

# EE648 (cc761-M) DSP II

Session 8 (live: 2/4/99)

## Outline

- Multichannel adaptive equalization for digital communications
- prelude to:
  - space-time processing
  - Perfect Reconstruction Filter Banks

$$H_{RF}(\omega) = \sum_{\ell=1}^P g_{\ell} e^{-j 2\pi \left(\frac{\omega}{2\pi T_0}\right) \tau_{\ell}}$$

$$= \sum_{\ell=1}^P g_{\ell} e^{-j \omega \frac{\tau_{\ell}}{T_0}}$$

- Recall a null in  $Q(\omega)$  in  $|\omega| < (1-\beta)\pi$  causes noise amplification
  - requires longer equalizer
- How does a null arise?

- recall, received digi comm signal in multipath propagation:

$$x(t) = \sum_{k=0}^{N-1} a[k] q(t - kT_0)$$

• where:  $q(t) = \sum_{\ell=1}^P g_{\ell} p_{rc}(t - \tau_{\ell})$

- for  $g[n] = q(nT_0)$ , showed last time

$$Q(\omega) = H_{RF}\left(\frac{\omega}{2\pi T_0}\right) \quad \text{for } |\omega| < \omega_p$$

$$\omega_p = 2\pi \frac{1-\beta}{2T_0} / \frac{1}{T_0}$$

$$= (1-\beta)\pi$$

- e.g., consider  $P=2$  rays, with second ray arriving at  $\tau_2 = \frac{T_0}{2}$
  - assume WLOG that for direct path:  $\tau_1 = 0$  and  $g_1 = 1$
  - only the relative gain and time-delay of 2nd relative to direct-path ray that matters
- $$Q(\omega) = 1 + g_2 e^{-j \frac{\omega}{2}}$$

• Suppose  $|g_2| \approx 1 \Rightarrow |g_2| = 1$

$$Q(\omega) = 1 + e^{j(\angle g_2 - \frac{\omega}{2})}$$

$$|Q(\omega)| = 2 \left| \cos\left(-\frac{\omega}{4} + \frac{\angle g_2}{2}\right) \right| \quad \text{for } |\omega| < (1-\beta)\pi$$

$$= 0 \quad \text{when: } -\frac{\omega}{4} + \frac{\angle g_2}{2} = \frac{\pi}{2}$$

$$\omega_n = 2(-\pi + \angle g_2) \quad |\omega| < (1-\beta)\pi$$

$$= 2(\angle g_2 - \pi)$$

• e.g.,  $\angle g_2 = \pi \Rightarrow \omega_n = 0$

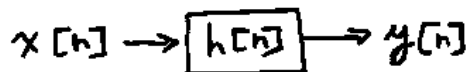
$\angle g_2 = \frac{7}{8}\pi \Rightarrow \omega_n = -\pi/4$

$\angle g_2 = \frac{3}{4}\pi \Rightarrow \omega_n = -\pi/2$

• See Hmwk2help.m at web site

• Preamble: multichannel signal processing (using conventional notation)

• Single-Input - Single Output (SISO) System and Inverse Filtering



• inverse filtering via an LTI filter



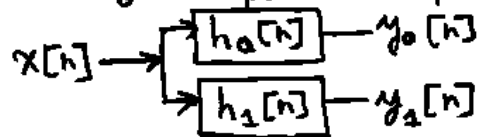
• require:  $h[n] * g[n] = \delta[n-D]$

$$H(z)G(z) = z^{-D} \Rightarrow G(z) = \frac{z^{-D}}{H(z)}$$

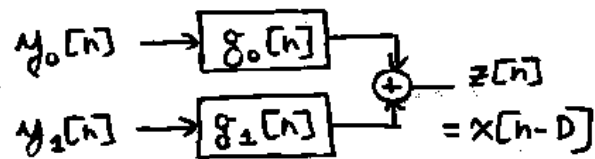
• if  $h[n]$  is FIR,  $g[n]$  must be IIR

