EE648 (CC761-M) DSP II

Session 8 (Live: 2/4/99)

<u>Outline</u>

- Multichannel adaptive equalization for digital communications
- prelude to:
  - space-time processing
  - Perfect Reconstruction Filter Banks

- recall, received digi comm signal in multipath propagation:

$$x(t) = \sum_{k=0}^{N-1} a[k]\, q(t - kT_0)$$

- where: $q(t) = \sum_{\ell=1}^{P} g_\ell\, p_{rc}(t - \tau_\ell)$

- for $q[n] = q(nT_0)$, showed last time

$$Q(\omega) = H_{RF}\left(\frac{\omega}{2\pi T_0}\right) \quad \text{for } |\omega| < \omega_p$$

$$\omega_p = 2\pi \frac{1-\beta}{2T_0} \Big/ \frac{1}{T_0}$$

$$= (1-\beta)\pi$$

$$H_{RF}(\omega) = \sum_{l=1}^{P} g_l e^{-j 2\pi \left(\frac{\omega}{2\pi T_0}\right)\tau_l}$$

$$= \sum_{l=1}^{P} g_l e^{-j \omega \frac{\tau_l}{T_0}}$$

- Recall a null in $Q(\omega)$ in $|\omega| < (1-\beta)\pi$ causes
  - noise amplification
  - requires longer equalizer
- How does a null arise?

- e.g., consider $P=2$ rays, with second ray arriving at $\tau_2 = \frac{T_0}{2}$
- assume WLOG that for direct path: $\tau_1 = 0$ and $g_1 = 1$
- only the <u>relative</u> gain and time-delay of 2nd relative to direct-path ray that matters

$$Q(\omega) = 1 + g_2 e^{-j\frac{\omega}{2}}$$

• Suppose $|g_2| \approx 1 \Rightarrow |g_2| = 1$

$$Q(\omega) = 1 + e^{j\left(\angle g_2 - \frac{\omega}{2}\right)}$$

for $|\omega| < (1-\beta)\pi$

$$|Q(\omega)| = 2\left|\cos\left(-\frac{\omega}{4} + \frac{\angle g_2}{2}\right)\right| \quad " \quad "$$

$= 0$ when $-\frac{\omega}{4} + \frac{\angle g_2}{2} = \frac{\pi}{2}$

$$\omega_n = 2\left(-\pi + \angle g_2\right) \qquad |\omega| < (1-\beta)\pi$$

$= 2\left(\angle g_2 - \pi\right)$

- e.g., $\angle g_2 = \pi \Rightarrow \omega_n = 0$

  $\angle g_2 = \frac{7}{8}\pi \Rightarrow \omega_n = -\pi/4$

  $\angle g_2 = \frac{3}{4}\pi \Rightarrow \omega_n = -\pi/2$

- See Hmwk2help.m at web site

- Preamble: multichannel signal processing (using conventional notation)
- Single-Input-Single Output (SISO) System and Inverse Filtering

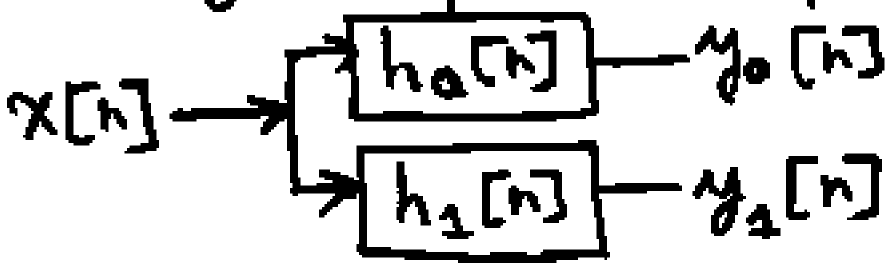
$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

- inverse filtering via an LTI filter

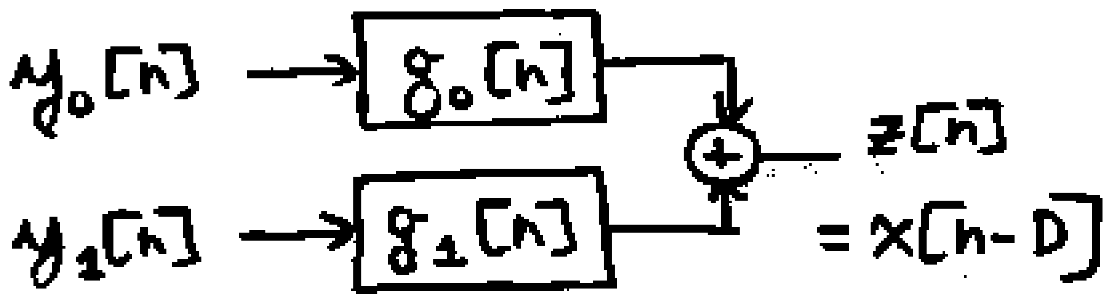
$$y[n] \longrightarrow \boxed{g[n]} \longrightarrow z[n] = x[n-D]$$

- require: $h[n] * g[n] = \delta[n-D]$

- $H(z)\,G(z) = z^{-D} \Rightarrow G(z) = \dfrac{z^{-D}}{H(z)}$
- if $h[n]$ is FIR, $g[n]$ must be IIR
- if $h[n]$ has zeroes outside unit circle, $g[n]$ is unstable
- alternatively, consider SIMO system
  Single Input - Multiple Output

