

EE648 (CC761-M) DSP II  
Session 6 (hive: 1/28/99)

Outline:

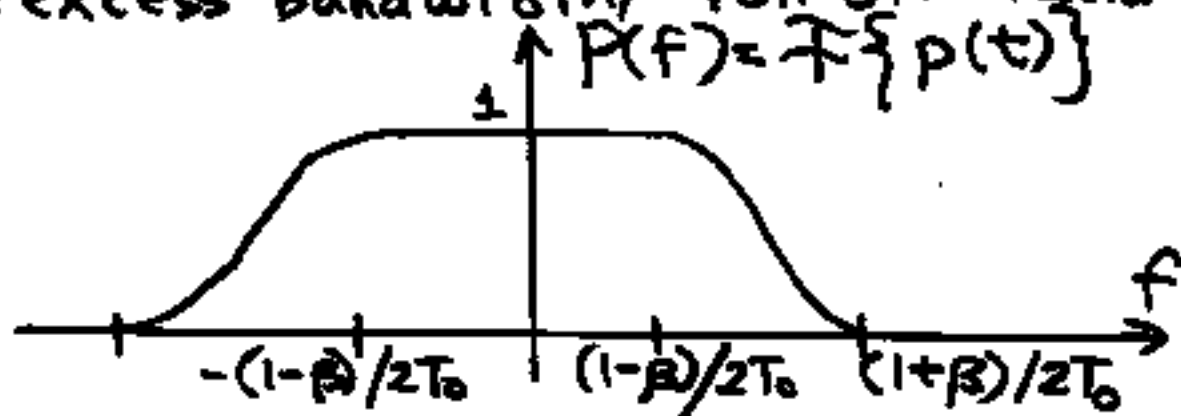
- Adaptive equalization of digital communication signals
- Sect. 12.1 of P&M 1<sup>st</sup> Ed.
  - pp. 861-864

- Overview of Digital Communications
- assume binary signalling for sake of simplicity
- message signal  $\Rightarrow$  sampled  
quantized  
binary encoded
- every  $T_0$  seconds, a pulse is transmitted - the  $k$ th pulse carries one bit of information

• typical pulse shaped used in narrowband digital communications

$$p(t) = \frac{\sin\left(\frac{\pi}{T_0} t\right)}{\pi \frac{t}{T_0}} \cdot \frac{\cos\left(2\pi\beta \frac{t}{T_0}\right)}{1 - \left(4\beta \frac{t}{T_0}\right)^2}$$

$\beta = \text{excess bandwidth / roll-off factor}$



• if  $k$ -th info. bit is a "1", transmit:

$$a[k] p(t - kT_0) \quad \text{with } a[k] = +A$$

• if  $k$ -th info. bit is a "0", transmit:

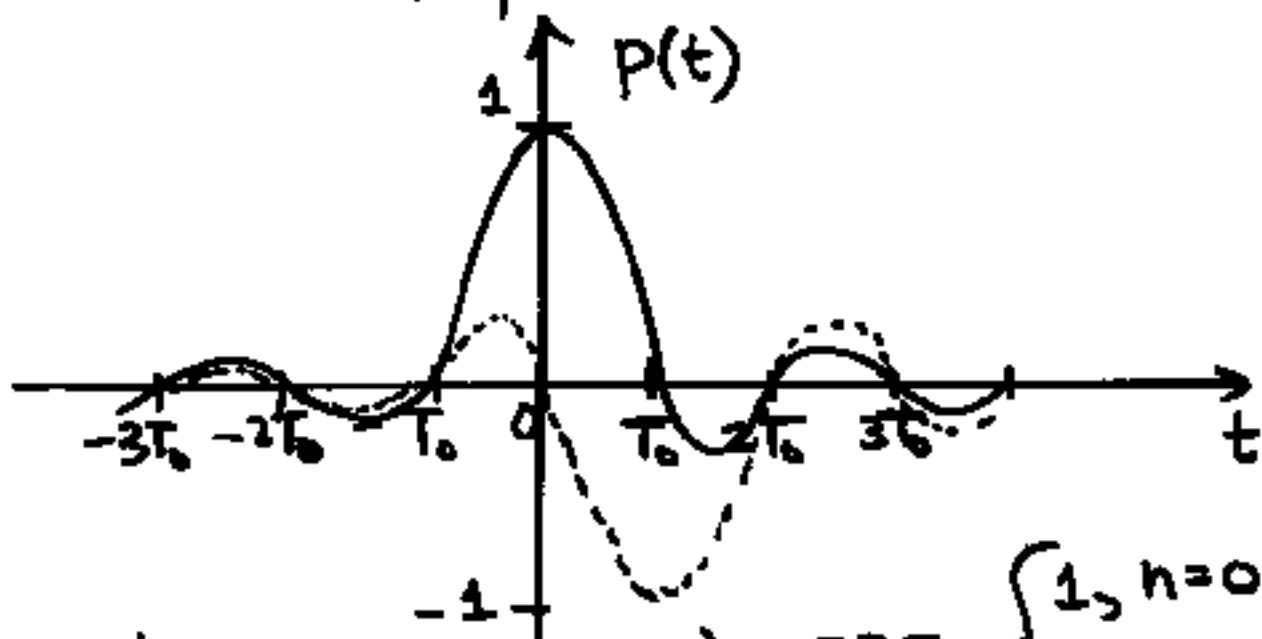
$$a[k] p(t - kT_0) \quad \text{with } a[k] = -A$$

