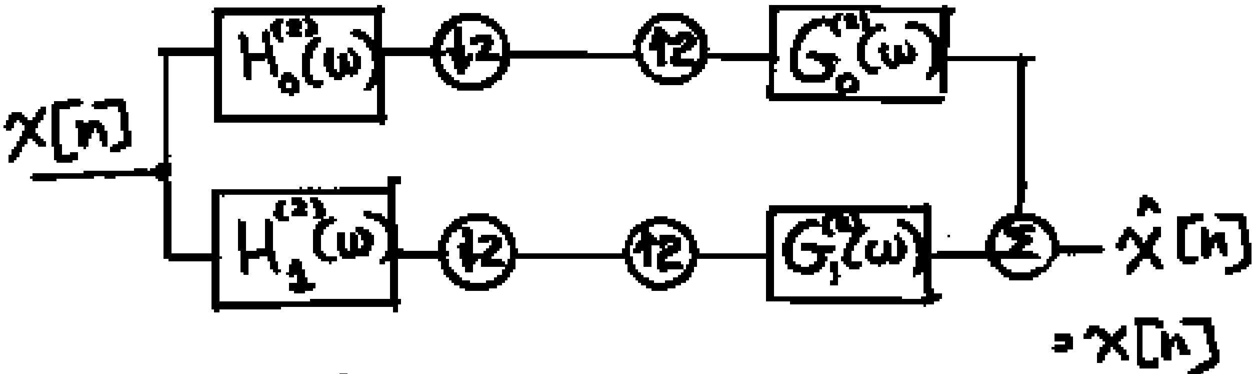
EE648 (CC761-M) DSP II

Session 21 (hive: 3/30/99)

Outline:

- Filters having raised cosine spectrum satisfying power symmetry property
  - Sect. 5.3 of V.
- Construction / Synthesis / Design of M-channel uniform Filter Banks
  - Sect. 5.4 of V. from Tree-Structured Filter Banks – Sect. 5.8

- Recap of Ideal 2-channel QMF Bank:
  - superscript (2) signifies 2 channels



- design $h[n]$ - symmetric (linear phase) satisfying:

$$|H(\omega)|^2 + |H(\omega-\pi)|^2 = 1 \quad \forall \omega$$

$$h_0^{(2)}[n] = h[n] \quad ; \quad g_0^{(2)}[n] = h[n]$$

$$h_1^{(2)}[n] = (-1)^n h[n] \quad ; \quad g_1^{(2)}[n] = -(-1)^n h[n]$$

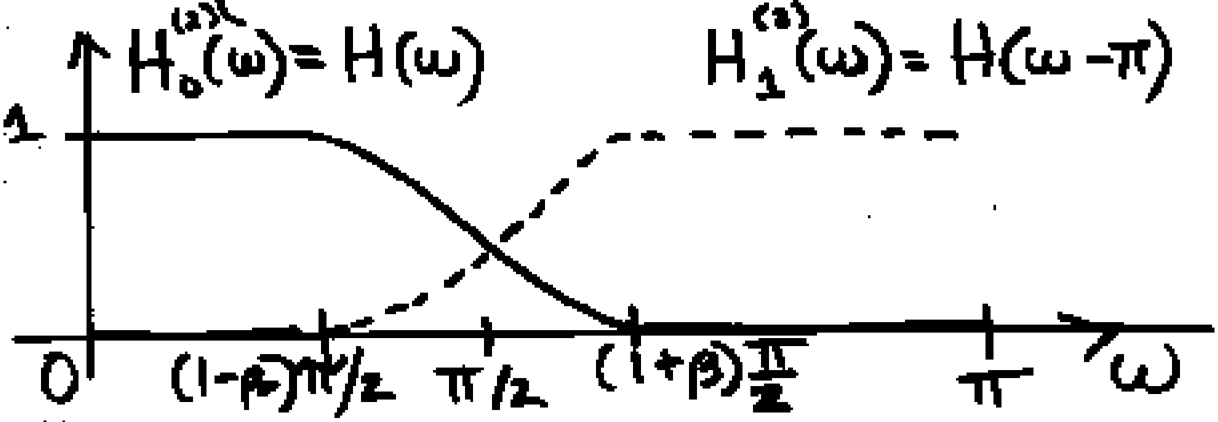
- Class of filters satisfying power symmetry constraint:

$$|H(\omega)|^2 + |H(\omega-\pi)|^2 = 1$$

- square-root raised cosine spectrum with impulse response

$$h[n] = \frac{2\beta \cos\left[(1+\beta)\pi\frac{(n+.5)}{2}\right]}{\pi\left(1-4\beta^2(n+.5)^2\right)} + \frac{\sin\left[(1-\beta)\pi\frac{(n+.5)}{2}\right]}{\pi(n+.5)\left(1-4\beta^2(n+.5)^2\right)}$$