• if  $h[n]$  has zeroes outside unit circle,  $g[n]$  is unstable

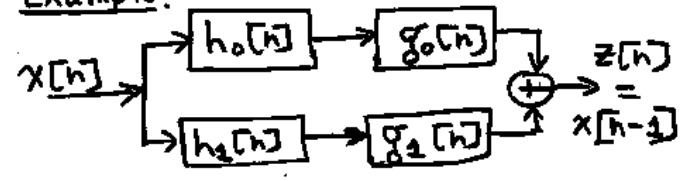
• alternatively, consider SIMO System  
Single Input - Multiple Output



- where  $h_0[n]$  and  $h_1[n]$  are FIR filters of the same "length"
- inverse filtering may be effected via a MISO system



- can show as long as  $H_0(z)$  and  $H_1(z)$  do not share a common zero,  $z[n] = x[n-D]$  may be achieved with FIR filters  $g_0[n]$  and  $g_1[n]$  of the same "length" as  $h_0[n]$  and  $h_1[n]$
- Example.



• Given:  $h_0[n] = \delta[n] + \delta[n-1] = \{1, 1\}$   
 $h_1[n] = \delta[n] - \delta[n-1] = \{1, -1\}$

• Find:  $g_0[n] = \{g_0[0], g_0[1]\}$   
 $g_1[n] = \{g_1[0], g_1[1]\}$

• satisfying:  
 $h_0[n] * g_0[n] + h_1[n] * g_1[n] = \delta[n-1]$

- Consider matrix representation of convolution,  $y[n] = x[n] * h[n]$

$$\begin{bmatrix}
 h[0] & 0 & 0 & \dots & 0 & 0 & \dots \\
 h[1] & h[0] & 0 & 0 & \dots & 0 & 0 & \dots \\
 h[2] & h[1] & h[0] & 0 & \dots & 0 & 0 & \dots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 h[m-1] & h[m-2] & \dots & h[0] & 0 & \dots & 0 & \dots \\
 0 & h[m-1] & \dots & h[2] & h[1] & \dots & 0 & \dots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 0 & 0 & 0 & \dots & 0 & 0 & h[m-1] & \dots
 \end{bmatrix}
 \begin{bmatrix}
 x[0] \\
 x[1] \\
 \vdots \\
 x[l-1]
 \end{bmatrix}
 =
 \begin{bmatrix}
 y[0] \\
 y[1] \\
 \vdots \\
 y[m+l-2]
 \end{bmatrix}$$

$$\left[ \begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{array} \right] \begin{bmatrix} g_0[0] \\ g_0[1] \\ g_1[0] \\ g_1[1] \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

• Solve using Matlab  $x = A \setminus b$

• Or set  $g_0[0] = 1$

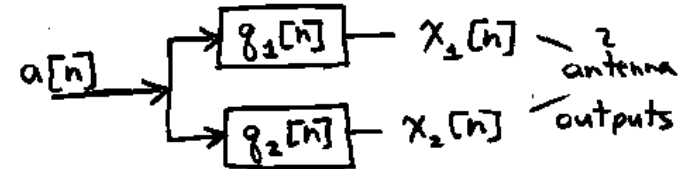
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} g_0[1] \\ g_1[0] \\ g_1[1] \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$g_0[n] = \{1, -0.5\} \quad g_1[n] = \{-1, -0.5\}$$

• check via Matlab

• Application of SISO Concepts to Adaptive Channel Equalization

• one way to realize 2 channels in digi comm is to have spatially separated antennas at the recvr



$$g_i[n] = \sum_{l=1}^P g_{il} p_{rc}(t - \tau_{il}) \Big|_{t=nT_0}$$

$i = 1, 2$

• if the antennas are "close"

$$|g_{12}| \approx |g_{21}| \quad l = 1, \dots, P$$

$$\tau_{12} \approx \tau_{21} \quad \text{" "}$$

$$\angle g_{12} \neq \angle g_{21} \quad \text{See Two Channel.m at web site}$$