underline{\Delta}^{-1} & -\underline{\Delta}^{-1}\underline{C}\,\underline{D}^{-1} \\ -\underline{D}^{-1}\underline{B}\,\underline{\Delta}^{-1} & \underline{D}^{-1}\underline{B}\,\underline{\Delta}^{-1}\underline{C}\,\underline{D}^{-1} + \underline{D}^{-1} \end{bmatrix}$$

$$\underline{\Delta} = \underline{A} - \underline{C}\,\underline{D}^{-1}\underline{B}$$

- Note: $\hat{\underline{R}}^{-1} = \left(\underline{U}^H \underline{R}_{xx} \underline{U} \right)^{-1}$

$$= \underline{U}^{-1} \underline{R}_{xx}^{-1} \left(\underline{U}^H \right)^{-1}$$

- but, by design: $\underline{U}^H \underline{U} = \underline{I}_{M \times M}$

$$\Rightarrow \underline{U}^{-1} = \underline{U}^H$$

$$\hat{\underline{R}}^{-1} = \underline{U}^H \hat{\underline{R}}_{xx}^{-1} \underline{U}$$

$$\underline{R}_{xx}^{-1} = \underline{U} \hat{\underline{R}}^{-1} \underline{U}^H$$

- To use this, define:

$$\underline{U} = \left[\underline{a}_{1n} \,\vdots\, \underline{B} \right] \qquad \underline{a}_{1n} = \frac{\underline{a}_1}{\|\underline{a}_1\|}$$

- $$\hat{\underline{R}} = \underline{U}^H \hat{\underline{R}}_{xx} \underline{U}$$

$$= \begin{bmatrix} \underline{a}_{1n}^H \\ \underline{B}^H \end{bmatrix} \hat{\underline{R}}_{xx} \left[\underline{a}_{1n} \;\; \underline{B} \right]$$

$$= \begin{bmatrix} \underline{a}_{1n}^H \hat{\underline{R}}_{xx} \underline{a}_{1n} & \underline{a}_{1n}^H \hat{\underline{R}}_{xx} \underline{B} \\ \underline{B}^H \hat{\underline{R}}_{xx} \underline{a}_{1n} & \underline{B}^H \hat{\underline{R}}_{xx} \underline{B} \end{bmatrix}$$

• for our case:

$$\begin{bmatrix} a & \underline{b}^H \\ \underline{b} & \underline{D} \end{bmatrix}^{-1} = \frac{1}{a-\underline{b}^H\underline{D}^{-1}\underline{b}}\left[\begin{array}{c|c} 1 & -\underline{b}^H\underline{D}^{-1} \\ \hline -\underline{D}^{-1}\underline{b} & \underline{D}^{-1}\underline{b}\,\underline{b}^H\underline{D}^{-1} \\ & +(a-\underline{b}^H\underline{D}^{-1}\underline{b})\underline{D}^{-1} \end{array}\right]$$

• to use block matrix inversion lemma

• Specify: $\underline{D}^{-1} = \underline{R}_{uu}^{-1} = \left(\underline{B}^H\hat{\underline{R}}_{xx}\underline{B}\right)^{-1}$

See Direct AOA.m at web site

$a = \underline{a}_{1n}^H\hat{\underline{R}}_{xx}\underline{a}_{1n}$

$\underline{b} = \underline{B}^H\hat{\underline{R}}_{xx}\underline{a}_{1n}$