EE648 (CC761-M) DSP II

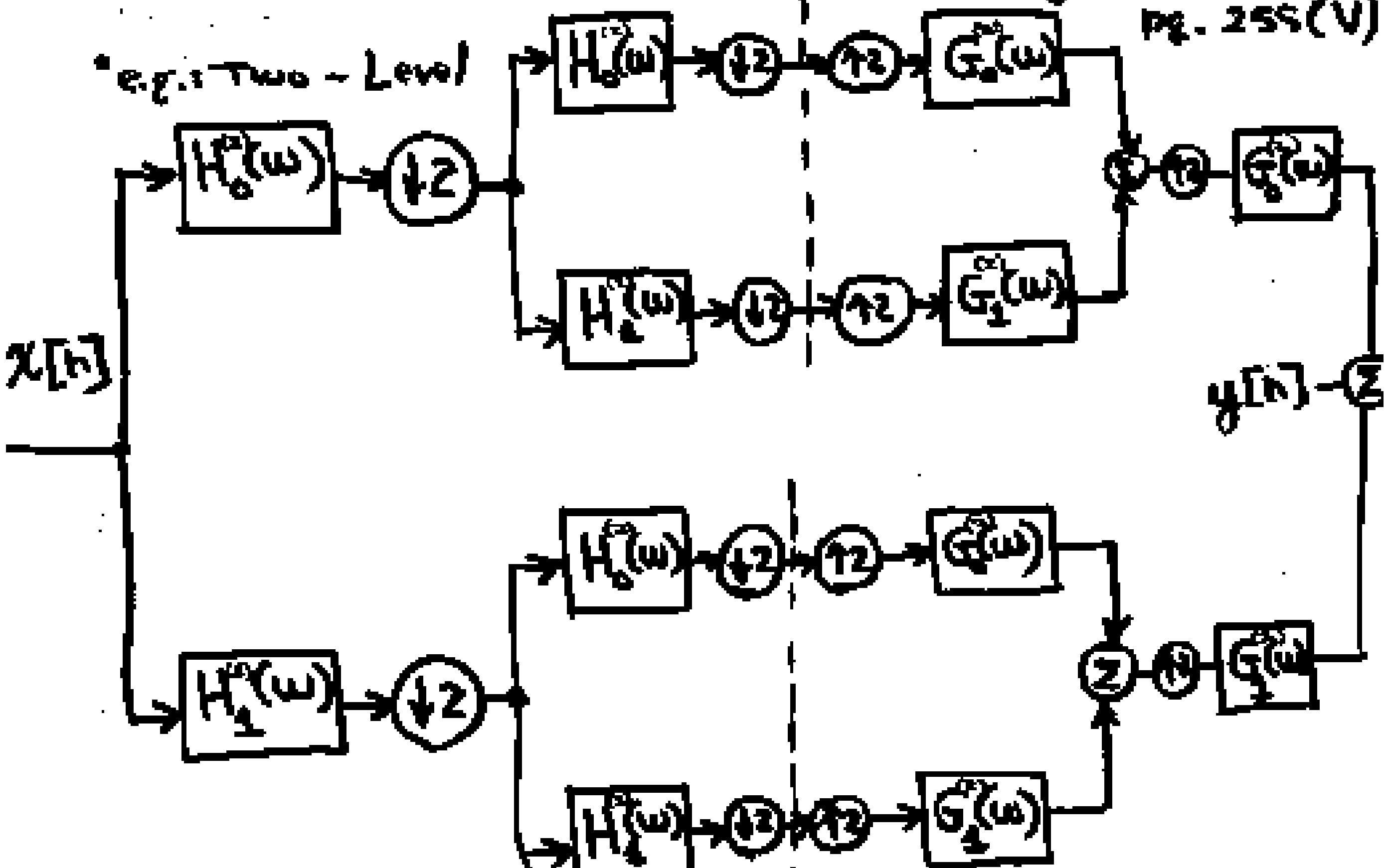
Session 22 (Live: 4/1/99)

Outline

- Synthesis of M-channel uniform filter banks from tree-structured filter banks – Sect. 5.8 of Y.
- Analysis of M-channel PR filter banks – Sect. 5.4 of Y.