•  $k = 0, 1, \dots, N-1$  } transmitting a burst of  $N$  bits

• transmitted signal (linear modulation)

$$s(t) = \sum_{k=0}^{N-1} a[k] p(t - kT_0)$$

- for current US TDMA cellular standard,  $\beta = 0.35$



• note:  $p[n] = p(nT_0) = \delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$

• Note: bandwidth of  $s(t)$  is  $\frac{1+\beta}{2T_0}$

• where:  $\frac{1}{T_0}$  is the bit rate

• Ideally, the Nyquist rate is

$$2 \left\{ \frac{1+\beta}{2T_0} \right\} = \frac{1+\beta}{T_0} \quad \text{where: } 0 < \beta < 1$$

• Consider sampling at bit rate  
nonetheless — sub-Nyquist sampling  
— assume synchronization

• thus:

$$\begin{aligned} s[n] &= s(nT_0) \\ &= \sum_{k=0}^{N-1} a[k] p(nT_0 - kT_0) \\ &= \sum_{k=0}^{N-1} a[k] p[n-k] \\ &= \sum_{k=0}^{N-1} a[k] f[n-k] \\ &= a[n] \end{aligned}$$

- despite :
  - pulses overlapping in time (to have as high data rate in a given bandwidth)
  - sub-Nyquist sampling
- can nonetheless recover the transmitted info. bits

- However, when multipath exists (reflections off buildings, etc.), the received signal is

$$x(t) = \sum_{l=1}^p g_l s(t - \tau_l)$$

- where:
  - $g_l$ : gain of  $l$ -th multipath
  - $\tau_l$ : delay of " "

- $x(t)$  may be alternatively expressed as

$$x(t) = s(t) * \left\{ \sum_{k=1}^P g_k \delta(t - \tau_k) \right\}$$

$$x(t) = \sum_{k=0}^{N-1} a[k] p(t - kT_0) * \left\{ \sum_{k=1}^P g_k \delta(t - \tau_k) \right\}$$

$$= \sum_{k=0}^{N-1} a[k] q(t - kT_0)$$

$$q(t) = \sum_{k=1}^P g_k p(t - \tau_k)$$

$$\begin{aligned}
& \sum_r P(t - kT_0) * \delta(t - \tau_r) g_r \\
&= \sum_r P(t) * \delta(t - kT_0) * \delta(t - \tau_r) g_r \\
&= \sum_r P(t - \tau_r) * \delta(t - kT_0) g_r \\
&= \left\{ \sum_r g_r P(t - \tau_r) \right\} * \delta(t - kT_0) \\
&= g(t) * \delta(t - kT_0) \\
&= g(t - kT_0)
\end{aligned}$$

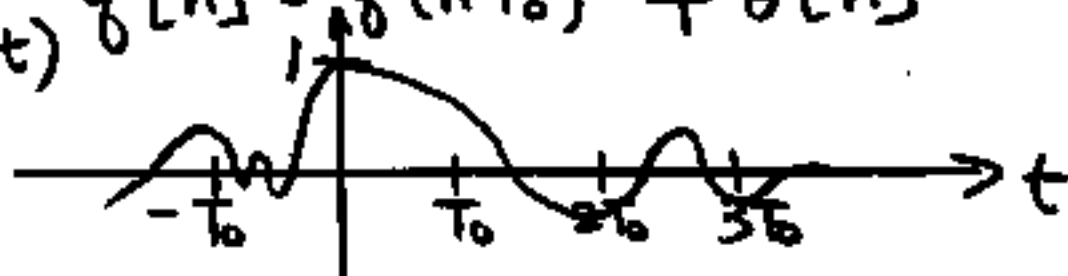
• received signal:

$$x(t) = \sum_{k=0}^{N-1} a[k] q(t - kT_0)$$

• where:  $q(t)$  is distorted pulse waveform due to multipath

$\Rightarrow$  Nyquist property is lost

$$q(t) \quad q[n] = q(nT_0) \neq \delta[n]$$



• sampling at bit rate:

$$X[n] = x(nT_0)$$

$$= \sum_{k=0}^{N-1} a[k] g(nT_0 - kT_0)$$

$$= \sum_{k=0}^{N-1} a[k] g[n-k]$$

$$= a[n] * g[n]$$

$$= \sum_{k=-M_1}^{M_2} g[k] a[n-k] = g(nT_0)$$

where:

$$g[n]$$

$$= g(nT_0)$$

- $a[n] \rightarrow \boxed{g[n]} \rightarrow x[n]$

- Equalization is about how to determine  $a[n]$  given  $x[n]$

- a Zero-Forcing (ZF) Equalizer does this by means of inverse filtering:

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] = \hat{a}[n]$$

- where:  $g[n] * h[n] = \delta[n]$

• where ideally:

$$h[n] \neq 0 \text{ for } -M_1 < n < \infty$$

• Sidenote example:

$$g[n] = \delta[n] - a\delta[n-1]$$

$$Q(z) = 1 - az^{-1} = \frac{z-a}{z}$$

• Inverse System:

$$H(z) = \frac{1}{Q(z)} = \frac{z}{z-a} \Rightarrow h[n] = a^n u[n]$$

$$g[n] * h[n] = \delta[n]$$

- adaptive filtering MMSE criterion

$$\text{Min}_{h[n]} E \left\{ \left[ a[n] - \sum_{k=-M_1}^{N_2} h[k] x[n-k] \right]^2 \right\}$$

- where:  $M_2 < N_2 < \infty$
- where:  $g[n] \neq 0$  for  $-M_1 < n < M_2$
- initially assume training sequence
- practically implement via LMS or RLS - see FIR equalizer.m at course web site