EE648 (CC761-M) DSP II
Session 6 (hive: 1/28/99)

<u>Outline</u>:

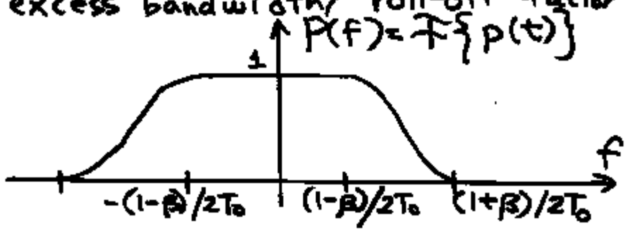
- Adaptive equalization of digital communication signals
- Sect. 12.1 of P&M 1st Ed.
  - pp. 861-864

- Overview of Digital Communications
- assume binary signalling for sake of simplicity
- message signal ⇒ sampled
  quantized
  binary encoded
- every T_0 seconds, a pulse is transmitted – the k-th pulse carries one bit of information

- typical pulse shaped used in narrowband digital communications

$$p(t) = \frac{\sin\left(\frac{\pi}{T_0}t\right)}{\pi\frac{t}{T_0}} \cdot \frac{\cos\left(2\pi\beta\frac{t}{T_0}\right)}{1-\left(4\beta\frac{t}{T_0}\right)^2}$$

β = excess bandwidth/ roll-off factor



- if k-th info. bit is a "1", transmit:

$a[k]\,p(t-kT_0)$ with $a[k]=+A$

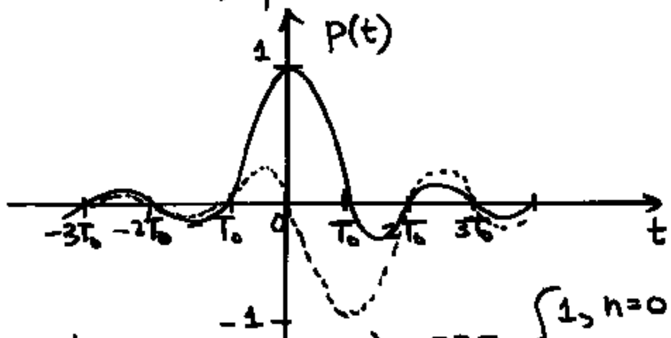
- if k-th info. bit is a "0", transmit:

$a[k]\,p(t-kT_0)$ with $a[k]=-A$

- $k=0,1,\ldots,N-1$ } transmitting a burst of N bits
- transmitted signal (linear modulation)

$$s(t)=\sum_{k=0}^{N-1} a[k]\,p(t-kT_0)$$

- for current US TDMA cellular standard, $\beta = 0.35$



- note: $p[n] = p(nT_0) = \delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$

- Note: bandwidth of $s(t)$ is $\dfrac{1+\beta}{2T_0}$
- where: $\dfrac{1}{T_0}$ is the bit rate
- Ideally, the Nyquist rate is

$$2\left\{\frac{1+\beta}{2T_0}\right\} = \frac{1+\beta}{T_0} \qquad \text{where: } 0 < \beta < 1$$

- Consider sampling at bit rate nonetheless — sub-Nyquist sampling
  - assume synchronization

- thus:

$$\begin{aligned} s[n] &= s(nT_0) \\ &= \sum_{k=0}^{N-1} a[k]\, p(nT_0 - kT_0) \\ &= \sum_{k=0}^{N-1} a[k]\, p[n-k] \\ &= \sum_{k=0}^{N-1} a[k]\, \delta[n-k] \\ &= a[n] \end{aligned}$$

- despite:
  - pulses overlapping in time (to have as high data rate in a given bandwidth)
  - sub-Nyquist sampling
- can nonetheless recover the transmitted info. bits

- However, when multipath exists (reflections off buildings, etc.), the received signal is

$$x(t) = \sum_{\ell=1}^{P} g_\ell \, s(t-\tau_\ell)$$

- where:
  - g_ℓ: gain of ℓ-th multipath
  - τ_ℓ: delay of " "
- $x(t)$ may be alternatively expressed as

$$x(t) = s(t) * \left\{ \sum_{\ell=1}^{P} g_\ell \, \delta(t-\tau_\ell) \right\}$$

$$x(t) = \sum_{k=0}^{N-1} a[k] \, p(t-kT_0) * \left\{ \sum_{\ell=1}^{P} g_\ell \, \delta(t-\tau_\ell) \right\}$$

$$= \sum_{k=0}^{N-1} a[k] \, q(t-kT_0)$$

$$q(t) = \sum_{\ell=1}^{P} g_\ell \, p(t-\tau_\ell)$$

$$\sum_{\ell} p(t-kT_0) * \delta(t-\tau_\ell) \, g_\ell$$

$$= \sum_{\ell} p(t) * \delta(t-kT_0) * \delta(t-\tau_\ell) \, g_\ell$$

$$= \sum_{\ell} p(t-\tau_\ell) * \delta(t-kT_0) \, g_\ell$$

$$= \left\{ \sum_{\ell} g_\ell \, p(t-\tau_\ell) \right\} * \delta(t-kT_0)$$

$$= q(t) * \delta(t-kT_0)$$

$$= q(t-kT_0)$$

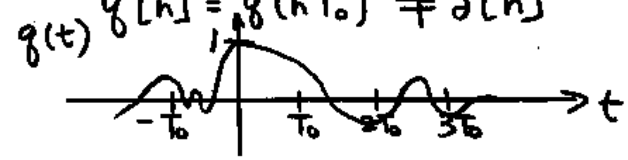
- received signal:

$$x(t) = \sum_{k=0}^{N-1} a[k] \, q(t-kT_0)$$

- where: $q(t)$ is distorted pulse waveform due to multipath

$\Rightarrow$ Nyquist property is lost

$$q[n] = q(nT_0) \neq \delta[n]$$



• sampling at bit rate:

$$
\begin{aligned}
x[n] &= x(nT_0) \\
&= \sum_{k=0}^{N-1} a[k]\, q(nT_0 - kT_0) \\
&= \sum_{k=0}^{N-1} a[k]\, q[n-k] \\
&= a[n] * q[n] \\
&= \sum_{k=-M_1}^{M_2} q[k]\, a[n-k]
\end{aligned}
$$

where: $q[n] = q(nT_0)$

• $a[n] \rightarrow \boxed{q[n]} \rightarrow x[n]$

• Equalization is about how to determine $a[n]$ given $x[n]$

• a Zero-Forcing (ZF) Equalizer does this by means of inverse Filtering:

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] = \hat{a}[n]$

• where: $q[n] * h[n] = \delta[n]$

• where ideally:

$h[n] \neq 0$ for $-M_1 < n < \infty$

• Sidenote example:

$$q[n] = \delta[n] - a\,\delta[n-1]$$

$$Q(z) = 1 - a z^{-1} = \frac{z-a}{z}$$

• Inverse System:

$$H(z) = \frac{1}{Q(z)} = \frac{z}{z-a} \Rightarrow h[n] = a^n u[n]$$

$$q[n] * h[n] = \delta[n]$$

• adaptive filtering MMSE criterion

$$\min_{h[n]} E\left\{\left[a[n] - \sum_{k=-M_1}^{N_2} h[k]\, x[n-k]\right]^2\right\}$$

• where: $M_2 < N_2 < \infty$

• where: $q[n] \neq 0$ for $-M_1 < n < M_2$

• initially assume training sequence

• practically implement via LMS or RLS - see FIRequalizer.m at course web site