EE648 (CC761-M) DSP II
Session 7 (Live: 2/2/99)

Outline:

- Add'l Treatment of Adaptive Equalization for Digital Communications
- See Hmwk2help.m at course web site

- Recall, model:

$$\underbrace{a[n]}_{\text{info. symbols}} \longrightarrow \boxed{q[n]} \longrightarrow x[n]$$

$$q[n] = \sum_{k=1}^{P} g_k \, p_{rc}(t-\tau_k)\Big|_{t=nT_0}$$

$$\neq 0 \quad \text{for} \quad -M_1 < n < M_2$$

- where $\frac{1}{T_0}$ = symbol rate
- Zero Forcing (ZF) Equalizer :

- effects inverse filtering

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n] = \hat{a}[n]$$

- where, ideally:

$$q[n] * h[n] = \delta[n]$$

- adaptive filter MMSE criterion:

$$\min_{h[n]} E\left\{\left[a[n] - \sum_{k=-M_1}^{N_2} h[k]\, x[n-k]\right]^2\right\}$$

- where $M_2 < N_2 < \infty$

- implement via LMS or RLS
- initially transmit training data
- then send data
- to track moderate time-variations in the channel, use decision-directed mode to update the equalizer as the channel evolves with time

- Frequency domain analysis of multipath effects:

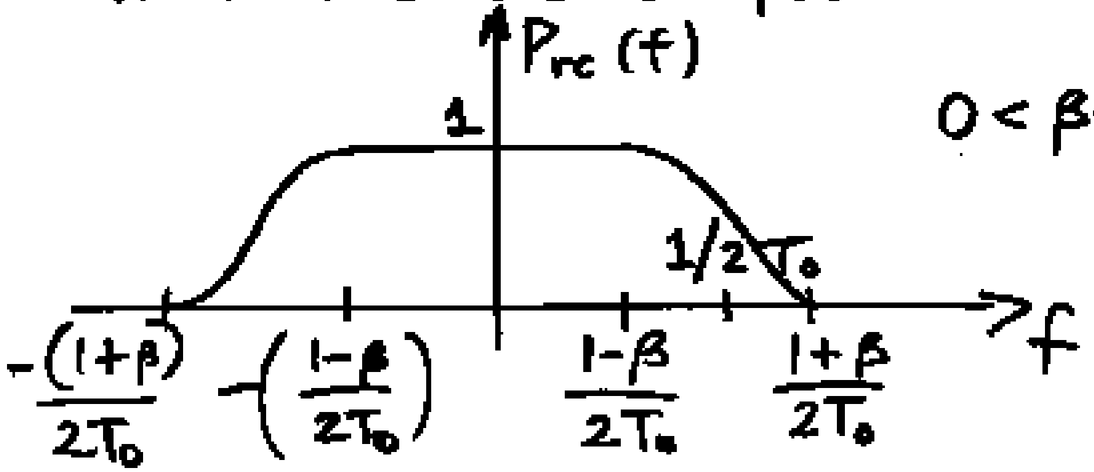
$$q(t) = \sum_{l=1}^{P} g_l \, p_{rc}(t-\tau_l)$$

$$= p_{rc}(t) * h_{RF}(t)$$

- where $h_{RF}(t)$ is the impulse response Radio-Frequency channel

$$h_{RF}(t) = \sum_{l=1}^{P} g_l \, \delta(t-\tau_l)$$

- $Q(f) = P_{rc}(f)\, H_{RF}(f)$
- where: $H_{RF}(f) = \sum_{\ell=1}^{P} g_\ell\, e^{-j2\pi f \tau_\ell}$
- recall: raised-cosine spectrum