$$H(\omega)=\begin{cases}1, & |\omega|<(1-\beta)\frac{\pi}{2}\\ \cos\left(\frac{1}{2\beta}\left[|\omega|-(1-\beta)\frac{\pi}{2}\right]\right), & (1-\beta)\frac{\pi}{2}<|\omega|<(1+\beta)\frac{\pi}{2}\\ 0, & (1+\beta)\frac{\pi}{2}<|\omega|<\pi\end{cases}$$



• for $(1-\beta)\frac{\pi}{2} < \omega < (1+\beta)\frac{\pi}{2}$:

(<u>note</u>: $|\omega - \pi| = \pi - \omega$)

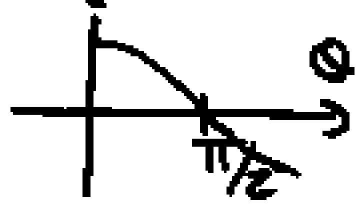
$$|H(\omega)|^2 + |H(\omega-\pi)|^2 =$$

$$= \cos^2\left(\frac{1}{2\beta}\left(\omega - (1-\beta)\frac{\pi}{2}\right)\right)$$

$$+ \cos^2\left(\frac{1}{2\beta}\left(\pi - \omega - (1-\beta)\frac{\pi}{2}\right)\right)$$

• <u>Recall:</u> $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$

$$\cos\left(\frac{\pi}{2} + \Delta\theta\right) = -\cos\left(\frac{\pi}{2} - \Delta\theta\right)$$



$$= \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\omega}{\beta} - \frac{\pi}{2\beta} + \frac{\pi}{2}\right)$$

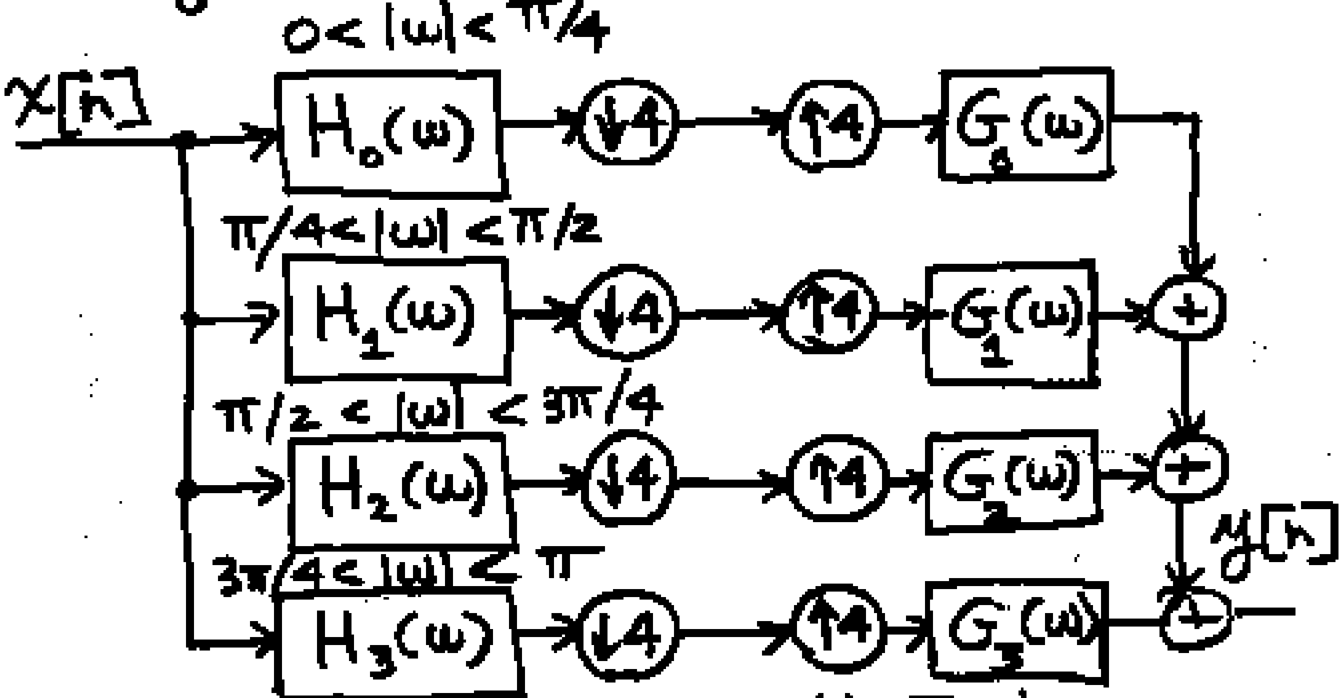
$$\frac{1}{2} + \frac{1}{2}\cos\left(-\frac{\omega}{\beta} + \frac{\pi}{2\beta} + \frac{\pi}{2}\right)$$

