

EE648 (CCT61-M) DSP II
 Session 7 (Live: 2/2/99)

Outline:

- Add'l Treatment of Adaptive Equalization for Digital Communications
- See Hmwk2 help.m at course web site

- effects inverse filtering

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] = \hat{a}[n]$$

- where, ideally:

$$g[n] * h[n] = \delta[n]$$

- adaptive filter MMSE criterion:

$$\min_{h[n]} E \left\{ \left[a[n] - \sum_{k=-M_1}^{N_2} h[k] x[n-k] \right]^2 \right\}$$

- where $M_2 < N_2 < \infty$

- Recall, model:

$$a[n] \rightarrow \boxed{g[n]} \rightarrow x[n]$$

info. symbols

$$g[n] = \sum_{k=1}^P g_k \text{Pr}(t - \tau_k) \Big|_{t=nT_0}$$

$$\neq 0 \text{ for } -M_2 < n < M_2$$

- where $\frac{1}{T_0} = \text{symbol rate}$

- Zero Forcing (ZF) Equalizer:

- implement via LMS or RLS

- initially transmit training data

- then send data

- to track ^{moderate} time-variations in the channel, use decision-directed mode to update the equalizer as the channel evolves with time

- Frequency domain analysis of multipath effects:

$$g(t) = \sum_{l=1}^P g_l p_{rc}(t - \tau_l)$$

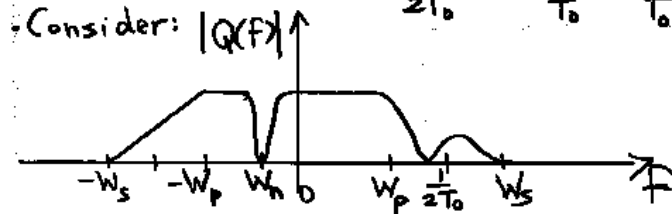
$$= p_{rc}(t) * h_{RF}(t)$$

- where $h_{RF}(t)$ is the impulse response Radio-Frequency channel

$$h_{RF}(t) = \sum_{l=1}^P g_l \delta(t - \tau_l)$$

- define: $W_p = \frac{1-\beta}{2T_0}$
- $W_s = \frac{1+\beta}{2T_0}$

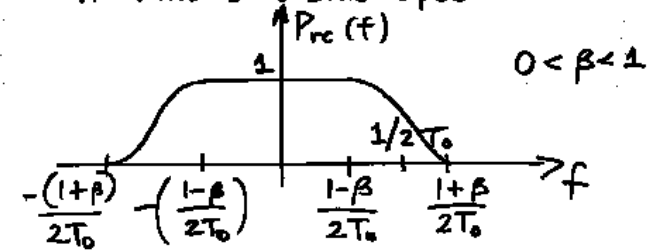
- Nyquist rate for $g(t)$ is same as that for $p_{rc}(t)$: $2 \frac{1+\beta}{2T_0} = \frac{1+\beta}{T_0} > \frac{1}{T_0}$



- $Q(f) = P_{rc}(f) H_{RF}(f)$

- where: $H_{RF}(f) = \sum_{l=1}^P g_l e^{-j2\pi f \tau_l}$

- recall: raised-cosine spectrum



- With symbol-rate sampling, $f_s = \frac{1}{T_0}$:

$$g[n] = g(nT_0) \xleftrightarrow{\text{DTFT}} Q(\omega) = \sum_{k=-\infty}^{\infty} Q\left(\frac{\omega}{2\pi T_0} (\omega - k2\pi)\right)$$

- for $-\pi < \omega < -\omega_p$: $\omega_p = \frac{2\pi}{T_0} \frac{1-\beta}{2}$
- aliasing occurs:

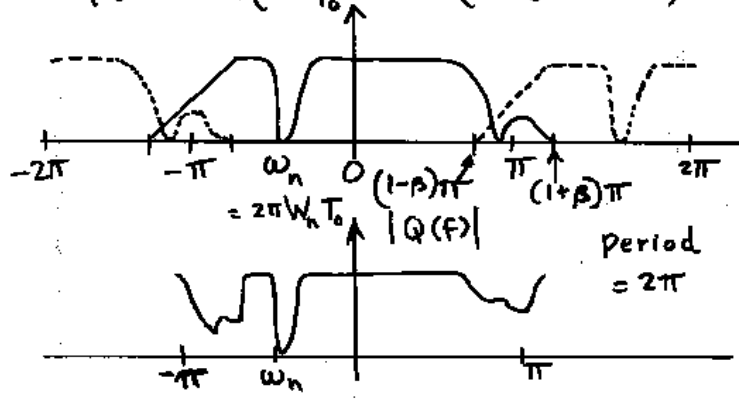
$$Q(\omega) = Q\left(\frac{\omega}{2\pi T_0}\right) + Q\left(\frac{\omega}{2\pi T_0} (\omega + 2\pi)\right) = (1-\beta)\pi < \pi$$

- for $|\omega| < \omega_p$: no aliasing

$$Q(\omega) = Q\left(\frac{\omega}{2\pi T_0}\right) = H_{RF}\left(\frac{\omega}{2\pi T_0}\right)$$

• for $\omega_p < \omega < \pi$: • aliasing

$$Q(\omega) = Q\left(\frac{\omega}{2\pi T_0}\right) + Q\left(\frac{1}{2\pi T_0}(\omega - 2\pi)\right)$$

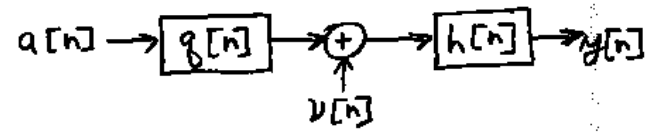


• Any null in the analog frequency range $|f| < W_p = \frac{1-\beta}{2T_0}$ caused by multipath effects (represented via multiplication by $H_{PE}(f)$) causes a corresponding null in $Q(\omega)$ in the range $|\omega| < (1-\beta)\pi$

- This causes two major problems
 1. Noise Enhancement
 2. Requires Increased Equalizer Length

1. Noise Enhancement

• Note: noise is additive at receiver due to thermal noise in receiver electronics



- noise, $v[n]$, typically has broad spectrum
- popular model: AWGN
 - Additive White Gaussian Noise

• Since $|Q(\omega)| \underbrace{|H(\omega)|}_{\text{equalizer}} = 1$

• then $|H(\omega)| = \frac{1}{|Q(\omega)|}$

- a null at $\omega = \omega_n$ in $Q(\omega)$, causes $H(\omega)$ to approach ∞ at $\omega = \omega_n$
- in turn, causes noise energy in the region of $\omega = \omega_n$ to be amplified

2. Increased Equalizer Length

A null in $Q(\omega)$ means $Q(z)$ has a zero near the unit circle

- $H(z) = \frac{1}{Q(z)} \Rightarrow$ implies $H(z)$ has a pole close to unit circle
- thus requires a long equalizer length to do effect inverse filtering

• Very simplistic example

$$q[n] = \delta[n] - a\delta[n-1]$$

$$Q(z) = 1 - az^{-1} = \frac{z-a}{z}$$

$$H(z) = \frac{1}{Q(z)} = \frac{z}{z-a}$$

$$h[n] = a^n u[n]$$

- as $|a| \rightarrow 1$, takes longer $|h[n]|$ to decay to zero \Rightarrow necessitates longer equalizer

- See Hmwk2 assignment and Hmwk2help.m at web site

$$\begin{aligned} q(t) &= P_{rc}(t) + g_e P_{rc}\left(t - \frac{T_0}{2}\right) \\ &= P_{rc}(t) * \underbrace{\left\{ \delta(t) + g_e \delta\left(t - \frac{T_0}{2}\right) \right\}}_{\text{simple two-ray multipath model}} \end{aligned}$$