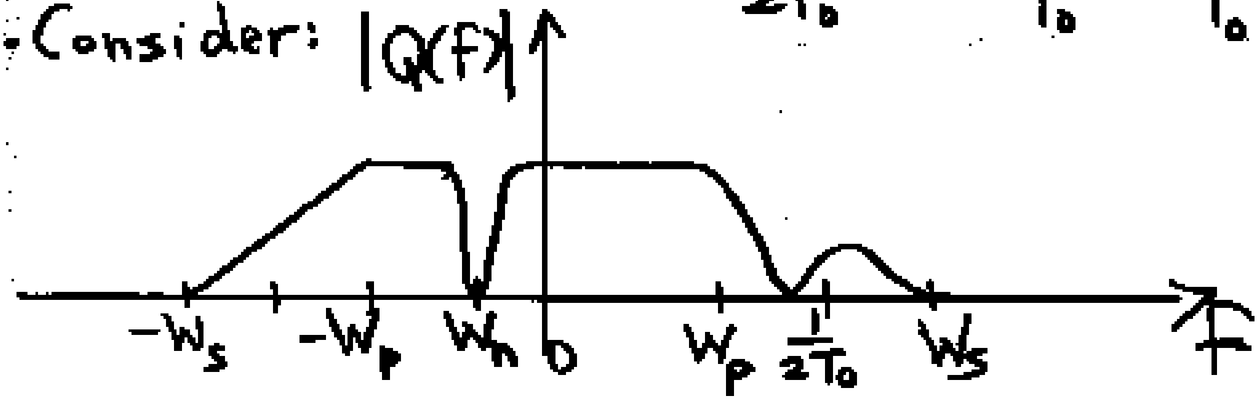


• define: $W_p = \frac{1-\beta}{2T_0}$

$W_s = \frac{1+\beta}{2T_0}$

• Nyquist rate for $q(t)$ is same as that for $p_{rc}(t)$: $2 \frac{1+\beta}{2T_0} = \frac{1+\beta}{T_0} > \frac{1}{T_0}$

• Consider:



• With symbol-rate sampling, $f_s = \frac{1}{T_0}$:

$$g[n] = g(nT_0) \xleftrightarrow{\text{DTFT}} Q(\omega) = \sum_{k=-\infty}^{\infty} Q\left(\frac{1}{2\pi T_0}(\omega - k2\pi)\right)$$

• for $-\pi < \omega < -\omega_p$: $\quad \omega_p = \dfrac{2\pi}{\frac{1}{T_0}} \dfrac{1-\beta}{2T_0}$

• aliasing occurs: $\quad = (1-\beta)\pi$

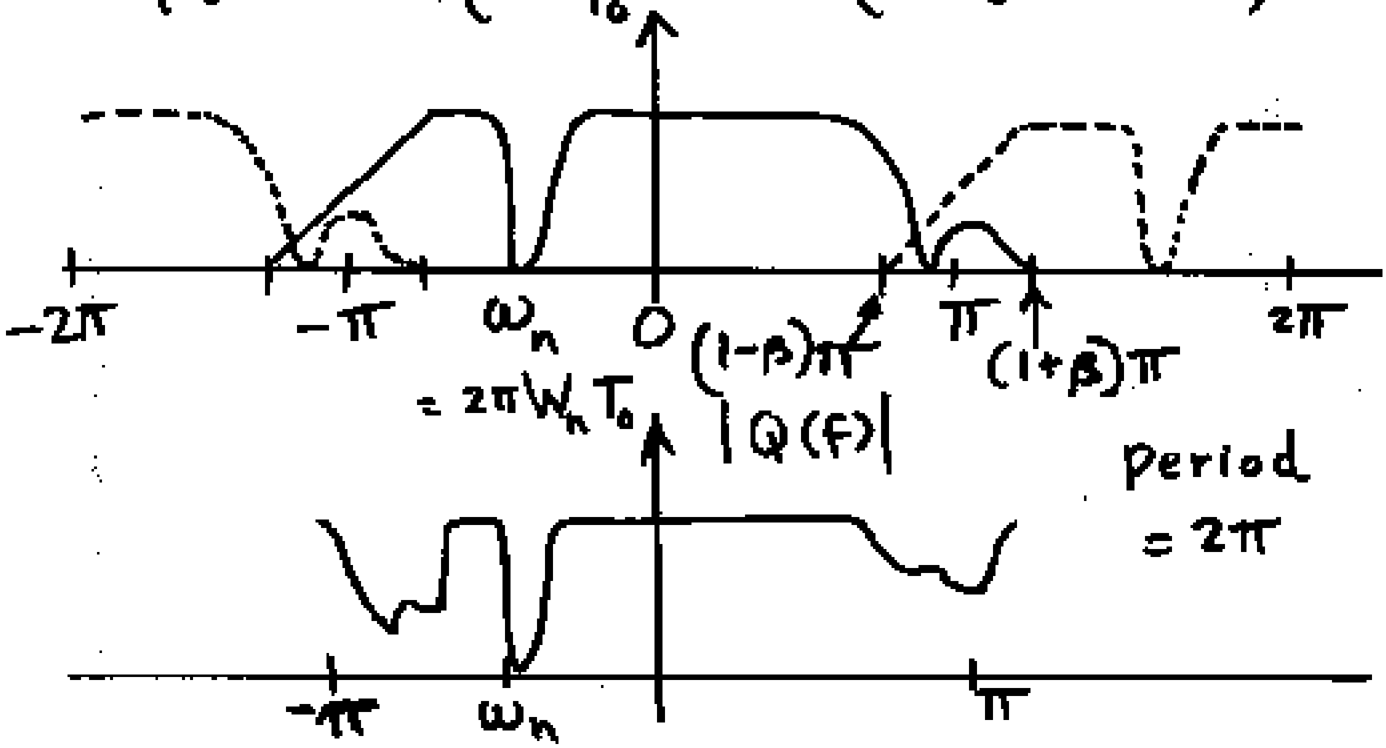
$$Q(\omega) = Q\left(\frac{\omega}{2\pi T_0}\right) + Q\left(\frac{1}{2\pi T_0}(\omega + 2\pi)\right) \qquad < \pi$$