(<u>equate</u>: $\Delta\theta = \frac{\omega}{\beta} - \frac{\pi}{2\beta}$)

$= 1$ for all ω

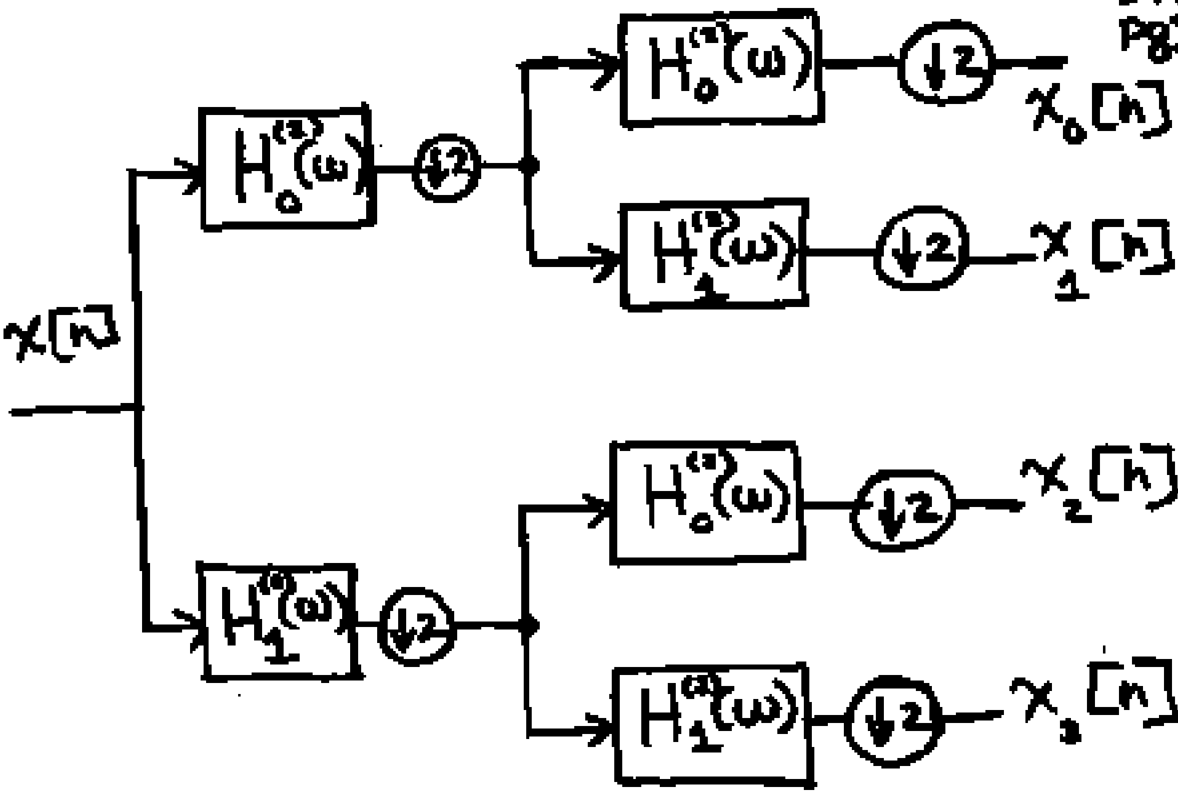
- See demo PRRCeg.m at course web site

- M- channel PR Filter Bank
- e.g. : M = 4



- See Fig. 5.8-1 in V. Text

• Tree-Structured Filter Bank - Fig. 5.8-1 Pg. 255 ✓



- to prove equivalence between tree-structured filter bank and M-channel filter bank (where M is a power of 2), need to use Noble's Identities

I. $x[n] \rightarrow (\downarrow 2) \rightarrow \boxed{H(\omega)} \rightarrow y[n]$

same I/O with :

$x[n] \rightarrow \boxed{H(2\omega)} \rightarrow (\downarrow 2) \rightarrow y[n]$