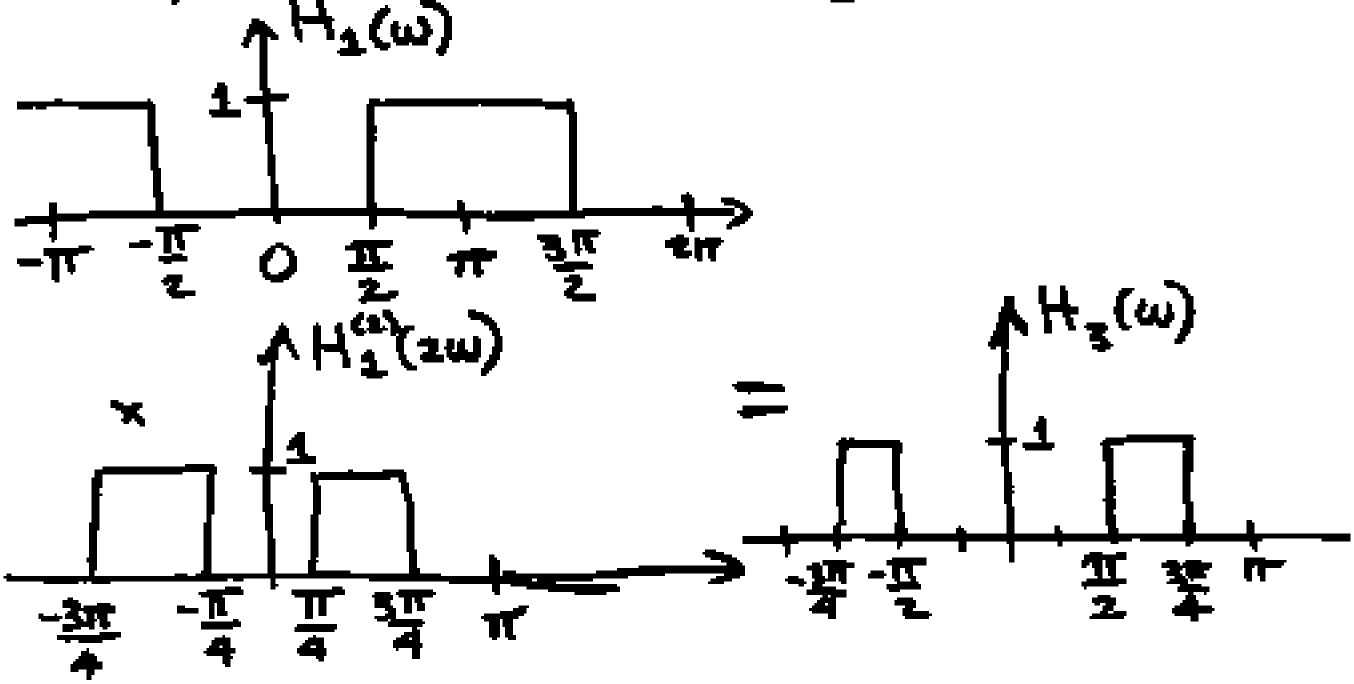
- Tree-Structured Filter Bank - Fig. 5.8-1 on pg. 255 (V)

