EE648 (CC761-M) DSP II
Session 18 (live: 3/11/99)

<u>Outline</u>:

- MUSIC/ESPRIT for
  - estimation of frequencies of 1-D temporal sinewaves superimposed in noise
  - Time-delay estimation
  - P&M Sects. 12.5.2, 12.5.3, 12.5.4

- Frequency Estimation via MUSIC and ESPRIT

$$x[n] = \sum_{k=1}^{P} c_k e^{j\omega_k n} \qquad n = 0, 1, \ldots, N-1$$

- $c_k = A_k e^{j\theta_k}, \quad k = 1, \ldots, P$

$$\underline{x}[n] = \left[x[n], x[n+1], \ldots, x[n+M-1]\right]^T$$

$$\hat{\underline{R}}_{xx} = \frac{1}{N-M+1} \sum_{n=0}^{N-M} \underline{x}[n]\,\underline{x}^H[n]$$

- define:

$$\underset{M\times 1}{\underline{s}(\omega_k)} = \left[1, e^{j\omega_k}, e^{j2\omega_k}, \ldots, e^{j(M-1)\omega_k}\right]^T$$

$$\underline{x}[n] = \sum_{k=1}^{P} c_k e^{j\omega_k n}\, \underline{s}(\omega_k)$$

- Recall for linear array of equi-spaced antennas - narrowband case

$$\underline{x}[n] = \sum_{k=1}^{P} s_k[n]\, \underline{s}(\mu_k)$$

- Use of ESPRIT/MUSIC to estimate relative time-delays of overlapping echoes
- application: range estimation in radar
- radar emits a pulse:

$$p_T(t) = A(t)\cos(2\pi f_c t + \phi(t)) \qquad t \in [0, T_p]$$

- e.g.:

$$A(t) = e^{-t^2/\sigma^2} \qquad \phi(t) = \frac{a}{2} t^2$$

- linear FM Gaussian envelope chirp

- echoes return to radar site

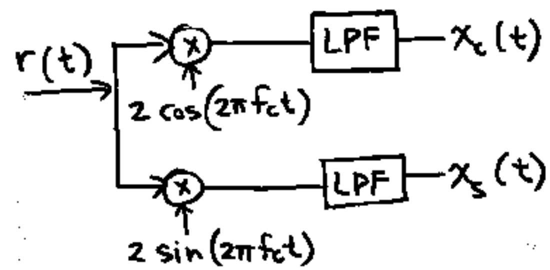
$$\tau_i = \frac{2R_i}{c}$$

R_i = range of i-th "target"

c = speed of E-M waves

- for closely-spaced "targets", the echoes return overlapping in time

- front-end receiver:



- $x(t) = x_c(t) + j\, x_s(t)$

- For P closely-spaced targets (e.g., point scatterers on an aircraft OR multiple airplanes flying in formation)

$$x(t) = \sum_{i=1}^{P} z_i\, A(t-\tau_i)\, e^{j\phi(t-\tau_i)}$$

z_i = complex amp. $\Rightarrow$ prop. to cross-sectional area of i-th scatterer

- Consider Fourier Transform of $x(t)$

$$X(f) = \sum_{i=1}^{P} z_i\, S_T(f)\, e^{-j2\pi f \tau_i}$$

where:

$$S_T(f) = \mathcal{F}\{A(t)\, e^{j\phi(t)}\}$$

$\Rightarrow$ Known a-priori

define: $s_b(t) = A(t)\, e^{j\phi(t)}$

- Dividing by $S_T(f)$ on both sides

$$Y(f) = \frac{X(f)}{S_T(f)} = \sum_{i=1}^{P} z_i \, e^{j 2\pi f \tau_i}$$

- in-practice: $x(t)$ is sampled - then compute N-pt. FFT
- Ultimately:

$$Y(k) = \frac{X(k)}{S_T(k)} = \sum_{i=1}^{P} z_i \, e^{-j 2\pi k \frac{\tau_i}{N}}$$

$$k = 0, 1, \dots, N-1$$

- in practice, $s_b(t) = A(t)e^{j\phi(t)}$ is digitized and an N pt. FFT is computed a-priori
- Ultimately, apply Music/ESPRIT to estimate $\omega_i = 2\pi \frac{\tau_i}{N}$

$$i = 1, \dots, P$$