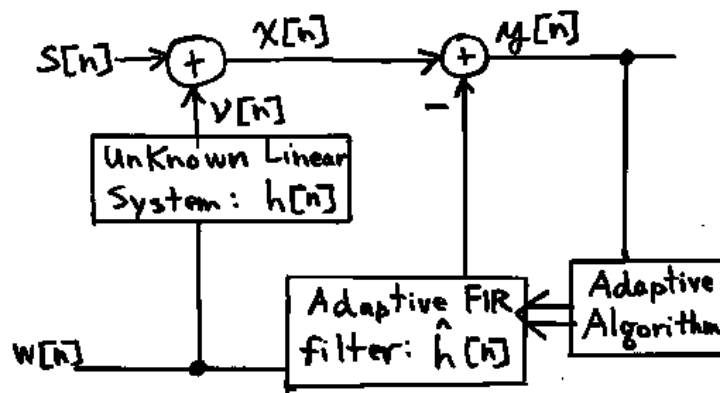


EE648 (CC761-M) DSP II

Session 5 (hive: 1/26/99)

Outline:

- Adaptive Noise Cancellation
- Adaptive Suppression of Narrowband Interference in a Wideband Signal
- Sect. 12.1 of 1st Ed. of P+M



- Application: Adaptive Noise Cancellation
- specifically: car phone embedded in steering wheel
- speech masked by "car noise"
- approach: place "mike" at point in car to pick up "car noise" only
- form an estimate of noise contaminating speech and subtract it off

- $S[n]$: Speech only (unobservable)
- $W[n]$: noise-only observed at "remote" sensor ("mike")
- $V[n]$: filtered version of $w[n]$ that corrupts desired speech (unobservable)

$$V[n] = W[n] * h[n]$$
- $X[n] = S[n] + V[n] \Rightarrow$ picked up by "mike" in steering wheel

• Choose $\hat{h}[n]$ to minimize

$$E\{y^2[n]\} = E\left\{\left[x[n] - \sum_{k=0}^{M-1} \hat{h}[n] w[n-k]\right]^2\right\}$$

$$= E\left\{\left[x[n] - \hat{\mathbf{h}}_M^T \underline{\mathbf{w}}[n]\right]^2\right\}$$

• where:

$$\hat{\mathbf{h}}_M = [\hat{h}[0], \hat{h}[1], \dots, \hat{h}[M-1]]^T$$

$$\underline{\mathbf{w}}[n] = [w[n], w[n-1], \dots, w[n-(M-1)]]^T$$

$$= E\left\{\left[s[n] + (\mathbf{h} - \hat{\mathbf{h}}_M)^T \underline{\mathbf{w}}[n]\right]^2\right\}$$

$$= E\{s^2[n]\} + 2(\mathbf{h} - \hat{\mathbf{h}}_M)^T E\{s[n] \underline{\mathbf{w}}[n]\} + (\mathbf{h} - \hat{\mathbf{h}}_M)^T E\{\underline{\mathbf{w}}[n] \underline{\mathbf{w}}[n]^T\} (\mathbf{h} - \hat{\mathbf{h}}_M)$$

$$= E\{s^2[n]\} + (\mathbf{h} - \hat{\mathbf{h}}_M)^T \mathbf{R}_{ww} (\mathbf{h} - \hat{\mathbf{h}}_M)$$

• thus: $\hat{\mathbf{h}}_M^{\text{opt}} = \mathbf{h}$ since $\mathbf{R}_{ww} = E\{\underline{\mathbf{w}}[n] \underline{\mathbf{w}}[n]^T\}$ is positive definite

• Show: $\hat{\mathbf{h}}_M^{\text{opt}} = \mathbf{h}$

• Recall: $x[n] = s[n] + v[n]$

$$= s[n] + w[n] * h[n]$$

$$= s[n] + \mathbf{h}^T \underline{\mathbf{w}}[n]$$

• Note: Assume $s[n]$ and $w[n]$ are independent random processes and $E\{w[n]\} = 0$

$$E\left\{\left[s[n] + \mathbf{h}^T \underline{\mathbf{w}}[n] - \hat{\mathbf{h}}_M^T \underline{\mathbf{w}}[n]\right]^2\right\}$$