- e.g.: Two-Level

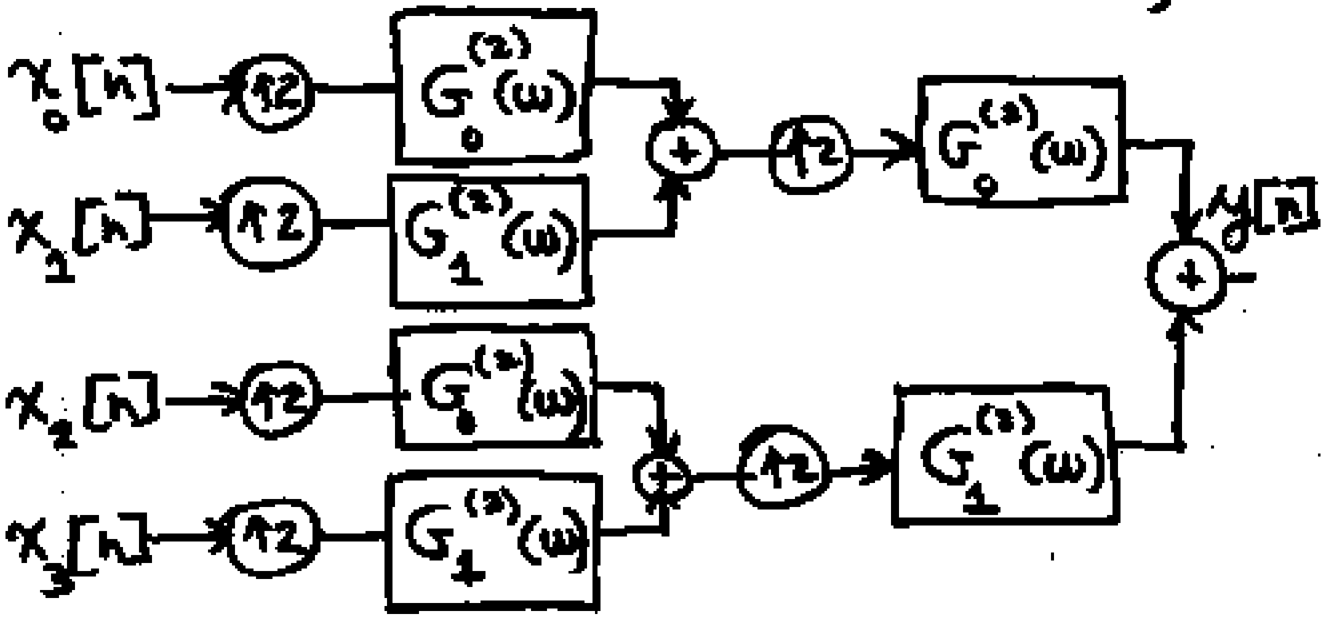


- as long as $\{H_0(\omega), H_1(\omega), G_0(\omega), G_1(\omega)\}$ form ideal QMF, over-all system is PR

• What frequency band is passed by $H_3(\omega) = H_1^{(a)}(\omega)\, H_1^{(a)}(2\omega)$?



