EE648 (CC761-M) Sp'99

Session 1 (Live: 1/12/99)

Outline:

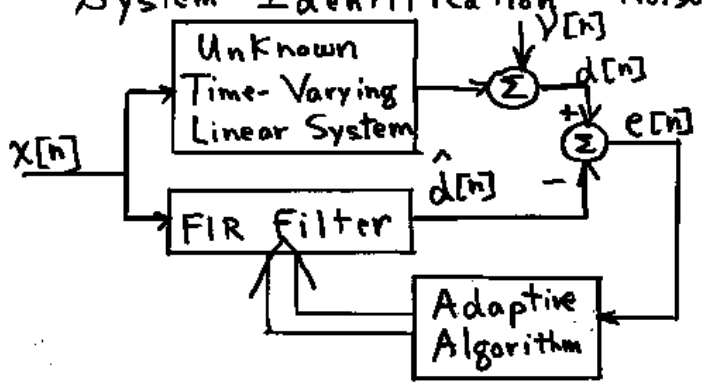
- Intro. to Adaptive Filtering
  - System Identification Application
  - Minimum Mean Square Error (MMSE) Criterion
- Main Reference: Chap. 12 of 1st Edition of Reference Text by Proakis & Manolakis

- Other references:

Advanced Digital Signal Processing, Authors: Proakis, Rader, Ling, and Nikias. Publisher: Macmillan. 1992. ISBN: 0-02-396841-9 Chap. 6 Sects. 6.1 and 6.2

Adaptive Filter Theory, Author: Simon Haykin, Publisher: Prentice-Hall. Chaps. 1, 3, and 5

System Identification



- Given $d[n]$ and $x[n]$
- determine $h[k], k = 0, 1, \dots, M-1$ to minimize

$$E\{e^2[n]\} = E\{(d[n] - \hat{d}[n])^2\}$$

$$= E\left\{\left(d[n] - \sum_{k=0}^{M-1} h[k]\, x[n-k]\right)^2\right\}$$

- vector notation:

$$\underline{h}_M = [h[0], h[1], \dots, h[M-1]]^T$$

$$\underline{x}[n] = [x[n], x[n-1], \dots, x[n-(M-1)]]^T$$

- $\hat{d}[n] = \underline{h}_M^T\,\underline{x}[n] = \underline{x}^T[n]\,\underline{h}_M$

$$= \sum_{k=0}^{M-1} h[k]\,x[n-k]$$

- $E\{e^2[n]\} = E\left\{\left(d[n] - \underline{h}_M^T\,\underline{x}[n]\right)^2\right\}$

$$= E\{d^2[n]\} - 2\,\underline{h}_M^T E\{d[n]\,\underline{x}[n]\} + \underline{h}_M^T\, E\{\underline{x}[n]\,\underline{x}^T[n]\}\,\underline{h}_M$$

$$= E\{d^2[n]\} - 2\,\underline{h}_M^T\,\underline{r}_{dx} + \underline{h}_M^T\,\underline{R}_{xx}\,\underline{h}_M$$

$$\underline{r}_{dx} = E\{d[n]\,\underline{x}[n]\} = \begin{bmatrix} E\{d[n]\,x[n]\} \\ E\{d[n]\,x[n-1]\} \\ \vdots \\ E\{d[n]\,x[n-(M-1)]\} \end{bmatrix}$$

$M \times 1$ vector

$$\underline{R}_{xx} = \begin{bmatrix} r_{xx}[0] & r_{xx}[1] & \cdots & r_{xx}[M-1] \\ r_{xx}[1] & r_{xx}[0] & \cdots & r_{xx}[M-2] \\ r_{xx}[2] & r_{xx}[1] & \cdots & r_{xx}[M-3] \\ \vdots & \vdots & & \vdots \\ r_{xx}[M-1] & r_{xx}[M-2] & r_{xx}[1] & r_{xx}[0] \end{bmatrix}$$

Toeplitz-Symmetric $M \times M$ matrix

- where: $r_{xx}[\ell] = E\{x[n]\,x[n-\ell]\}$

autocorrelation sequence for $x[n]$ assuming stationarity

- recall: completing the square of a quadratic polynomial

$$f(x) = ax^2 + bx + c \qquad (a>0)$$

$$= ax^2 + bx + \left(\frac{b}{2\sqrt{a}}\right)^2 + c - \left(\frac{b}{2\sqrt{a}}\right)^2$$

$$= a\left(x + \frac{b}{2a}\right)^2 + c - \left(\frac{b}{2a}\right)^2$$

- since $a>0$, minimum of $f(x)$ occurs with $x = -\frac{b}{2a}$

- $E\{e^2[n]\} = E\{d^2[n]\} - 2\,\underline{h}_M^T\,\underline{r}_{dx} + \underline{h}_M^T\,\underline{R}_{xx}\,\underline{h}_M + \underline{r}_{dx}^T\,\underline{R}_{xx}^{-1}\,\underline{r}_{dx} - \underline{r}_{dx}^T\,\underline{R}_{xx}^{-1}\,\underline{r}_{dx}$

$$= E\{d^2[n]\} - \underline{r}_{dx}^T\,\underline{R}_{xx}^{-1}\,\underline{r}_{dx} + \left(\underline{R}_{xx}\underline{h}_M - \underline{r}_{dx}\right)^T \underline{R}_{xx}^{-1} \left(\underline{R}_{xx}\underline{h}_M - \underline{r}_{dx}\right)$$

Minimized when: $\underline{h}_M = R_{xx}^{-1}\,\underline{r}_{dx}$