• Proof: $x[n] \rightarrow (\downarrow 2) \rightarrow \boxed{H(\omega)} \rightarrow y[n]$

$$Y(\omega) = H(\omega)\left\{\frac{1}{2}X\left(\frac{\omega}{2}\right) + \frac{1}{2}X\left(\frac{\omega-2\pi}{2}\right)\right\}$$

$$= \frac{1}{2}H(\omega)X\left(\frac{\omega}{2}\right) + \frac{1}{2}H(\omega)X\left(\frac{\omega}{2}-\pi\right)$$

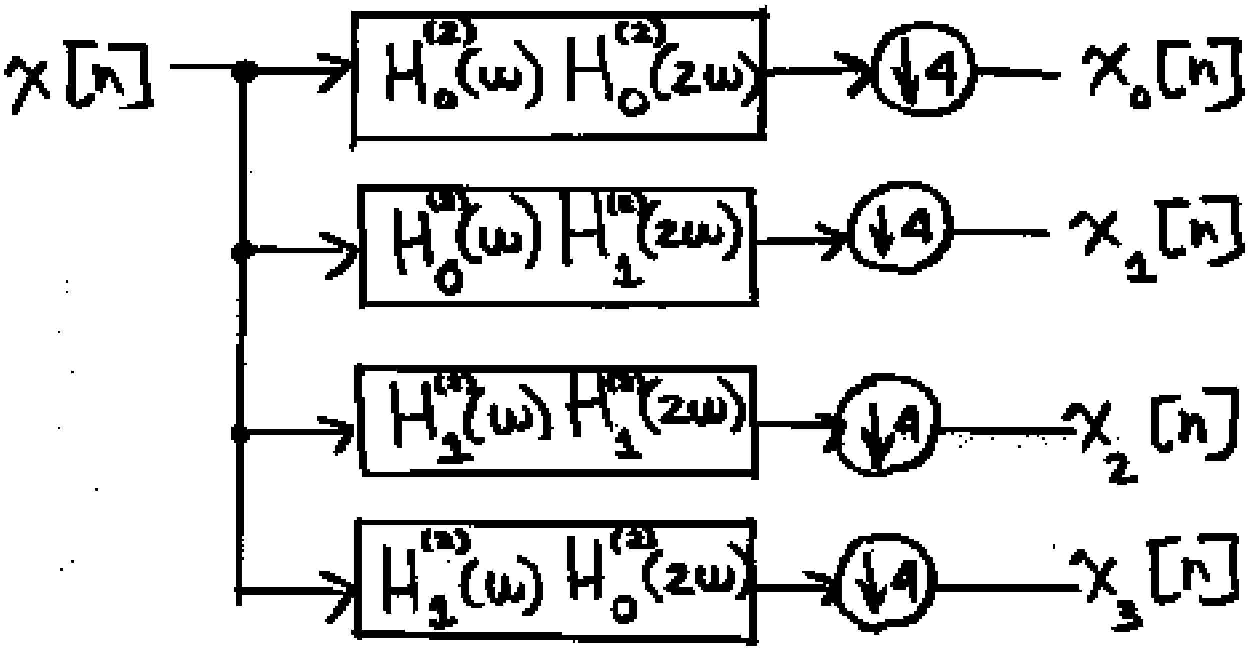
$x[n] \rightarrow \boxed{H(2\omega)} \xrightarrow{z[n]} (\downarrow 2) \rightarrow y[n]$

$$Y(\omega) = \frac{1}{2}Z\left(\frac{\omega}{2}\right) + \frac{1}{2}Z\left(\frac{\omega-2\pi}{2}\right)$$

• where: $Z(\omega) = H(2\omega)X(\omega)$

$$Y(\omega) = \frac{1}{2}H(\omega)X\left(\frac{\omega}{2}\right) + \frac{1}{2}\underbrace{H(\omega-2\pi)}_{H(\omega)}X\left(\frac{\omega-2\pi}{2}\right)$$

• thus, the analysis portion of the tree-structured filter bank



- What is the impulse response associated with the DTFT

$$H_1(\omega) = H_0^{(2)}(\omega)\, H_1^{(2)}(\omega) \quad ?$$

- Answer:

$$h_1[n] = h_0^{(2)}[n] * z_1[n]$$

  - where: $z_1[n] = \begin{cases} h_1^{(2)}\left[\frac{n}{2}\right], & n \text{ even} \\ 0 & , n \text{ odd} \end{cases}$

- What frequency band is passed by

$$H_2(\omega) = H_0^{(2)}(\omega)\, H_1^{(2)}(2\omega) \quad ?$$

- Recall: superscript (2) denotes 2-channel QMF-PR filter bank

• If $H_0^{(2)}(\omega)$ is an Ideal LPF