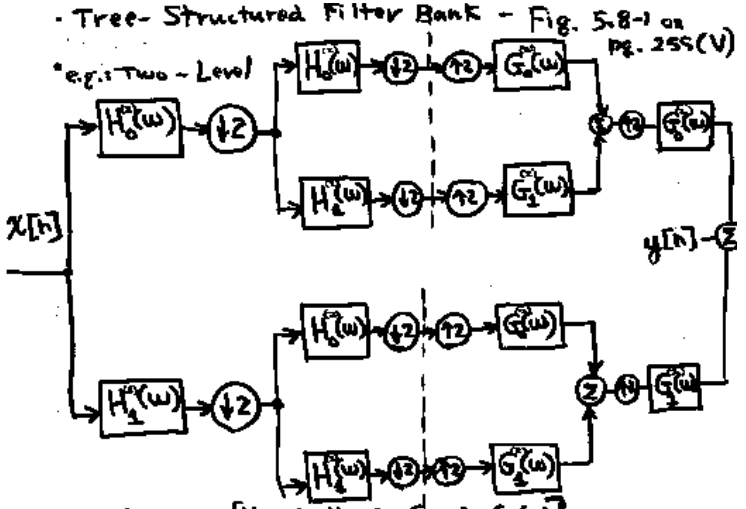
EE648 (CC761-M) DSP II

Session 22 (hive: 4/1/99)

Outline

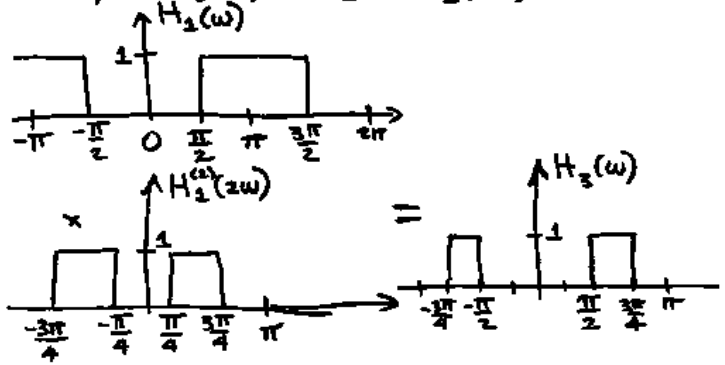
- Synthesis of M-channel uniform filter banks from tree-structured filter banks - Sect. 5.8 of V.
- Analysis of M-channel PR filter banks - Sect. 5.4 of V.

- Tree-Structured Filter Bank - Fig. 5.8-1 on pg. 255(V)

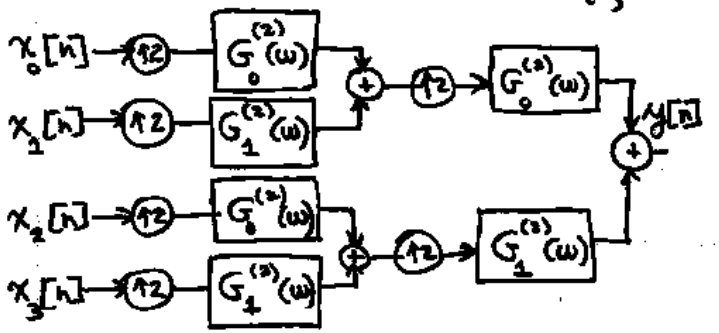


- as long as $\{H_0(\omega), H_1(\omega), G_0(\omega), G_1(\omega)\}$ form ideal QMF, over-all system is PR

- What frequency band is passed by $H_3(\omega) = H_1^{(1)}(\omega) H_1^{(1)}(2\omega)$?



- Synthesis section of tree-structured filter bank = e.g., $M = 2^2$



• to show equivalence to M=4 channel filter bank, use Noble's second identity

II. $x[n] \rightarrow \boxed{H(\omega)} \rightarrow (\uparrow 2) \rightarrow y[n]$ (A)

• same I/O relationship with

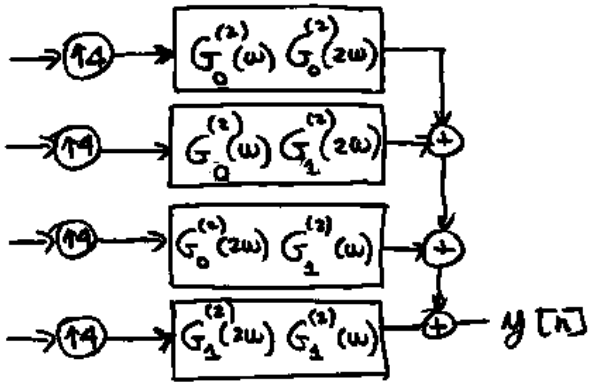
$x[n] \rightarrow (\uparrow 2) \rightarrow \boxed{H(2\omega)} \rightarrow y[n]$ (B)

(A) $Y(\omega) = H(2\omega)\,X(2\omega)$

(B) $Y(\omega) = H(2\omega)\,X(2\omega)$

$\Big\}$ SAME I/O

• equivalent M=4 uniform filter bank:



• See PR 4 chan. m (based on $h[n] = \{1,1\}$) and PR RC 4 chan. m (based on $h[n]$ having a raised cosine spectrum) at course web site

• Analysis of M-channel filter bank

$$X_m(\omega) = \sum_{\ell=0}^{M-1} H_m\left(\frac{\omega - \ell 2\pi}{M}\right) X\left(\frac{\omega - \ell 2\pi}{M}\right)$$

$$W_m(\omega) = X_m(M\omega) = \sum_{\ell=0}^{M-1} H_m(\omega - \ell 2\pi) X(\omega - \ell 2\pi)$$

$$Y(\omega) = \sum_{m=0}^{M-1} G_m(\omega) W_m(\omega)$$

$$= \sum_{m=0}^{M-1} G_m(\omega) \sum_{\ell=0}^{M-1} H_m(\omega - \ell 2\pi) X(\omega - \ell 2\pi)$$

$$Y(\omega)=\sum_{\ell=0}^{M-1} X\left(\omega-\ell\tfrac{2\pi}{M}\right)\sum_{m=0}^{M-1} G_m(\omega)H_m\left(\omega-\ell\tfrac{2\pi}{M}\right)$$

· define: $F_\ell(\omega)=\sum_{m=0}^{M-1} G_m(\omega)H_m\left(\omega-\ell\tfrac{2\pi}{M}\right)$

$$Y(\omega)=F_0(\omega)X(\omega)+\sum_{\ell=1}^{M-1} F_\ell(\omega)X\left(\omega-\ell\tfrac{2\pi}{M}\right)$$

· desire: $|F_0(\omega)|=$ constant
$\angle F_0(\omega)$ - linear $\geq \forall\omega$

$F_\ell(\omega)=0 \quad \forall\omega$
for $\ell=1,\ldots,M-1$