EE648 (CC761-M) DSP II

Session 27 (Live: 4/20/99)

Outline

- Continuous-Time Wavelets
  - Sect 11.3 of V. Text

- Two primary features of Continuous-Time Wavelets

- Feature 1. Nonuniforms filters banks
  - instead of achieving different center freq's. via modulation of a lowpass filter =>
    alternatively achieved frequency scaling of a bandpass filter

- Consider: $h_k(t) = a^{-k/2} h(a^{-k} t)$

k, integer

$a > 1$ (e.g., $a=2$)

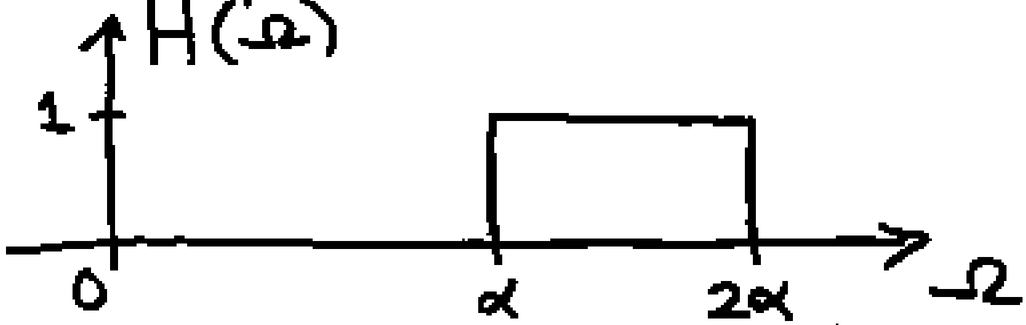
- recall: $f(at) \overset{CTFT}{\longleftrightarrow} \frac{1}{|a|} F\left(\frac{\Omega}{a}\right)$

$\Omega = 2\pi f$ where f is in Hz

- thus: $H_k(\Omega) = a^{-\frac{k}{2}} \frac{1}{a^{-k}} H\left(\frac{\Omega}{a^{-k}}\right)$

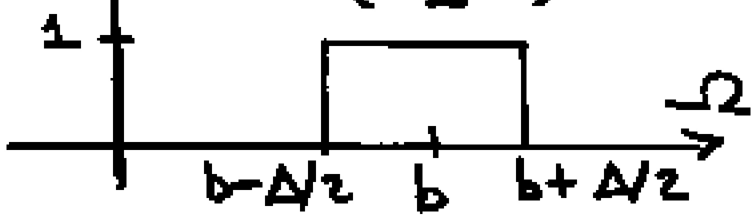
$= a^{k/2} H(a^k \Omega)$

- Example. $a=2$



$$H(\Omega) = \mathrm{rect}\left(\frac{\Omega - \frac{3\alpha}{2}}{\alpha}\right)$$

