EE648 (CC761-M) DSP II

Session 14 (Live: 2/25/99)

Outline

- Data-adaptive Spatial Filtering
  - switch to decision-directed mode
- Eigenstructure Techniques for Spatial Spectrum Estimation

- In the matlab demo DirectAOA.m the spatial filter is first adapted using known direction of desired signal - no training sequence
- done for first N_s symbols
- then switches over to decision-directed mode, i.e., error term uses symbol decision rather than "raw" output of spatial filter

- in decision-directed mode, we update spatial filter $\underline{h}$ directly not $\underline{w}$ as in $\underline{h} = \underline{a}_1 + \underset{M\times(M-1)}{\underline{B}}\ \underset{(M-1)\times 1}{\underline{w}}$
- just prior to switching to decision-directed (DD) mode, RLS has a current estimate of

$$\hat{\underline{R}}^{-1}_{uu}[n] = \left(\underline{B}^H \hat{\underline{R}}_{xx}[n] \underline{B}\right)^{-1}$$

- Recall: $\hat{\underline{R}}_{xx}[n] = \sum_{n=0}^{N-1} \underline{x}[n]\underline{x}^H[n]$

$$\hat{\underline{R}}_{uu}[n] = \sum_{n=0}^{N-1} \underline{u}[n]\underline{u}^H[n]$$

where: $\underline{u}[n] = \underline{B}^H \underline{x}[n]$

$$= \sum_{n=0}^{N-1} \underline{B}^H \underline{x}[n]\underline{x}^H[n]\underline{B}$$

$$\hat{\underline{R}}_{uu}[n] = \underline{B}^H \hat{\underline{R}}_{xx}[n] \underline{B}$$

• DD mode requires initial estimate of $\hat{\underline{R}}_{xx}^{-1}[n]$

• How do we compute this from $\hat{\underline{R}}_{uu}^{-1}[n] = \left(\underline{B}^H \hat{\underline{R}}_{xx}^{-1}[n]\, \underline{B}\right)^{-1}$

• Use block matrix inversion lemma

$$\begin{bmatrix} \underline{A} & \underline{C} \\ \underline{B} & \underline{D} \end{bmatrix}^{-1} = \begin{bmatrix} \underline{\Delta}^{-1} & -\underline{\Delta}^{-1}\underline{C}\,\underline{D}^{-1} \\ -\underline{D}^{-1}\underline{B}\,\underline{\Delta}^{-1} & \underline{D}^{-1}\underline{B}\,\underline{\Delta}^{-1}\underline{C}\,\underline{D}^{-1}+\underline{D}^{-1} \end{bmatrix}$$

$$\underline{\Delta} = \underline{A} - \underline{C}\,\underline{D}^{-1}\underline{B}$$

• Note: $\hat{\underline{R}}^{-1} = \left(\underline{U}^H \underline{R}_{xx} \underline{U}\right)^{-1}$

$$= \underline{U}^{-1} \underline{R}_{xx}^{-1} \left(\underline{U}^H\right)^{-1}$$

• but, by design: $\underline{U}^H \underline{U} = \underline{I}_{M\times M}$

$$\Rightarrow \underline{U}^{-1} = \underline{U}^H$$

$$\hat{\underline{R}}^{-1} = \underline{U}^H \hat{\underline{R}}_{xx}^{-1} \underline{U}$$

$$\underline{R}_{xx}^{-1} = \underline{U}\, \hat{\underline{R}}^{-1} \underline{U}^H$$

• To use this, define:

$$\underline{U} = [\underline{a}_{1n} \,\vdots\, \underline{B}] \qquad \underline{a}_{1n} = \frac{\underline{a}_1}{\|\underline{a}_1\|}$$

• $\hat{\underline{R}} = \underline{U}^H \hat{\underline{R}}_{xx} \underline{U}$

$$= \begin{bmatrix} \underline{a}_{1n}^H \\ \underline{B}^H \end{bmatrix} \hat{\underline{R}}_{xx} [\underline{a}_{1n} \ \ \underline{B}]$$

$$= \begin{bmatrix} \underline{a}_{1n}^H \hat{\underline{R}}_{xx} \underline{a}_{1n} & \underline{a}_{1n}^H \hat{\underline{R}}_{xx} \underline{B} \\ \underline{B}^H \hat{\underline{R}}_{xx} \underline{a}_{1n} & \underline{B}^H \hat{\underline{R}}_{xx} \underline{B} \end{bmatrix}$$

• for our case:

$$\begin{bmatrix} a & \underline{b}^H \\ \underline{b} & \underline{D} \end{bmatrix} = \frac{1}{a - \underline{b}^H \underline{D}^{-1} \underline{b}} \left[\begin{array}{c|c} 1 & -\underline{b}^H \underline{D}^{-1} \\ \hline -\underline{D}^{-1}\underline{b} & \underline{D}^{-1}\underline{b}\,\underline{b}^H \underline{D}^{-1} \\ & +(a-\underline{b}^H\underline{D}^{-1}\underline{b})\underline{D}^{-1} \end{array}\right]$$

• to use block matrix inversion lemma

• Specify: $\underline{D}^{-1} = \underline{R}_{uu}^{-1} = \left(\underline{B}^H \hat{\underline{R}}_{xx} \underline{B}\right)^{-1}$

See Direct AOA.m at web site

$$a = \underline{a}_{1n}^H \hat{\underline{R}}_{xx} \underline{a}_{1n}$$

$$\underline{b} = \underline{B}^H \hat{\underline{R}}_{xx} \underline{a}_{1n}$$