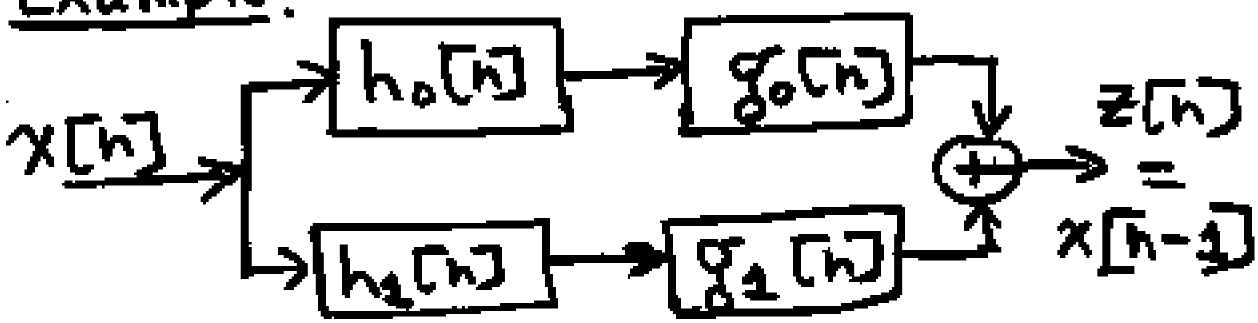


- where $h_0[n]$ and $h_1[n]$ are FIR filters of the same "length"
- inverse filtering may be effected via a MISO system



• can show as long as $H_0(z)$ and $H_1(z)$ do <u>not</u> share a common zero, $z[n] = x[n-D]$ may be achieved with FIR filters $g_0[n]$ and $g_1[n]$ of the same "length" as $h_0[n]$ and $h_1[n]$

• <u>Example</u>.



- Given: $h_0[n] = \delta[n] + \delta[n-1] = \{1, 1\}$

  $h_1[n] = \delta[n] - \delta[n-1] = \{1, -1\}$

- Find: $g_0[n] = \{g_0[0], g_0[1]\}$

  $g_1[n] = \{g_1[0], g_1[1]\}$

- satisfying:

  $$h_0[n] * g_0[n] + h_1[n] * g_1[n] = \delta[n-1]$$

• Consider matrix representation of convolution, $y[n] = \underbrace{x[n]}_{L} * \underbrace{h[n]}_{M}$

$$
\begin{bmatrix}
h[0] & 0 & 0 & 0 & \cdots & & 0 \\
h[1] & h[0] & 0 & 0 & \cdots & & 0 \\
h[2] & h[1] & h[0] & 0 & \cdots & & 0 \\
\vdots & \vdots & \vdots & & & & \\
h[M-1] & h[M-2] & \cdots & h[0] & 0 & \cdots & 0 \\
0 & h[M-1] & \cdots & h[1] & h[0] & \cdots & 0 \\
\vdots & \vdots & & \vdots & & & \\
0 & 0 & 0 & \cdots & 0 & 0 & h[M-1]
\end{bmatrix}
\begin{bmatrix}
x[0] \\ x[1] \\ \vdots \\ x[L-1]
\end{bmatrix}
=
\begin{bmatrix}
y[0] \\ y[1] \\ \vdots \\ y[M+L-2]
\end{bmatrix}
$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{array}\right] \left[\begin{array}{c} g_0[0] \\ g_0[1] \\ \hline g_1[0] \\ g_1[1] \end{array}\right] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

- Solve using Matlab $x = A\backslash b$
- Or set $g_0[0] = 1$

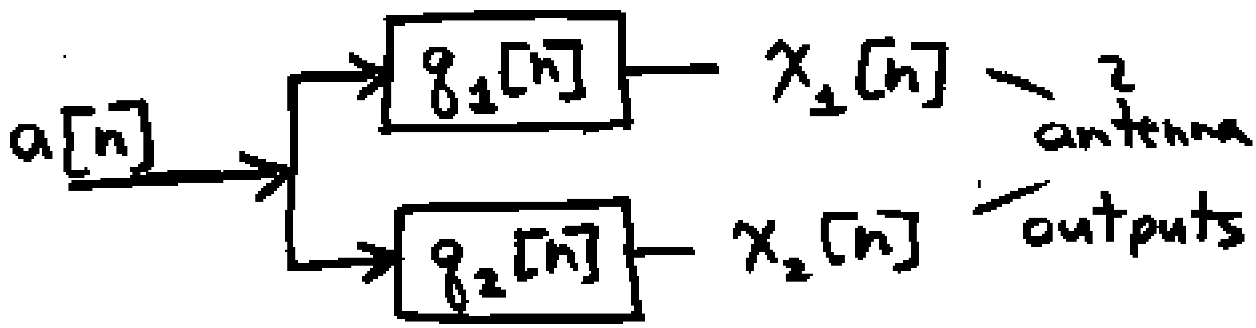
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} g_0[1] \\ g_1[0] \\ g_1[1] \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$g_0[n] = \{\underset{\uparrow}{1}, -.5\} \qquad g_1[n] = \{-1, -.5\}$$

- check via Matlab

- Application of SIMO Concepts to Adaptive Channel Equalization

- one way to realize 2 channels in digi comm is to have spatially separated antennas at the rec'vr



$$r_i[n] = \sum_{l=1}^{P} g_{il}\, p_{rc}(t - \tau_{il}) \Big|_{t = nT_0}$$

$i = 1, 2$

- if the antennas are "close"

$$|g_{1l}| \approx |g_{2l}| \qquad l = 1, \ldots, P$$

$$\tau_{1l} \approx \tau_{2l} \qquad \text{"} \qquad \text{"}$$

$$\angle g_{1l} \neq \angle g_{2l}$$

See TwoChannel.m at web site