- where: $\mathrm{rect}\left(\frac{x-b}{\Delta}\right)$

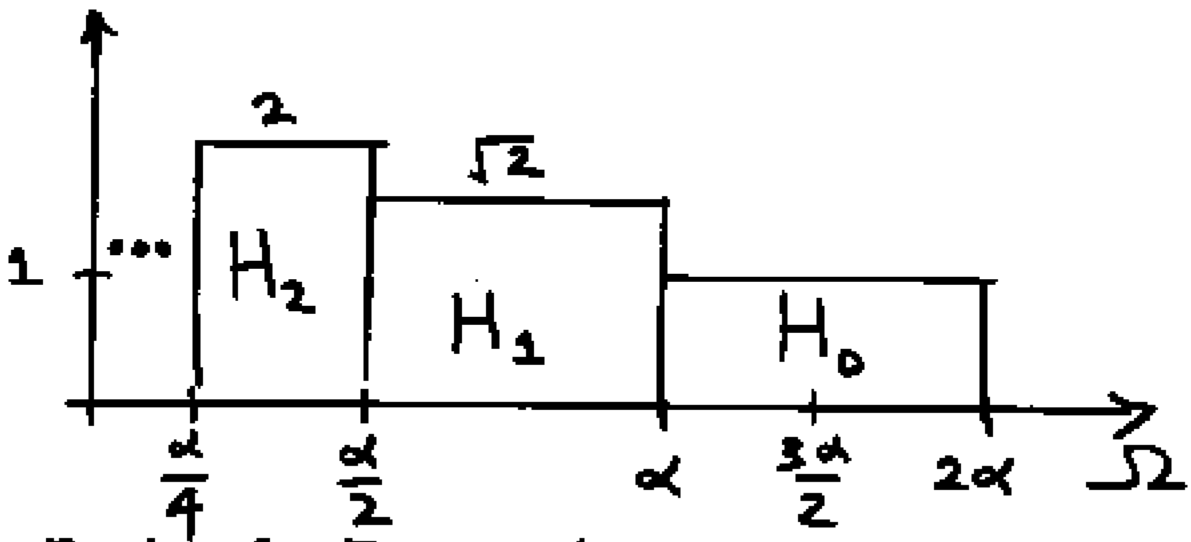


$$H_k(\Omega) = a^{k/2}\,\mathrm{rect}\left(\frac{a^k\Omega - \frac{3\alpha}{2}}{\alpha}\right)$$

$$= a^{k/2}\,\mathrm{rect}\left(\frac{\Omega - a^{-k}\frac{3\alpha}{2}}{a^{-k}\alpha}\right)$$

• consider $a=2$:

$$H_k(\Omega) = 2^{\frac{k}{2}}\,\mathrm{rect}\left(\frac{\Omega - \left(\frac{1}{2}\right)^k\frac{3\alpha}{2}}{\left(\frac{1}{2}\right)^k\alpha}\right)$$



- End of Example
- $H(\Omega)$ has to be bandpass
- note: amplitude scaling by $a^{-k/2}$ insures $\int_{-\infty}^{\infty} |h_k(t)|^2 dt$ indep of k

- Feature 2. Nonuniform sampling
- Since bandwidth of $H_k(\Omega)$ is smaller for large k $\Rightarrow$ can sample at correspondingly lower rate
- in time domain, width of $h_k(t)$ is larger for increasing k $\Rightarrow$ so we can "slide" $h_k(t)$ by correspondingly larger step size

• k-th analysis filter output (as a function of τ)

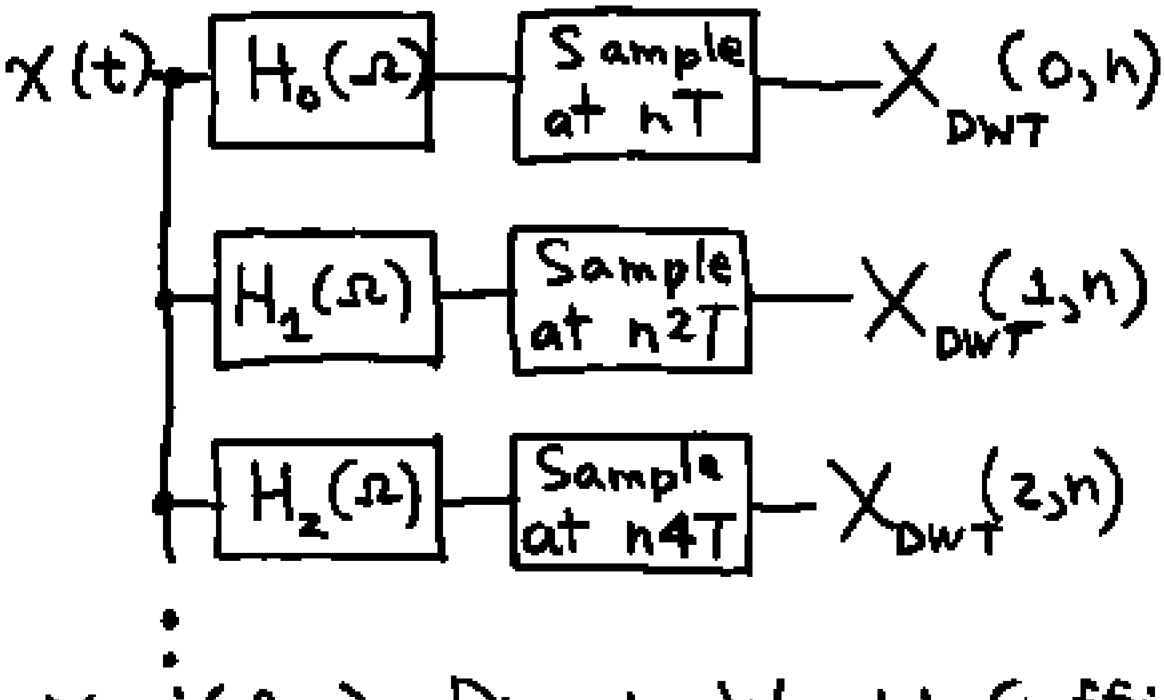
$$x_k(\tau) = \underbrace{a^{-k/2}}\int_{-\infty}^{\infty} x(t)\, h\left(a^{-k}(\tau - t)\right) dt$$