• in implementing either LMS or RLS;

• $x[n]$: plays role of desired signal $d[n]$

• $y[n]$: plays role of error signal $e[n]$

• See summary of LMS/RLS for Adaptive Noise Cancellation delineated in Hmwk. 1 write-up

• Summary of LMS/RLS for Adaptive Noise Cancellation

• for RLS (with $w = \gamma$)

$$y[n, n-1] = x[n] - \underline{h}_M^T[n-1] \underline{w}[n]$$

$$\underline{f}[n] = \underline{R}_{ww}^{-1}[n-1] \underline{w}[n]$$

$$\mu[n] = \gamma + \underline{w}^T[n] \underline{f}[n]$$

$$\underline{K}_M[n] = \underline{f}[n] / \mu[n]$$

$$\underline{h}_M[n] = \underline{h}_M[n-1] + y[n, n-1] \underline{K}_M[n]$$

$$\underline{R}_{ww}^{-1}[n] = \frac{1}{\gamma} \left\{ \underline{R}_{ww}^{-1}[n-1] + \underline{K}_M[n] \underline{f}^T[n] \right\}$$

Go to Step 1

• for LMS: $y[n] = x[n] - \underline{h}_M^T[n] \underline{w}[n]$
 $\underline{h}_M[n+1] = \underline{h}_M[n] + \mu y[n] \underline{w}[n]$

• See demo NoiseCancel.m and WordCancel.m at course web site

• Suppression of Narrowband Interference in a wideband

Signal $x[n] = s[n] + i[n]$

information signal (wideband) + interference (narrowband)

• Assumption 1: $s[n]$ and $i[n]$ are independent random processes

• $s[n]$: "wideband" signal

• power spectral density, $S_{ss}(\omega)$ is generally non-negligible over a wide range of frequencies

• as a result, the autocorrelation $r_{ss}[m]$ is highly localized near $m=0$

• Extreme case: white noise

$$r_{ss}[m] = \sigma_w^2 \delta[m] \xleftrightarrow{\text{DTFT}} S_{ss}(\omega) = \sigma_w^2 \text{ for all } \omega$$

- assumption: $r_{ss}[m] \approx 0$ for $|m| \geq D$
where D is not too large

- In contrast:

- $i[n]$: "narrowband" signal

- $S_{ii}(\omega)$ is highly concentrated in frequency: $S_{ii}(\omega) \approx 0$ except over $\omega_0 - \Delta\omega < \omega < \omega_0 + \Delta\omega$ where $\Delta\omega$ is small

- as a result: $r_{ii}[m]$ is generally nonzero for a large range of m

- Extreme case: $i[n] = e^{j(\omega_0 n + \Theta)}$ $\neq n$
 $r_{ii}[m] = e^{j\omega_0 m} \xleftrightarrow{\text{DTFT}} \int_{ii} S_{ii}(\omega) = 2\pi \delta(\omega - \omega_0)$
for $-\pi < \omega < \pi$

- in particular: $r_{ii}[m] \neq 0$ for $D \leq m \leq D+M$

- specifically, assumption is $i[n+D]$ is highly correlated with $i[n], i[n-1], \dots, i[n-(M-1)]$

- whereas $s[n+D]$ is uncorrelated with $s[n], s[n-1], \dots, s[n-(M-1)]$

- criterion for suppressing $i[n]$:

$$\text{Min}_{h[n]} E \left\{ \left[x[n+D] - \sum_{k=0}^{M-1} h[k] x[n-k] \right]^2 \right\}$$

- thus: $x[n+D]$ plays role of "desired" signal

$$\text{Min}_{h[n]} E \left\{ \left[x[n+D] - \underline{h}_M^T \underline{x}[n] \right]^2 \right\}$$

Define: $\tilde{h}[n] = \begin{cases} 1, & n=0 \\ 0, & n=1, \dots, D-1 \\ -h[n-D], & n=D, \dots, D+M-1 \end{cases}$

- the objective may be expressed as

$$\text{Min}_{\tilde{h}[n]} E \left\{ \left[\sum_{k=0}^{D+M-1} \tilde{h}[k] x[n+D-k] \right]^2 \right\}$$

- See Demo CancelTone.m at web site