• Synthesis section of tree-structured filter bank: e.g., $M=2^2$



- to show equivalence to $M=4$ channel filter bank, use Noble's second identity

II. $x[n] \rightarrow \boxed{H(\omega)} \rightarrow (\uparrow 2) \rightarrow y[n]$ (A)

- same I/O relationship with

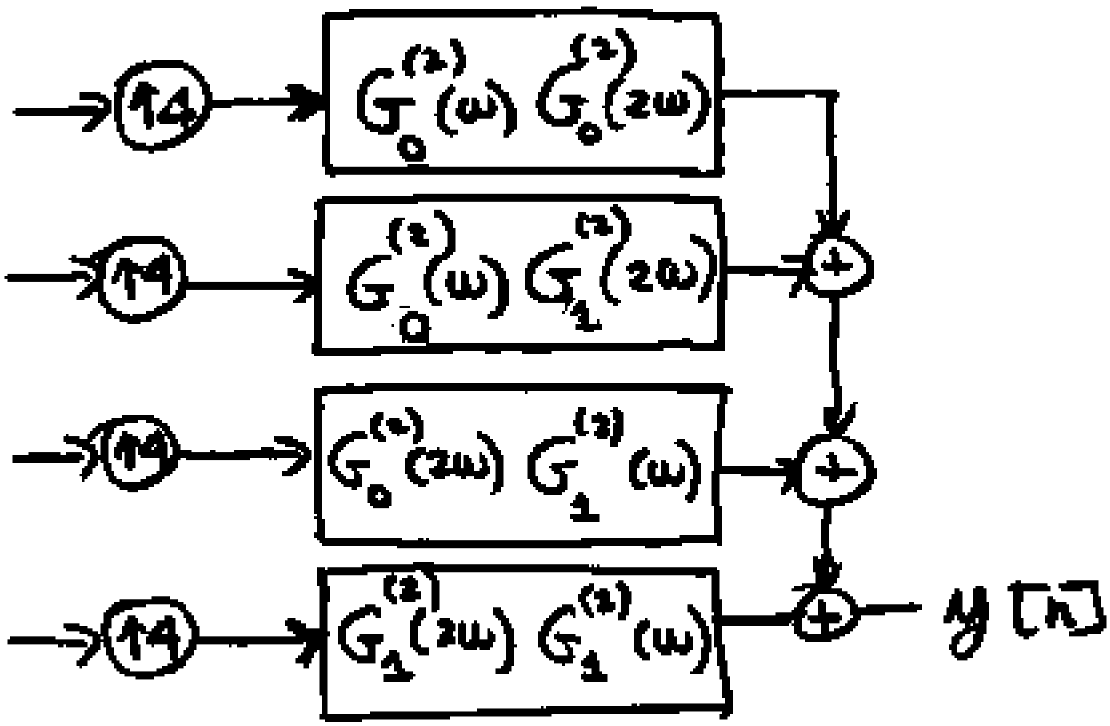
$x[n] \rightarrow (\uparrow 2) \rightarrow \boxed{H(2\omega)} \rightarrow y[n]$ (B)

(A) $Y(\omega) = H(2\omega)\, X(2\omega)$

(B) $Y(\omega) = H(2\omega)\, X(2\omega)$

SAME I/O

• equivalent $M=4$ uniform filter bank:



- See PR4chan.m (based on $h[n] = \{1, 1\}$) and PR RC4chan.m (based on $h[n]$ having a raised cosine spectrum) at course web site

• Analysis of M-channel filter bank

$$X_m(\omega) = \sum_{\ell=0}^{M-1} H_m\left(\frac{\omega - \ell 2\pi}{M}\right) X\left(\frac{\omega - \ell 2\pi}{M}\right)$$

$$W_m(\omega) = X_m(M\omega)$$
$$= \sum_{\ell=0}^{M-1} H_m\left(\omega - \ell \frac{2\pi}{M}\right) X\left(\omega - \ell \frac{2\pi}{M}\right)$$

$$Y(\omega) = \sum_{m=0}^{M-1} G_m(\omega) W_m(\omega)$$
$$= \sum_{m=0}^{M-1} G_m(\omega) \sum_{\ell=0}^{M-1} H_m\left(\omega - \ell \frac{2\pi}{M}\right) X\left(\omega - \ell \frac{2\pi}{M}\right)$$

$$Y(\omega) = \sum_{l=0}^{M-1} X\left(\omega - l\frac{2\pi}{M}\right) \sum_{m=0}^{M-1} G_m(\omega) H_m\left(\omega - l\frac{2\pi}{M}\right)$$

· define: $$F_l(\omega) = \sum_{m=0}^{M-1} G_m(\omega) H_m\left(\omega - l\frac{2\pi}{M}\right)$$

$$Y(\omega) = F_0(\omega) X(\omega) + \sum_{l=1}^{M-1} F_l(\omega) X\left(\omega - l\frac{2\pi}{M}\right)$$

• desire: $\left.\begin{array}{l} |F_0(\omega)| = \text{constant} \\ \angle F_0(\omega) - \text{linear} \end{array}\right\} \forall \omega$

$F_\ell(\omega) = 0 \quad \forall \ \omega$

for $\ell = 1, \dots, M-1$