• sampling at $\tau = n a^{k} T$

$$X_{DWT}(k,n) = \int_{-\infty}^{\infty} x(t)\, h_k\left(n a^{k} T - t\right) dt$$

$$= a^{-k/2}\int_{-\infty}^{\infty} x(t)\, h\left(nT - a^{-k} t\right) dt$$

since $h_k(t) = a^{-\frac{k}{2}} h\left(a^{-k} t\right)$



$X_{DWT}(k,n)$: Discrete Wavelet Coefficients

See Fig. 11.3-3 in V. Text

- General Def'n of Wavelet Transform
  - Continuous Wavelet Transform

$$X_{CWT}(p,q) = \frac{1}{\sqrt{|p|}} \int_{-\infty}^{\infty} x(t)\, g\left(\frac{t-q}{p}\right) dt$$

  - p, q are continuous, real-valued
- This reduces to $X_{DWT}(k,n)$ with

$$\underbrace{p = a^k \;;\; q = n a^k T}_{\text{evaluate CWT at discrete points in 2-D plane}} \;;\; g(t) = h(-t)$$

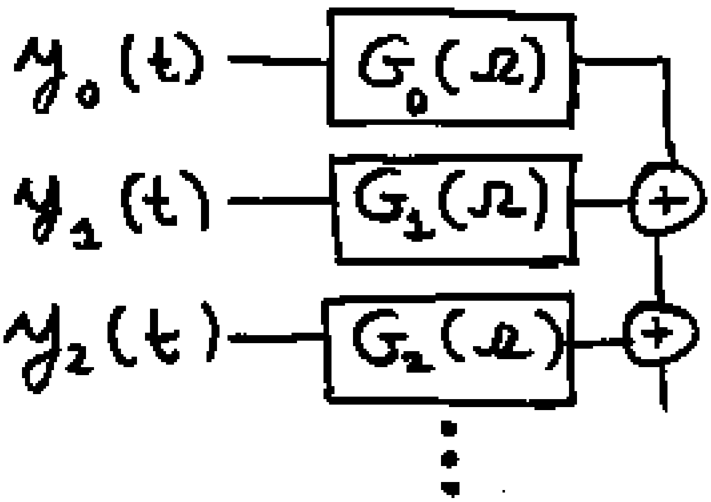
- CWT: maps $x(t)$ into 2-D fn. $X_{CWT}(p,q)$
- DWT: maps $x(t)$ into 2-D <u>sequence</u> $X_{DWT}(k,n)$
- Inverse Discrete Wavelet Transform

$$x(t) = \sum_{k} \sum_{n} X_{DWT}(k,n) \underbrace{\psi_{kn}(t)}_{\text{basis fns.}}$$

- filter bank interpretation of reconstructing $x(t)$ from $X_{DWT}(k,n)$

- must view input to each synthesis filter as

$$y_k(t) = \sum_n X_{DWT}(k,n)\,\delta_a(t - n a^k T)$$



$$\hat{x}(t) = \sum_{k} \sum_{n} X_{DWT}(k,n)\, g_{k}(t - na^{k}T)$$

- analysis filters derived from single filter $\Rightarrow$ same true for synthesis filters

$$g_k(t) = a^{-\frac{k}{2}}\, g(a^{-k}t)$$

$$\hat{x}(t) = \sum_{k} \sum_{n} X_{DWT}(k,n)\, \underbrace{a^{-\frac{k}{2}} g\left(a^{-k}(t - na^{k}T)\right)}_{\psi_{kn}(t)}$$