• for $|\omega| < \omega_p$: no aliasing

$$Q(\omega) = Q\left(\frac{\omega}{2\pi T_0}\right) = H_{RF}\left(\frac{\omega}{2\pi T_0}\right)$$

• for $\omega_p < \omega < \pi$:   • aliasing

$$Q(\omega) = Q\left(\frac{\omega}{2\pi T_0}\right) + Q\left(\frac{1}{2\pi T_0}(\omega - 2\pi)\right)$$

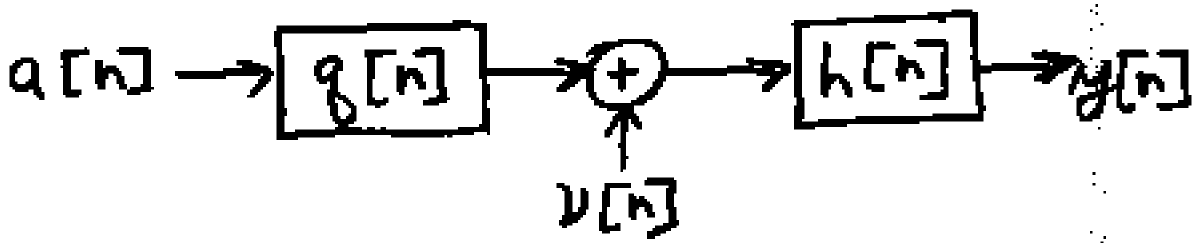


- Any null in the analog frequency range $|f| < W_p = \frac{1-\beta}{2T_0}$ caused by multipath effects (represented via multiplication by $H_{RF}(f)$) causes a corresponding null in $Q(\omega)$ in the range $|\omega| < (1-\beta)\pi$

- This causes two major problems
  1. Noise Enhancement
  2. Requires Increased Equalizer Lengths

1. Noise Enhancement

- Note: noise is additive at receiver due to thermal noise in r'cvr electronics



- noise, $\nu[n]$, typically has broad spectrum
  - popular model: AWGN
    - Additive White Gaussian Noise

- Since $|Q(\omega)| \underbrace{|H(\omega)|}_{\text{equalizer}} = 1$

- then $|H(\omega)| = \dfrac{1}{|Q(\omega)|}$

- a null at $\omega = \omega_n$ in $Q(\omega)$, causes $H(\omega)$ to approach ∞ at $\omega = \omega_n$

- in turn, causes noise energy in the region of $\omega = \omega_n$ to be amplified

2. Increased Equalizer Length

A null in $Q(\omega)$ means $Q(z)$ has a zero near the unit circle

- $H(z) = \dfrac{1}{Q(z)} \Rightarrow$ implies $H(z)$ has a pole close to unit circle
- thus requires a long equalizer length to do effect inverse filtering

- Very simplistic example

$$q[n] = \delta[n] - a\,\delta[n-1]$$

$$Q(z) = 1 - a z^{-1} = \frac{z-a}{z}$$

$$H(z) = \frac{1}{Q(z)} = \frac{z}{z-a}$$

$$h[n] = a^n u[n]$$

- as $|a| \to 1$, takes longer $|h[n]|$ to decay to zero $\Rightarrow$ necessitates longer equalizer

- See Hmwk2 assignment and Hmwk2help.m at web site

$$q(t) = p_{rc}(t) + g_2\, p_{rc}\left(t - \frac{T_0}{2}\right)$$

$$= p_{rc}(t) * \underbrace{\left\{ \delta(t) + g_2\, \delta\left(t - \frac{T_0}{2}\right) \right\}}_{\text{simple two-ray multipath model}}$$